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Abstract

The present paper considers Dickey-Fuller-type unit root tests which account for a structural break occurring at an unknown point in time. The break is modelled by an innovational outlier approach. Provided that the break date is estimated correctly, the exact invariance to a mean and a slope shift holds for these tests under the null hypothesis. An erroneous estimation of the break date leads to considerable spurious rejections of the null hypothesis in small samples. In this paper, test procedures are developed using a components representation of the data generating process. In contrast to the conventionally used approaches, these tests enable the identification of the true break date and ensure the invariance property of the corresponding test statistics. Monte Carlo simulations of size and power testify the favorable properties of the developed tests.

KEYWORDS: Unit root tests, structural break, endogenous break date estimation, innovational outlier models, spurious rejections, component representation.

1 Introduction

Perron (1989) analyzes the properties of classical unit root tests in the presence of a structural change. There he is able to show that ignoring a structural break in the trend function leads to a remarkable reduction of power of unit root tests. For this reason Perron develops a Dickey-Fuller-(DF-)type unit root test which explicitly accounts for a break with known break date. He distinguishes two testing approaches, that differ in their assumptions about the adjustment process towards the new equilibrium after a shock. The additive outlier (AO) model assumes an instantaneous adjustment, whereas in the innovational outlier (IO) model the adjustment takes place gradually.

However, the assumption of a known break date is criticized because of its tendency to favor the alternative hypothesis of stationarity. Consequently many authors treat the break date as unknown, identifying the timing of the break endogenously, as, e.g., Christiano (1992), Zivot and Andrews (1992) or Banerjee, Lumsdaine and Stock (1992). Vogelsang and Perron (1998) develop the asymptotic and finite distributions of the test statistics based on the AO- and IO-model, taking various methods for determining the break date into account. They show that the IO-model based statistics are exactly invariant to a break in the constant, the slope or both, provided the break date is estimated correctly. In the case of an incorrect identification of the break date, the invariance property no longer holds in finite samples, and for a slope shift not even asymptotically. The conventionally used methods for choosing the break date have the disadvantage that they do not even asymptotically identify the correct timing of the break in the IO-case.

Because the test statistics are asymptotically invariant to an intercept shift no matter whether the true break date is identified or not, this seems to be the less problematic case. Though, Nunes, Newbold and Kuan (1997), Harvey, Leybourne and Newbold (2001) as well as Lee and Strazicich (2001) provide evidence of a considerable number of spurious rejections of the null hypothesis in usually encountered sample sizes, for which the erroneous estimation seems to be responsible. Modified methods for break date estimation proposed by Harvey et al. (2001) and Lee et al. (2001) are not completely satisfying.

In this paper an unobserved component representation of the data generating process (DGP) serves as a starting point to generate modified Perron-type test statistics based on the t-value of the AR coefficient. Using these test statistics it is possible to identify the true break date even in small samples. Moreover, the modified test statistics show the desirable invariance property to an intercept and slope shift. The distributions are identical to those when the timing of the break is given exogenously.

The present paper is structured as follows. In the next section the conventionally used test equations based on the IO-appoach are presented. Thereafter, various methods found in the literature for selecting the true break date and the corresponding problem of spurious rejections of the null hypothesis is discussed. The modified test statistics based on the representation of the DGP in form of a component model will be generated in section 3. In sections 4 and 5 the finite size and power will be analyzed using Monte Carlo techniques testifying the favorable properties of the new tests. The results are summarized in section 6.

2 Innovational outlier models and spurious rejections

2.1 Innovational outlier models

The presentation of the IO-models is based on the papers of Vogelsang and Perron (1998) and Perron and Vogelsang (1992). These papers distinguish between the following model specification for the IO-model: a model allowing for a change in the intercept for non-trending data (model (0)), for an intercept shift for trending data (model (1)), for a break in intercept and slope (model (2)) as well as solely for a slope shift (model (3)).

The IO-model assumes a gradual adjustment process after a break in the trend function takes place. In consideration of the immense possibilities of a smooth adjustment, Perron restricts the adjustment process to be identical for shocks to the trend function as those to the innovation process.

Under the null hypothesis of a unit root, the time series y_t is generated by the following models:

$$\text{model (0)} \quad : \quad y_t = y_{t-1} + \Psi^*(L)(\theta D(T_B)_t + e_t), \tag{1}$$
$$\text{model (1)} \quad : \quad y_t = y_{t-1} + \beta + \Psi^*(L)(\theta D(T_B)_t + e_t), \tag{2}$$
$$\text{model (2)} \quad : \quad y_t = y_{t-1} + \beta + \Psi^*(L)(\theta D(T_B)_t + \gamma DU_t + e_t), \tag{3}$$
$$\text{model (3)} \quad : \quad y_t = y_{t-1} + \beta + \Psi^*(L)(\gamma DU_t + e_t), \tag{4}$$

with $e_t \sim iid(0, \sigma_e^2)$. The break dummies are defined as follows: $D(T_B)_t = 1(t = T_B + 1)$ and $DU_t = 1(t > T_B)$. $1(.)$ is the indicator function and T_B symbolizes the break date. The lag polynomial $\Psi(L)$ can be factored as $\Psi(L) = A(L)^{-1}B(L) = (1-\rho L)^{-1}A^*(L)^{-1}B(L) = (1-\rho L)^{-1}\Psi^*(L)$. $A(L)$ and $B(L)$ are lag polynomials of order $p+1$ and q respectively. Since $A(L)$ can be factored as $A(L) = (1-\rho L)A^*(L)$, $A^*(L)$ is a polynomial in L of order p. It is assumed that $A^*(L)$ and $B(L)$ have all roots outside the unit circle. The magnitudes of the possible intercept or slope breaks are measured by the parameters θ and γ. An intercept break occurring at period T_B in model (0) has an immediate impact of θ and a long-run impact of $\Psi^*(1)\theta$ on y_t.

Under the alternative hypothesis H_1 of a stationary time series, the considered models are given by:

$$\text{model (0)} \quad : \quad y_t = \alpha + \Psi(L)(\theta DU_t + e_t), \tag{5}$$
$$\text{model (1)} \quad : \quad y_t = \alpha + \beta t + \Psi(L)(\theta DU_t + e_t), \tag{6}$$
$$\text{model (2)} \quad : \quad y_t = \alpha + \beta t + \Psi(L)(\theta DU_t + \gamma DT_t + e_t), \tag{7}$$
$$\text{model (3)} \quad : \quad y_t = \alpha + \beta t + \Psi(L)(\gamma DT_t + e_t), \tag{8}$$

with $DT_t = 1(t > T_B)(t - T_B)$. The immediate and long-run effect of an intercept shift in model (0) is now θ and $\Psi(1)\theta$, respectively.

Both hypotheses can be nested in the following test regressions:

$$y_t = \alpha + \theta DU_t + \delta D(T_B)_t + \rho y_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t, \quad (9)$$

$$y_t = \alpha + \theta DU_t + \beta t + \delta D(T_B)_t + \rho y_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t, \quad (10)$$

$$y_t = \alpha + \theta DU_t + \beta t + \gamma DT_t + \delta D(T_B)_t + \rho y_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t. (11)$$

For model (3), the equations (4) and (8) will not be nested. Instead a procedure of Zivot and Andrews (1992) and Banerjee et al. (1992) will be adopted, that uses the following regression:[1]

$$y_t = \alpha + \beta t + \gamma DT_t + \rho y_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t. \quad (12)$$

The unit root hypothesis can be tested with reference to the t-value of the test on $H_0 : \rho = 1$ (with the implicit assumption of $\theta = \gamma = 0$). In the remainder of the paper, these t-statistics will be denoted by $t_{\hat{\rho}}$.

2.2 Break date estimation and spurious rejections

The t-statistics for the test of a unit root depend on two generally unknown parameters: the break date T_B and the lag parameter k. Regarding the choice of k, the literature has by now reached consensus on selecting the lag parameter by data-dependent procedures.

In order to determine the break date T_B, usually two methods are considered in the literature. According to the first method, used by Zivot and Andrews (1992), Banerjee et al. (1992) or Perron and Vogelsang (1992) inter alia, T_B is that point in time that minimizes the t-statistic for a test on $\rho = 1$. This choice of T_B corresponds to that break date which most likely rejects the null hypothesis of a unit root:

$$\hat{T}_B^1 = \arg\min t_{\hat{\rho}}.$$

The second method, which originates from Christiano (1992), chooses T_B by considering statistics that test for the significance of the break dummies. For model (0) and (1) T_B is chosen in such a way, that the absolute value of the t-statistic for testing the significance of the break dummy θ is maximized. In the case of model (2) and (3) T_B is chosen using the t-statistic for testing $\gamma = 0$.[2] For model (2) T_B can be additionally selected with reference to the maximum

[1] It has to be noted that this test regression for model (3) is qualitatively different from those for model (0) to (2) because it does not allow for a break under the null hypothesis. Thus, it is a joint test of the null of a unit root and no break.

[2] Perron modifies the second methods insofar as he minimizes or maximizes the value (not the absolute value) of the break dummy t-statistic depending on the direction of the break in the trend function. This implies the a priori restriction of the direction of the break. Using this additional information increases the power of the test.

of the F-statistic that jointly tests for the significance of the intercept and slope dummy. That is formally:

$$\hat{T}_B^2 = \begin{cases} \arg\max |t_{\hat{\theta}}| & \text{, for model (0) and (1)} \\ \arg\max |t_{\hat{\gamma}}| & \text{, for model (2)} \\ \arg\max F_{\hat{\gamma},\hat{\theta}} & \text{, for model (2)} \end{cases}$$

The distribution of the test statistics differs according to the methods for selecting the break date and the considered model specification. Vogelsang and Perron (1998) develop the respective finite sample and asymptotic distributions under the null. There it is shown that the test statistics are exactly invariant to an intercept and slope shift given the correct estimation of the break date.[3] However, if the true break date is not identified, the invariance property of $t_{\hat{\rho}}$ to θ and γ no longer holds for finite samples, the invariance to γ not even asymptotically. Because the proposed methods for choosing the break date do not even asymptotically identify the break date correctly, the invariance of $t_{\hat{\rho}}$ is only asymptotically present with respect to θ. But Harvey, Leybourne and Newbold (2001) and Lee and Strazicich (2001) encounter a considerable number of spurious rejections of the unit root hypothesis in finite samples just for this case.

In their analysis of the IO-models (1) and (2) Lee and Strazicich (2001) are able to confirm that the two presented methods for choosing the break date generally identify $T_B - 1$ as the break date and the frequency increases with the break magnitude. Furthermore, they show that the usage of the incorrect break date leads to a biased estimate of the parameter ρ. This bias $(\rho - \hat{\rho})$ reaches its maximum in $T_B - 1$ leading to a minimal t-statistic $t_{\hat{\rho}}$. This explains the spurious rejections. Even under the alternative hypothesis the break date is estimated incorrectly.

Harvey, Leybourne and Newbold (2001) propose a new procedure for selecting the break date that they apply for the IO-models (0) and (1). Because $T_B - 1$ is generally chosen, they add 1 to the break date estimated with the second method concerning the break dummy variable:

$$\hat{T}_B^3 = 1 + \arg\max |t_{\hat{\theta}}|. \tag{13}$$

This modification leads to superior test properties.

In contrast, Lee and Strazicich (2001) propose a new method for selecting T_B based on the Schwartz Bayesian Criterion (SBC):

$$\hat{T}_B^4 = \arg\min SBC, \tag{14}$$

$$SBC(T_B) = \ln \hat{\sigma}^2(T_B) + (k + c)\ln(T)/T, \tag{15}$$

with $c = 5$ for IO-model (1) and $c = 6$ for IO-model (2). The parameter k stands for the number of lagged differences in the test regressions. Choosing T_B with the SBC, it is possible to estimate the break date accurately. However, the distribution of the tests based on the SBC depends on further parameters as the magnitude and the timing of the break. A completely satisfying procedure apparently does not yet exist. But, the test generated in the next section proposes a solution for the problem of spurious rejections.

[3] This is not true for model (3). Model (3) will not longer be considered explicitly, cf. Perron (1994:119).

3 Representing the DGP as an unobserved component model

In the unit root literature it is common to represent the DGP as an unobserved component model. Advantages of this representation are discussed in, e.g., Schmidt and Phillips (1992). Using this approach, the test regressions for the models (0) to (2) are not regarded as the DGPs, but merely as regression equations to generate test statistics. As in Perron (1989), the unit root hypothesis is tested using the t-statistic for testing $\rho = 1$ in the modified test regressions.

3.1 Unobserved component model and modified test regressions

The DGP of the time series y_t consists of a deterministic and a stochastic component:

$$\begin{aligned} y_t &= d_t + u_t, \\ u_t &= \rho u_{t-1} + \varepsilon_t, \\ \varepsilon_t &= \Psi^*(L) e_t = A^*(L)^{-1} B(L) e_t, \end{aligned}$$

with $e_t \sim iid(0, \sigma_e^2)$. The lag polynomials are defined as in the former section.

The IO-models (0) to (2) differ only in the specification of the deterministic component. In contrast to the IO-models, the AO-approach starts from an unobserved component model, e.g. Perron and Rodriguez (2003). This representation approach is now applied to the IO-case:

$$\begin{aligned} \text{model (0)} &: \quad d_t = \alpha + \Psi^*(L)\theta DU_t, \\ \text{model (1)} &: \quad d_t = \alpha + \beta t + \Psi^*(L)\theta DU_t, \\ \text{model (2)} &: \quad d_t = \alpha + \beta t + \Psi^*(L)(\theta DU_t + \gamma DT_t). \end{aligned}$$

The dummy variables DU_t, DT_t and $D(T_B)_t$ are defined as before. The use of the same lag polynomial $\Psi^*(L)$ as in the error process assures an identical adjustment process after the occurrence of „big shocks" (that affect the trend function) and „regular shocks". This structural form serves as a starting point to develop test regressions for the IO-models. The reduced form for model (0) is as follows:

$$y_t = \rho y_{t-1} + \alpha^* - \phi\theta DU_{t-1} + \theta D(T_B)_t + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t \qquad (16)$$

with $\alpha^* = \Psi^*(1)^{-1}\alpha(1-\rho)$ and $\phi = (\rho - 1)$.

Comparing the new test regression (16) with the usually employed test regression (9), which is reproduced here for ease of comparison:

$$y_t = \rho y_{t-1} + \alpha + \theta DU_t + \delta D(T_B)_t + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t, \qquad (17)$$

two points are worth noting. First, the meaning of the parameter θ has changed. In (16) it is the coefficient of the impulse dummy $D(T_B)_t$ and in (17) the coefficient of the dummy variable DU_t. This is relevant when selecting the break date with the second method concerning the t-statistic of θ. Second, the dummy variable DU_t now appears lagged in (16). However, this difference does not seem to be essential. This can be seen by reshaping equation (16):

$$y_t = \rho y_{t-1} + \alpha^* - \phi\theta DU_t + \rho\theta D(T_B)_t + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t, \qquad (18)$$

in which DU_t no longer appears lagged.[4] The test regressions are identical in such a way as they yield the same t-value $t_{\hat{\rho}}$ when the same break date is given. That is, the distributions of $t_{\hat{\rho}}$ based on (16) and (18) are identical, provided that the same method for estimating the break date is used. However, in order to test the unit root hypothesis test regression (16) can be preferred because the relevant parameter θ stands isolated before the impulse dummy $D(T_B)_t$. In this regard the difference is essential.

The modified test regression for model (1) is:

$$y_t = \rho y_{t-1} + \alpha_1^* + \beta^* t - \phi\theta DU_{t-1} + \theta D(T_B)_t + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t, \qquad (19)$$

with $\beta^* = \Psi^*(1)^{-1}\beta(1-\rho)$ and $\alpha_1^* = \Psi^*(1)^{-1}\alpha(1-\rho) + \beta(1-\rho)\Psi^{*\prime}(1)^{-1}$, $\Psi^{*\prime}(1)^{-1}$ being the mean lag[5].

In contrast, the test regression (10) found in Vogelsang and Perron (1998) is:

$$y_t = \rho y_{t-1} + \alpha + \beta t + \theta DU_t + \delta D(T_B)_t + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t. \qquad (20)$$

Again, the meaning of the relevant parameter θ is changed and the dummy variable DU_t enters in lagged form.

For model (2) qualitatively similar conclusions can be drawn. The new test regression has the following form:

$$\begin{aligned} y_t &= \rho y_{t-1} + \alpha_2^* + \beta_1^* t + \gamma DU_t - \phi\theta DU_{t-1} - \phi\gamma DT_{t-1} + \\ &\quad + \theta D(T_B)_t + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t, \qquad (21) \\ y_t &= \rho y_{t-1} + \alpha_2^* + \beta_1^* t + (\gamma+\theta) D(T_B)_t + (\gamma - \phi\theta) DU_{t-1} - \\ &\quad - \phi\gamma DT_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t, \qquad (22) \end{aligned}$$

with $\beta_1^* = \Psi^*(1)^{-1}\beta(1-\rho)$ and $\alpha_2^* = \Psi^*(1)^{-1}\alpha(1-\rho) + \beta(1-\rho)\Psi^{*\prime}(1)^{-1}$.

Comparing (22) with the conventionally used test regression (11):

$$y_t = \rho y_{t-1} + \alpha + \beta t + \theta DU_t + \gamma DT_t + \delta D(T_B)_t + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t, \qquad (23)$$

shows the difference in the time reference of the trend dummy variable DT_t. Moreover, the parameter γ can be found in (21) in connection with the dummy variable DU_t, not with the trend dummy DT_t as in (23).

[4] For the models (1) and (2), the kind of reshaping can be realized analogously.
[5] Cf. Assenmacher (2002:248) and Perron (1994:126).

3.2 Break date estimation in the modified test regressions

Presumably, the reason for the spurious rejections is related to the erroneous break date estimation. By conducting method 2 one has to take care of the changed meaning of the break dummy coefficients. The selection method per se does not change and is for model (0) and (1):

$$\hat{T}_B^{2a} = \arg\max |t_{\hat{\theta}}|.$$

One has to bear in mind that θ now is the coefficient of the impulse dummy $D(T_B)_t$. As can be seen in equations (17) and (18), Perron chooses the break date using the following rule:

$$\hat{T}_B^{\text{Perron}} = \arg\max |t_{\widehat{\phi\theta}}|.$$

The relevant parameter θ is "diluted" and under the null hypothesis of $\phi = 0$ equal or near zero.

For model (2) several methods for estimating the break date are possible:

$$\begin{aligned} \hat{T}_B^{2b} &= \arg\max |t_{\hat{\gamma}}|, \\ \hat{T}_B^{2c} &= \arg\max |t_{\widehat{(\gamma+\theta)}}| \text{ or} \\ \hat{T}_B^{2d} &= \arg\max F_{\hat{\gamma},\hat{\theta}}. \end{aligned}$$

$\hat{T}_B^{2b}$ is the favored method in the literature for model (2) basing its choice on the parameter of the slope break dummy γ. In the modified version of the test regression, equation (22), γ is not isolated, but can be identified.

In any case, it is preferable to focus on both the intercept as well as the slope break. The coefficient of the impulse dummy $D(T_B)_t$ in (22) represents the sum of both parameters θ and γ, on which $\hat{T}_B^{2c}$ is based. Though, if both parameters have opposite signs, this approach can result in severe problems due to the additive conjunction of θ and γ. A similar approach is to choose the break date based on the joint test of $\gamma = \theta = 0$ using a F-statistic. This approach is already proposed and discarded by Vogelsang and Perron (1998). Harvey, Leybourne and Newbold (2001) explain the rejection with the fact, that the limiting distribution of $t_{\hat{\rho}}$ using the F-statistic depends on nuisance parameters. Further difficulties with this approach can arise because generally only small break magnitudes for the slope are observed. According to Vogelsang and Perron (1998:1090), for many macroeconomic time series the magnitudes of an intercept shift are generally less than 5 standard deviations and of a slope shift less than 0.5 standard deviations of the innovation errors.

In the following section a simulation analysis will be conducted using the modified test regressions in conjunction with various break date estimation approaches.

4 Simulation of the finite sample critical values

The following simulation study is based on the prior works of Vogelsang and Perron (1998), Harvey, Leybourne and Newbold (2001) and Lee and Strazicich (2001). As is common in the cited papers, the lag parameter k is assumed to be zero. Because the test statistics are exactly invariant to y_0 and β under the null, these parameters are set to zero. For simplicity, it is imposed that the true intercept is zero as well. The true break date is exactly in the middle of the time series, i.e. for $T = 100$ the break date will be $T_B = 50$. All simulations are conducted using GAUSS. The trimming factor is 10%.

The DGP for model (0) has the reduced form:

$$y_t = \theta D(T_B)_t + y_{t-1} + e_t$$

and the following structural form:

$$\begin{aligned} y_t &= \alpha + \theta DU_t + u_t \\ u_t &= u_{t-1} + e_t. \end{aligned}$$

The error process e_t is standard normally distributed, i.e. $e_t \sim N(0, 1)$. The break magnitude varies over the following values: $\theta = 0$, 2, 2.5, 4, 5, 6, 8, 10, 20. The simulations are based on 50 000 replications with $T = 100$ observations using the standard and modified test regressions (9) and (16) respectively. In order to endogenously identify the break date for model (0) and (1), the following methods will be used:

$$\hat{T}_B^1 = \arg\max t_{\hat{\rho}} \quad \text{and} \quad \hat{T}_B^{2a} = \arg\max |t_{\hat{\theta}}|.$$

For model (1) also the selection procedure based on the SBC will be applied enabling a comparison with the results from Lee and Strazicich (2001).

The first method is standard in the literature. Because the results based on this method show such remarkable problems, they will only be reproduced for model (0). In table 1 it can be seen that these results are qualitatively similar to the case of using method 2 in conjunction with the standard test regressions.

The DGP for model (1) contains a trend and an intercept break of the size θ:

$$y_t = \beta + \theta D(T_B)_t + y_{t-1} + e_t$$

or

$$\begin{aligned} y_t &= \alpha + \beta t + \theta DU_t + u_t \\ u_t &= u_{t-1} + e_t. \end{aligned}$$

The simulations for model (1) are based on 10 000 replications, with $T = 100$ each. The unit root hypothesis is tested on the basis of the standard test regression (10) and the modified test regression (19). The true slope parameter β will be set to zero as already stated above. Otherwise, the specifications for model (0) are applied.

A break in the intercept and the slope is included in the DGP for model (2):

$$y_t = \beta + \theta D(T_B)_t + \gamma DU_t + y_{t-1} + \varepsilon_t$$

or in the structural form:

$$\begin{aligned} y_t &= \alpha + \beta t + \theta DU_t + \gamma DT_t + u_t \\ u_t &= u_{t-1} + e_t. \end{aligned}$$

For model (2) the standard test regression (11) will be applied with the conventional method 2: $\hat{T}_B^{2b} = \arg\max |t_{\hat{\gamma}}|$. In contrast, the modified test regression

(22) will be used with the methods 2c and 2d: $\hat{T}_B^{2c} = \arg\max |t_{\widehat{(\gamma+\theta)}}|$ and $\hat{T}_B^{2d} = \arg\max F_{\hat{\gamma},\hat{\theta}}$. The break parameter θ and γ are varied over the following values: $\theta = 0$, 2, 4, 6, 8, 10 and $\gamma = 0$, 0.5, 1, 2, 4, 10. Since the so far ignored model specification (3) is only a special case of model (2) for $\theta = 0$, model (3) will be considered in the simulation analysis implicitly. Otherwise, the specifications are the same as for model (1).

In each case, the empirical 5 percent critical values and the corresponding empirical size for the individual break magnitudes will be calculated. The empirical size is the rate of rejecting the null hypothesis using the critical value assuming no break, that is for model (0) $\theta = 0$. In the case of invariance to the magnitude of the break, the empirical size should be equal to the nominal size of 5 percent. The results can be found in tables 1, 4 and 8 for models (0), (1) and (2), respectively.

Moreover, the relative frequency of the estimated break dates are calculated. The results are reproduced in tables 2 and 3 for model (0), in tables 5 and 7 for model (1) and in tables 9, 10 and 11 for model (2) each differing in the used test regression and break date selection method. The information is to be interpreted as follows. For instance, the value of 0.24 and 24.08 in the row for T_B and $T_B - 1$ respectively in table 2 for $\theta = 5$ means, that the true break date is identified in 0.24 percent of the replications. In 24.08 percent of the cases $T_B - 1$ is incorrectly chosen as the true break date.

For all model specifications the tests based on the standard test regressions show the well known problems regarding the spurious rejections and the inability of estimating the correct break point. The empirical size converges to unity with the break magnitude increasing. As stated in the literature, generally period $T_B - 1$ is incorrectly identified as the break date.

In contrast, the tests based on the modified test regressions and using break point selection method 2 show for all the models (0), (1) and (2) stable critical values resulting in constant empirical sizes. For model (2), this is only true using method 2c, $\hat{T}_B^{2c} = \arg\max |t_{\widehat{(\gamma+\theta)}}|$. As expected, method 2d ($\hat{T}_B^{2d} = \arg\max F_{\hat{\theta},\hat{\gamma}}$) shows remarkable size distortions for small break magnitudes. Therefore, even for $\theta = 0$ and $\gamma \neq 0$, i.e. model (3), the empirical size remains stable using the test for model (2). Expecting a break only in the slope parameter, the test for model (2) can be applied.

It is worth noting that the distribution of $t_{\hat{\rho}}$ approaches the same distribution as with exogenously given break point. That is intuitively clear, since with increasing break magnitude it becomes easier to identify the true break date. In an extreme case of a very big structural change, the break date will be estimated correctly in 100 percent of the cases resembling the case of a known break date. For comparison, the 5 percent critical values for the exogenous test are summarized in table 12.

The exact invariance concerning the break magnitude of θ and γ stated in Vogelsang and Perron (1998:1084) can be found in the simulation result for the modified tests. Furthermore, the true break date can be identified accurately. For example, in the case of a structural break of $\theta = 4$ for models (0) and (1) and of $\theta = 4$ and $\gamma = 0.5$ for model (2), the modified testing procedure correctly estimates the true break point in 87.98 percent, 86.19 percent and 93.36 percent of the cases for model (0), (1) and (2), respectively. However, in the presence of even smaller breaks this procedure is capable of estimating the true break date.

The modified selection methods from Harvey, Leybourne and Newbold (2001) and Lee and Strazicich (2001) are only able to achieve one of these two favorable properties. Therefore, the method proposed by Harvey et al. (2001) leads to a stable empirical size reflecting the invariance property. Though, this method has difficulties in identifying the true break date. As in the aforementioned case of $\theta = 4$ and $\gamma = 0.5$, the frequency of estimating the true break date is simply 16.76 percent, 18.55 percent and 22.06 percent for the respective model. Regarding this selection method, there are some points worth noting. First, the distributions for $t_{\hat{\rho}}(\hat{T}_B^3)$ do not comply with those for the exogenous tests. Second, there does not seem to be an easy way to generalize it for multiple breaks. And third, as can be observed on the basis of the power simulations in the next section, the tests proposed in this paper are more powerful than those based on $\hat{T}_B^3$.[6]

Applying the modified approach proposed by Lee and Strazicich (2001) one is able to identify the true break date with almost the same accuracy as with the new procedure discussed in this paper. This can be seen in table 6. Though, it can also be seen from table 4 that the critical values vary with increasing break magnitude. Therefore, the distribution depends on nuisance parameters, which are the magnitude and the timing of the break. This proves that not only the frequency of accurately estimating the break point assures the invariance property. From a particular break magnitude onwards the break date using the SBC is identified in 100 percent of the time, thus it is not surprising that the distribution of $t_{\hat{\rho}}$ also converges towards the distribution of the exogenous test.[7] The empirical densities of $t_{\hat{\rho}}$ using the SBC and the modified tests for various break magnitudes are shown in figures 1 and 2.

Since the difference in distribution is already present in the case of no break, this difference is not due to the ability of identifying the true break point (because there is no break). To find the reason for the difference in the distribution using $\hat{T}_B^4 = \arg\min SBC$ and $\hat{T}_B^{2a} = \arg\max |t_{\hat{\theta}}|$, the simulation data will be grouped. Because of identical t-values $t_{\hat{\rho}}$ for equally chosen break dates, the difference can only arise for the cases in which the break date estimation differs forming one group of data with identical break date ($\hat{T}_B^{2a} = \hat{T}_B^4$) and the other group with different estimated break dates ($\hat{T}_B^{2a} \neq \hat{T}_B^4$). Figure 3 displays the simulation data for $\theta = 0$. The upper and middle right panel show the histogram of the estimated break dates for selection method 2 and 3, respectively, using the data of the group with $\hat{T}_B^{2a} \neq \hat{T}_B^4$. As expected, every potential break date is almost equally chosen. This shows that the observed difference between the distribution of $t_{\hat{\rho}}$ for the selection methods cannot arise from asymmetries in the estimation of the break points.[8] The mentioned difference is obvious in the corresponding distribution of $t_{\hat{\rho}}$ found in the left side panels. The lower

[6] Cf. the power simulations in Harvey et al. (2001:570), table 5, panel B for model (0) and (1).

[7] As stated above, the t-statistic $t_{\hat{\rho}}$ is identical using the standard and modified test regression for a given break date. So, by using the SBC it makes no difference whether it is based on the standard or the modified test regressions.

[8] Asymmetries can be observed for break magnitudes $\theta > 0$ using the SBC in form of a higher rate for break dates less than the true break date, especially those that are close to the true break date, that is $T_B - 1$ and $T_B - 2$. If the true break occurs relatively soon after the estimated break date, the standard test tends to exhibit more negative t-values, as stated by Kim et al. (2000). In spite of these asymmetries, it does not seem to be the primary cause for the difference in the distributions.

left panel shows the distribution of $t_{\hat{\rho}}$ for the group with identical break date estimates ($\hat{T}_B^{2a} = \hat{T}_B^4$). A comparison of the three distributions clarifies that on average the SBC yields more negative t-values when the estimated break dates differ. The distributions of the t-values using $\hat{T}_B^{2a}$ do not differ across both groups.

On the basis of the scatter diagram of the estimated break date and the corresponding t-value of $\hat{\rho}$ it can be analyzed if the t-value varies systematically with the chosen break date. The red symbols correspond to the coordinates ($\hat{T}_B^{2a}$, $t_{\hat{\rho}}$) and the blue symbols to the coordinates ($\hat{T}_B^4$, $t_{\hat{\rho}}$). It can be seen in the lower right panel of figure 3 that the range of the t-values do not change with the selected break date for each selection method. However, the blue symbols are on average below the red symbols.

Hence, in order to achieve both favorable test properties (invariance property and accurate break date estimation) both the selection method and the form of the test regression are relevant.

5 Power simulations

The power simulations are based on 5 000 replications with $T = 100$ observations each. The relevant AR-coefficient ρ is set to 0.8 for all the simulations. Besides the power, also the adjusted power of the test will be calculated. The power will be determined with the critical values assuming no break, i.e. $\theta = \gamma = 0$. In contrast, the adjusted power is based on the empirical 5 percent critical values for various break magnitudes calculated in the former section. The results can be found in table 13, 16 and 19 for the models (0), (1) and (2) respectively.

Lee and Strazicich (2001) show that the erroneous estimation of the break point occurs also under the alternative hypothesis. This is not the case using the test procedures based on the modified test regression. The results are summarized in tables 14 and 15 for the standard and modified versions of model (0). The corresponding results for model (1) are reproduced in tables 17 and 18, for model (2) in tables 20, 21 and 22.

The remarkable size distortions are reflected in the increasing power of the standard test converging to 100 percent. The adjusted power calculations show the true picture. The adjusted power of the modified test always[9] exceeds that of the standard procedure. In general, the power and adjusted power diminish across the model specification (0), (1) and (2) reflecting the necessity of choosing the composition of the deterministic component with some caution.

The performance of the modified procedure identifying the true break date is still excellent. For the case of $\theta = 4$ and $\gamma = 0.5$ the rate of estimating the true break point is 86.08 percent, 85.44 percent and 90.70 percent for model (0), (1) and (2) respectively.

6 Concluding remarks

The present paper considers innovational outlier unit root tests that treat the break point as unknown. Following the approach of representing the DGP as an unobserved component model, modified test regressions can be generated. Test

[9] For model (2), only in 33 of 36 cases.

statistics based on these modified test regressions do not show the problem of spurious rejections when a break occurs under the null hypothesis. It is shown that both the specification of the test regression and the method of selecting the break date are necessary to yield favorable test properties, that is the invariance to a level and slope shift and the accurate estimation of the true break date. The distribution of the modified test statistic equals that of the exogenous test.

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Figure 1: Estimated density of $t_{\hat{\rho}}$ using $\hat{T}_B^4 = \arg\min SBC$ for various break magnitudes of θ, standard (=modified) model (1)

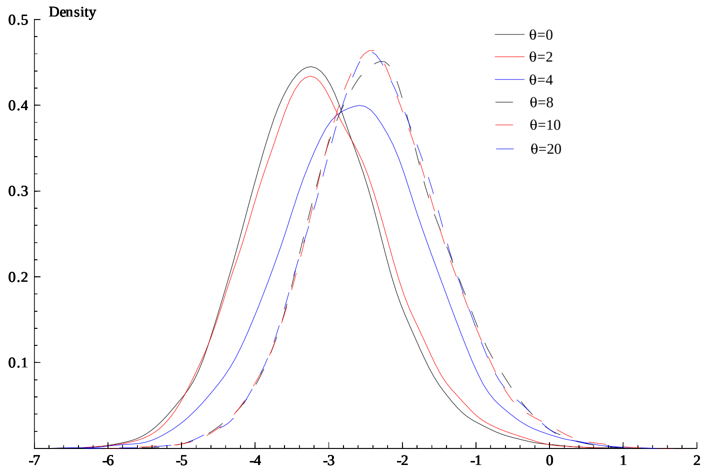

Figure 2: Estimated density of $t_{\hat{\rho}}$ using $\hat{T}_B^{2a} = \arg\max|t_{\hat{\theta}}|$ for various break magnitudes of θ, modified model (1)

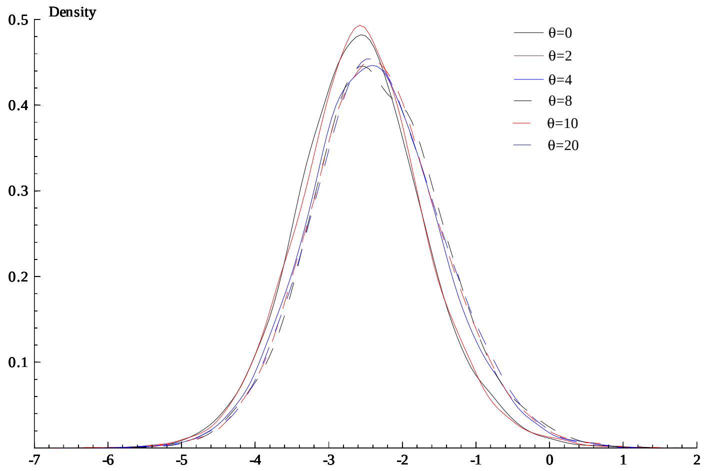

Table 1: Empirical 5 percent critical values and size for IO-model (0)

	standard				modified			
$\hat{T}_B$	$\arg\min t_{\hat{\rho}}$		$\arg\max\lvert t_{\hat{\theta}}\rvert$		$\arg\min t_{\hat{\rho}}$		$\arg\max\lvert t_{\hat{\theta}}\rvert$	
θ	$t_{\mathrm{cv},5\%}$	emp. size	$t_{\mathrm{cv},5\%}$	emp. size	$t_{\mathrm{cv},5\%}$	emp. size	$t_{\mathrm{cv},5\%}$	emp. size
0	-4.34	5.00	-4.33	5.00	-4.34	5.00	-3.45	5.00
2	-4.38	5.61	-4.37	5.54	-4.38	5.61	-3.47	5.13
2.5	-4.39	5.67	-4.37	5.63	-4.39	5.67	-3.45	4.93
4	-4.60	8.56	-4.59	8.53	-4.60	8.56	-3.42	4.57
5	-4.78	11.36	-4.77	11.41	-4.78	11.36	-3.40	4.40
6	-5.14	15.66	-5.13	15.67	-5.14	15.66	-3.38	4.20
8	-6.06	28.05	-6.06	28.22	-6.06	28.05	-3.38	4.29
10	-7.05	44.01	-7.05	44.26	-7.05	44.01	-3.39	4.32
20	-12.59	92.40	-12.59	92.50	-12.59	92.40	-3.39	4.26

Figure 3: Break magnitude $\theta = 0$; top left: distribution of $t_{\hat{\rho}}$ with $\hat{T}_B^{2a} = \arg\max |t_{\hat{\theta}}|$ and $\hat{T}_B^{2a} \neq \hat{T}_B^4$; top right: absolute frequency of estimated break date using $\hat{T}_B^{2a}$; middle left: distribution of $t_{\hat{\rho}}$ with $\hat{T}_B^4 = \arg\min SBC$ and $\hat{T}_B^{2a} \neq \hat{T}_B^4$; middle right: absolute frequency of estimated break date using $\hat{T}_B^4$; bottom left: distribution of $t_{\hat{\rho}}$ for $\hat{T}_B^{2a} = \hat{T}_B^4$; bottom right: scatter diagram of $(\hat{T}_B^{2a}, t_{\hat{\rho}})$ (red) and $(\hat{T}_B^4, t_{\hat{\rho}})$ (blue).

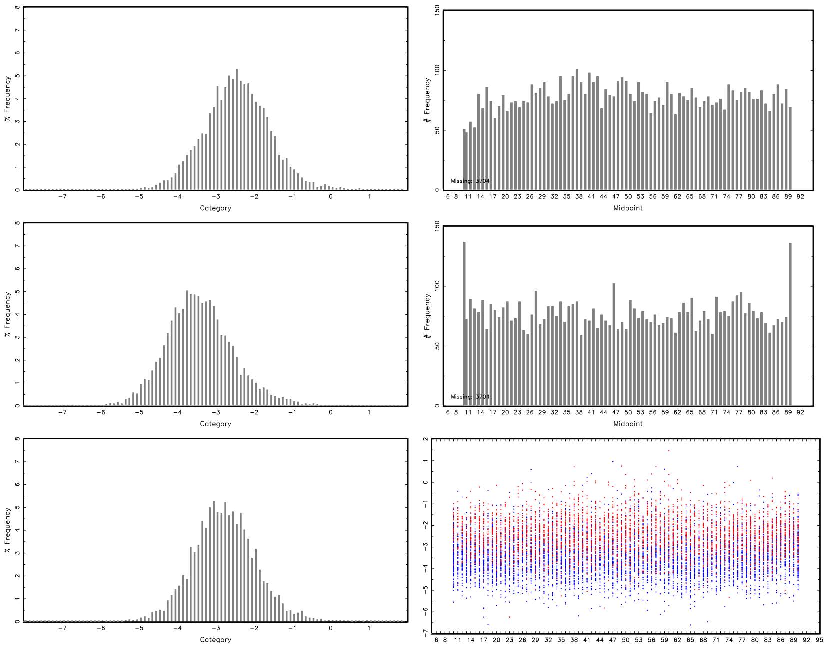

Table 2: Relative frequency of estimated break points, in percent

standard IO-model (0), $\hat{T}_B = \arg\max \lvert t_{\hat{\theta}} \rvert$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	43.21	41.74	41.46	37.69	33.81	29.65	20.59	12.95	0.32
$T_B - 4$	1.28	1.66	1.82	2.25	2.40	2.45	2.08	1.63	0.12
$T_B - 3$	1.38	1.94	2.21	2.88	3.28	3.56	3.34	2.67	0.17
$T_B - 2$	1.22	2.46	3.02	4.68	5.49	6.28	6.80	5.42	0.46
$T_B - 1$	1.32	5.68	7.56	16.76	24.08	32.69	50.93	67.62	98.58
T_B	1.35	0.66	0.45	0.23	0.24	0.18	0.13	0.16	0.03
$T_B + 1$	1.26	0.93	0.92	0.62	0.52	0.45	0.39	0.29	0.07
$T_B + 2$	1.33	0.92	0.82	0.52	0.41	0.34	0.25	0.15	0.02
$T_B + 3$	1.30	1.02	0.89	0.54	0.47	0.33	0.22	0.14	0.00
$T_B + 4$	1.32	1.10	0.93	0.63	0.41	0.34	0.23	0.14	0.00
$> T_B + 4$	45.03	41.90	39.91	33.19	28.89	23.74	15.04	8.84	0.02

Table 3: Relative frequency of estimated break points, in percent

modified IO-model (0), $\hat{T}_B = \arg\max \lvert t_{\hat{\theta}} \rvert$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	42.49	31.77	24.75	5.15	0.77	0.08	0.00	0.00	0.00
$T_B - 4$	1.27	0.96	0.81	0.18	0.03	0.00	0.00	0.00	0.00
$T_B - 3$	1.34	0.94	0.69	0.15	0.02	0.00	0.00	0.00	0.00
$T_B - 2$	1.27	0.88	0.76	0.18	0.04	0.00	0.00	0.00	0.00
$T_B - 1$	1.22	0.97	0.82	0.15	0.05	0.00	0.00	0.00	0.00
T_B	1.28	26.14	42.90	87.98	98.06	99.79	100.00	100.00	100.00
$T_B + 1$	1.24	0.99	0.73	0.15	0.03	0.00	0.00	0.00	0.00
$T_B + 2$	1.31	0.90	0.70	0.17	0.02	0.00	0.00	0.00	0.00
$T_B + 3$	1.24	0.96	0.72	0.16	0.01	0.00	0.00	0.00	0.00
$T_B + 4$	1.28	0.97	0.73	0.15	0.02	0.00	0.00	0.00	0.00
$> T_B + 4$	46.06	34.51	26.40	5.57	0.93	0.11	0.00	0.00	0.00

Table 4: Empirical 5 percent critical values and size for IO-model (1)

	standard		standard=modified		modified	
$\hat{T}_B$	$\arg\max \lvert t_{\hat{\theta}} \rvert$		$\arg\min SBC$		$\arg\max \lvert t_{\hat{\theta}} \rvert$	
θ	$t_{\text{cv},5\%}$	emp. size	$t_{\text{cv},5\%}$	emp. size	$t_{\text{cv},5\%}$	emp. size
0	-4.79	5.00	-4.60	5.00	-3.94	5.00
2	-4.81	5.34	-4.64	5.45	-3.95	5.10
2.5	-4.80	5.14	-4.61	5.18	-3.93	4.88
4	-5.00	7.65	-4.36	3.05	-3.83	3.89
5	-5.26	11.67	-4.03	1.53	-3.79	3.51
6	-5.70	16.38	-3.81	0.80	-3.75	3.42
8	-6.70	31.93	-3.76	0.52	-3.76	3.31
10	-7.83	50.27	-3.74	0.50	-3.74	2.97
20	-14.06	95.05	-3.75	0.52	-3.75	3.09

Table 5: Relative frequency of estimated break points, in percent

standard IO-model (1), $\hat{T}_B = \arg\max \lvert t_{\hat{\theta}} \rvert$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	44.84	42.69	41.59	35.73	30.87	25.13	15.69	8.40	0.12
$T_B - 4$	1.29	1.47	1.54	1.65	1.89	1.86	1.37	0.83	0.01
$T_B - 3$	1.38	1.69	2.20	2.74	2.45	3.00	2.47	1.63	0.06
$T_B - 2$	1.28	2.14	2.29	3.85	3.97	4.42	4.11	2.92	0.16
$T_B - 1$	1.26	5.84	7.83	18.55	28.26	38.44	58.64	75.71	99.21
T_B	1.24	0.52	0.29	0.03	0.01	0.00	0.00	0.00	0.00
$T_B + 1$	1.13	0.97	0.80	0.74	0.63	0.53	0.45	0.35	0.17
$T_B + 2$	1.5	0.89	0.90	0.54	0.44	0.40	0.25	0.21	0.02
$T_B + 3$	1.05	1.02	0.85	0.71	0.47	0.48	0.28	0.18	0.00
$T_B + 4$	1.10	0.99	1.05	0.81	0.63	0.38	0.35	0.22	0.01
$> T_B + 4$	44.28	41.78	40.66	34.65	30.38	25.36	16.39	9.55	0.24

Table 6: Relative frequency of estimated break points, in percent

standard (=modified) IO-model (1), $\hat{T}_B = \arg\min SBC$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	44.52	37.57	32.87	11.71	3.33	0.40	0.01	0.00	0.00
$T_B - 4$	1.15	1.06	0.93	0.36	0.07	0.00	0.00	0.00	0.00
$T_B - 3$	1.47	1.27	1.16	0.34	0.15	0.01	0.00	0.00	0.00
$T_B - 2$	1.12	1.36	1.38	0.62	0.13	0.04	0.00	0.00	0.00
$T_B - 1$	1.19	1.31	1.28	0.69	0.22	0.04	0.02	0.00	0.00
T_B	1.03	16.95	28.19	74.75	93.02	99.07	99.97	100.00	100.00
$T_B + 1$	1.35	0.60	0.43	0.08	0.03	0.00	0.00	0.00	0.00
$T_B + 2$	1.10	0.77	0.46	0.19	0.00	0.00	0.00	0.00	0.00
$T_B + 3$	1.19	0.80	0.59	0.17	0.02	0.00	0.00	0.00	0.00
$T_B + 4$	1.17	0.81	0.51	0.15	0.04	0.00	0.00	0.00	0.00
$> T_B + 4$	44.71	37.50	32.20	10.94	2.99	0.44	0.00	0.00	0.00

Table 7: Relative frequency of estimated break points, in percent

modified IO-model (1), $\hat{T}_B = \arg\max \lvert t_{\hat{\theta}} \rvert$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	43.94	33.89	27.60	6.31	1.14	0.07	0.00	0.00	0.00
$T_B - 4$	1.27	0.86	0.70	0.23	0.06	0.00	0.00	0.00	0.00
$T_B - 3$	1.23	0.88	0.81	0.18	0.03	0.00	0.00	0.00	0.00
$T_B - 2$	1.39	0.78	0.80	0.17	0.04	0.00	0.00	0.00	0.00
$T_B - 1$	1.43	0.98	0.73	0.19	0.02	0.00	0.00	0.00	0.00
T_B	1.30	25.28	40.38	86.19	97.65	99.84	100.00	100.00	100.00
$T_B + 1$	1.27	0.85	0.78	0.15	0.02	0.00	0.00	0.00	0.00
$T_B + 2$	1.03	0.96	0.70	0.17	0.02	0.00	0.00	0.00	0.00
$T_B + 3$	1.36	0.79	0.91	0.17	0.04	0.01	0.00	0.00	0.00
$T_B + 4$	1.20	0.99	0.72	0.10	0.01	0.00	0.00	0.00	0.00
$> T_B + 4$	44.58	33.74	25.87	6.14	0.97	0.07	0.00	0.00	0.00

Table 8: Empirical 5 percent critical values and size for IO-model (2)

		standard		modified			
	$\hat{T}_B$	$\arg\max \lvert t_{\hat{\gamma}} \rvert$		$\arg\max \lvert t_{\widehat{(\gamma+\theta)}} \rvert$		$\arg\max F_{\hat{\gamma}.\hat{\theta}}$	
θ	γ	$t_{\text{cv},5\%}$	emp. size	$t_{\text{cv},5\%}$	emp. size	$t_{\text{cv},5\%}$	emp. size
0	0	-4.83	5.00	-4.30	5.00	-4.95	5.00
	0.5	-4.98	6.97	-4.11	3.49	-4.91	4.50
	1	-5.35	15.65	-4.07	3.03	-5.13	7.20
	2	-6.49	65.97	-4.19	4.02	-5.94	15.78
	4	-8.98	98.31	-4.13	3.16	-5.65	6.18
	10	-14.29	99.86	-4.27	4.70	-4.28	0.81
2	0	-4.88	5.70	-4.32	5.13	-4.93	4.75
	0.5	-5.09	8.72	-4.18	3.82	-4.87	4.05
	1	-5.41	16.87	-4.18	3.88	-4.98	5.41
	2	-6.44	64.56	-4.20	4.02	-5.27	7.19
	4	-8.74	98.99	-4.26	4.42	-4.46	2.51
	10	-13.68	99.94	-4.24	4.20	-4.35	1.17
4	0	-5.23	10.02	-4.29	4.93	-4.70	2.99
	0.5	-5.44	14.25	-4.24	4.23	-4.59	2.51
	1	-5.64	22.36	-4.21	4.03	-4.52	2.25
	2	-6.34	62.98	-4.23	4.32	-4.45	2.21
	4	-8.40	99.36	-4.22	4.10	-4.33	1.33
	10	-12.99	99.95	-4.25	4.38	-4.32	1.22
6	0	-6.16	20.23	-4.27	4.71	-4.28	1.14
	0.5	-6.28	28.80	-4.31	5.06	-4.29	1.11
	1	-6.38	36.68	-4.24	4.26	-4.29	1.09
	2	-6.55	62.67	-4.24	4.26	-4.26	0.96
	4	-8.03	99.44	-4.26	4.40	-4.29	0.97
	10	-12.29	99.98	-4.26	4.40	-4.27	0.80
8	0	-7.34	35.72	-4.22	4.33	-4.25	0.85
	0.5	-7.54	50.80	-4.21	4.08	-4.25	0.64
	1	-7.59	57.80	-4.28	4.61	-4.27	0.79
	2	-7.54	70.06	-4.27	4.44	-4.26	0.71
	4	-7.89	99.49	-4.24	4.31	-4.26	0.71
	10	-11.66	99.99	-4.25	4.34	-4.26	0.87
10	0	-8.65	51.42	-4.27	4.60	-4.24	0.65
	0.5	-8.80	70.91	-4.25	4.48	-4.24	0.66
	1	-8.87	75.86	-4.25	4.42	-4.26	0.79
	2	-8.84	80.93	-4.28	4.63	-4.30	0.85
	4	-8.89	99.29	-4.21	4.10	-4.26	0.84
	10	-11.07	99.99	-4.25	4.47	-4.23	0.78

Table 9: Relative frequency of estimated break points, in percent

		standard IO-model (2), $\hat{T}_B = \arg\max \lvert t_{\hat{\gamma}} \rvert$										
T_B		< -4	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$> +4$
θ	γ											
0	0	43.62	1.54	1.60	1.68	1.75	1.46	1.58	1.48	1.39	1.40	42.50
	0.5	63.82	3.20	2.46	1.84	1.30	1.67	1.59	1.41	1.12	1.15	20.44
	1	86.00	3.81	1.73	0.88	0.57	0.83	0.79	0.55	0.56	0.35	3.93
	2	95.95	2.14	0.59	0.11	0.03	0.03	0.03	0.02	0.01	0.00	1.09
	4	97.27	1.00	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.60
	10	90.25	8.64	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14
2	0	40.44	1.57	1.76	2.45	5.39	2.00	1.12	1.15	1.41	1.20	41.51
	0.5	65.86	1.78	1.15	0.86	6.20	3.23	1.03	0.81	0.87	0.85	17.36
	1	87.56	1.39	0.68	0.32	4.23	0.93	0.42	0.32	0.32	0.26	3.57
	2	97.40	1.02	0.14	0.02	0.47	0.03	0.01	0.00	0.01	0.00	0.90
	4	98.41	0.64	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.88
	10	92.47	6.94	0.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
4	0	34.87	0.88	1.42	3.11	17.24	3.50	0.57	0.85	0.88	1.04	35.64
	0.5	55.26	0.68	0.87	1.09	22.06	3.82	0.33	0.31	0.40	0.48	14.70
	1	76.37	0.56	0.20	0.23	18.38	1.08	0.16	0.12	0.12	0.08	2.70
	2	94.10	0.47	0.07	0.00	4.51	0.03	0.02	0.00	0.00	0.00	0.80
	4	98.96	0.44	0.05	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.49
	10	94.36	5.18	0.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
6	0	24.86	0.60	0.97	2.54	32.38	4.71	0.76	0.73	0.85	0.87	30.73
	0.5	37.90	0.50	0.66	1.06	44.91	3.86	0.23	0.21	0.16	0.34	10.17
	1	54.46	0.18	0.16	0.25	42.29	0.96	0.02	0.07	0.04	0.02	1.55
	2	81.42	0.16	0.02	0.00	18.04	0.03	0.00	0.00	0.00	0.00	0.33
	4	97.65	0.47	0.00	0.00	1.60	0.00	0.00	0.00	0.00	0.00	0.28
	10	95.20	4.48	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
8	0	18.11	0.27	0.47	1.45	46.60	5.55	1.19	0.91	0.80	0.84	23.81
	0.5	20.29	0.29	0.28	0.57	66.64	3.47	0.25	0.18	0.20	0.19	7.64
	1	30.31	0.06	0.08	0.21	67.84	0.61	0.04	0.01	0.01	0.00	0.83
	2	54.87	0.06	0.00	0.00	44.83	0.05	0.00	0.00	0.00	0.00	0.19
	4	88.05	0.21	0.01	0.00	11.64	0.00	0.00	0.00	0.00	0.00	0.09
	10	95.23	4.31	0.38	0.00	0.07	0.00	0.00	0.00	0.00	0.00	0.01
10	0	12.26	0.19	0.14	0.67	57.47	6.00	1.81	1.11	0.87	0.92	18.56
	0.5	9.70	0.10	0.11	0.20	81.65	2.68	0.31	0.25	0.23	0.16	4.61
	1	14.21	0.02	0.05	0.05	84.95	0.36	0.01	0.01	0.01	0.01	0.32
	2	28.07	0.00	0.01	0.00	71.85	0.00	0.00	0.00	0.00	0.00	0.07
	4	64.52	0.19	0.00	0.00	35.24	0.00	0.00	0.00	0.00	0.00	0.05
	10	93.96	4.61	0.17	0.00	1.25	0.00	0.00	0.00	0.00	0.00	0.01

Table 10: Relative frequency of estimated break points, in percent

		modified IO-model (2), $\hat{T}_B = \arg\max \lvert t_{\widehat{(\gamma+\theta)}} \rvert$										
T_B		< -4	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$> +4$
θ	γ											
0	0	39.75	1.52	1.11	1.30	1.30	1.38	1.21	1.55	1.49	1.32	48.07
	0.5	33.46	1.08	1.16	1.24	1.15	2.17	2.03	2.13	2.48	2.41	50.69
	1	25.10	0.87	0.85	0.95	0.97	4.64	4.84	5.20	5.03	5.00	46.55
	2	8.28	0.50	0.37	0.43	0.48	15.29	15.40	15.29	12.44	8.77	22.75
	4	0.15	0.04	0.03	0.02	0.03	41.41	33.20	17.38	5.45	1.32	0.97
	10	0.00	0.00	0.00	0.00	0.00	95.89	4.11	0.00	0.00	0.00	0.00
2	0	29.22	0.95	1.18	1.03	0.98	26.22	0.97	1.18	0.88	1.03	36.36
	0.5	21.42	0.68	0.78	0.69	0.71	39.39	1.60	1.32	1.48	1.49	30.44
	1	12.56	0.51	0.54	0.56	0.57	53.96	3.00	2.74	2.64	2.36	20.56
	2	2.06	0.15	0.11	0.14	0.20	75.82	5.50	4.71	3.49	2.32	5.50
	4	0.01	0.00	0.00	0.00	0.00	91.69	5.24	2.29	0.61	0.10	0.06
	10	0.00	0.00	0.00	0.00	0.00	99.92	0.08	0.00	0.00	0.00	0.00
4	0	5.23	0.09	0.22	0.17	0.22	86.40	0.21	0.20	0.14	0.05	7.07
	0.5	2.29	0.11	0.06	0.13	0.12	93.36	0.14	0.18	0.19	0.14	3.28
	1	0.82	0.04	0.06	0.03	0.10	96.70	0.25	0.16	0.15	0.21	1.48
	2	0.06	0.00	0.01	0.01	0.01	99.10	0.24	0.24	0.10	0.08	0.15
	4	0.00	0.00	0.00	0.00	0.00	99.77	0.11	0.08	0.03	0.01	0.00
	10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
6	0	0.08	0.01	0.00	0.01	0.01	99.78	0.00	0.00	0.00	0.00	0.11
	0.5	0.03	0.01	0.00	0.01	0.00	99.88	0.01	0.00	0.00	0.00	0.06
	1	0.00	0.00	0.00	0.01	0.00	99.94	0.00	0.00	0.00	0.02	0.03
	2	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
8	0	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	0.5	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
10	0	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	0.5	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00

Table 11: Relative frequency of estimated break points, in percent

	modified IO-model (2), $\hat{T}_B = \arg\max F_{\hat{\gamma},\hat{\theta}}$											
T_B		< -4	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$> +4$
θ	γ											
0	0	41.64	1.23	1.62	1.77	1.49	1.52	1.60	1.79	1.57	1.59	44.18
	0.5	46.61	2.28	2.70	2.79	3.54	3.18	3.13	2.78	2.78	2.06	28.15
	1	44.96	3.63	3.93	5.04	8.72	8.05	5.35	4.19	2.93	2.42	10.78
	2	26.13	3.49	3.99	6.38	24.44	24.87	6.18	2.49	1.05	0.41	0.57
	4	3.37	1.33	0.96	1.38	44.90	47.41	0.62	0.03	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	49.96	50.04	0.00	0.00	0.00	0.00	0.00
2	0	35.46	1.60	1.49	1.74	2.63	18.31	0.81	0.86	1.07	1.23	34.80
	0.5	36.00	1.87	2.11	2.85	5.51	30.94	0.86	1.01	0.96	1.00	16.89
	1	27.17	2.13	2.47	3.85	10.52	46.19	0.77	0.87	0.77	0.61	4.65
	2	10.99	1.26	1.42	3.20	17.34	64.96	0.28	0.24	0.13	0.06	0.12
	4	0.84	0.23	0.22	0.41	22.45	75.85	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	23.86	76.14	0.00	0.00	0.00	0.00	0.00
4	0	10.15	0.59	0.62	1.11	1.72	75.28	0.16	0.20	0.15	0.27	9.75
	0.5	8.59	0.26	0.57	0.91	1.87	84.72	0.07	0.09	0.05	0.10	2.77
	1	4.52	0.29	0.40	1.00	2.09	91.17	0.06	0.03	0.03	0.03	0.38
	2	1.31	0.18	0.19	0.47	3.00	94.83	0.00	0.01	0.00	0.01	0.00
	4	0.03	0.02	0.05	0.04	3.41	96.45	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	3.28	96.72	0.00	0.00	0.00	0.00	0.00
6	0	0.46	0.04	0.03	0.05	0.21	98.88	0.01	0.00	0.00	0.01	0.31
	0.5	0.37	0.00	0.03	0.03	0.25	99.25	0.00	0.00	0.00	0.00	0.07
	1	0.14	0.05	0.03	0.03	0.20	99.55	0.00	0.00	0.00	0.00	0.00
	2	0.06	0.00	0.01	0.00	0.26	99.67	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.32	99.68	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.17	99.83	0.00	0.00	0.00	0.00	0.00
8	0	0.00	0.00	0.00	0.00	0.02	99.97	0.00	0.00	0.00	0.00	0.01
	0.5	0.00	0.00	0.00	0.00	0.03	99.97	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.04	99.96	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.03	99.97	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.02	99.98	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.01	99.99	0.00	0.00	0.00	0.00	0.00
10	0	0.00	0.00	0.00	0.00	0.01	99.99	0.00	0.00	0.00	0.00	0.00
	0.5	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.01	99.99	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.01	99.99	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.01	99.99	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.01	99.99	0.00	0.00	0.00	0.00	0.00

Table 12: 5 percent critical values in the case of an exogenously given break date

IO-model, T_B exogenous			
T	model (0)	model (1)	model (2)
100	-3.38	-3.77	-4.25
∞	-3.34	-3.76	-4.24

Source: Perron (1990, 1994), Vogelsang and Perron (1998), own calculations.

Table 13: Power and adjusted power, $\rho = 0.8$, 5 percent significance level, IO-model (0)

	standard		modified	
$\hat{T}_B$	$\arg\max \lvert t_{\hat{\theta}} \rvert$		$\arg\max \lvert t_{\hat{\theta}} \rvert$	
θ	power	power$_{adj}$	power	power$_{adj}$
0	40.60	40.60	68.18	68.18
2	44.88	42.42	54.98	54.00
2.5	47.48	45.12	54.10	54.10
4	62.38	49.72	59.32	61.00
5	74.50	50.90	60.96	64.04
6	85.06	54.66	63.50	67.92
8	96.80	54.94	63.18	67.38
10	99.46	57.90	64.32	67.54
20	100.00	65.18	62.72	66.30

Table 14: Relative frequency of estimated break points, in percent

standard IO-model (0), $\hat{T}_B = \arg\max \lvert t_{\hat{\theta}} \rvert$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	43.94	38.62	34.18	18.88	11.52	5.24	1.38	0.18	0.00
$T_B - 4$	0.90	3.46	3.40	4.32	3.20	1.98	0.92	0.14	0.00
$T_B - 3$	0.82	3.80	4.90	6.36	4.74	3.86	1.82	0.34	0.00
$T_B - 2$	0.98	5.62	8.02	11.58	10.96	8.28	4.02	1.20	0.00
$T_B - 1$	0.76	17.24	27.18	52.56	67.82	80.24	91.86	98.14	100.00
T_B	0.96	0.86	0.38	0.02	0.00	0.00	0.00	0.00	0.00
$T_B + 1$	1.00	0.84	0.48	0.04	0.00	0.00	0.00	0.00	0.00
$T_B + 2$	1.04	1.02	0.66	0.20	0.06	0.02	0.00	0.00	0.00
$T_B + 3$	0.74	1.28	0.96	0.12	0.04	0.00	0.00	0.00	0.00
$T_B + 4$	0.80	0.86	0.90	0.36	0.06	0.00	0.00	0.00	0.00
$> T_B + 4$	48.06	26.50	18.94	5.56	1.60	0.38	0.00	0.00	0.00

Table 15: Relative frequency of estimated break points, in percent

modified IO-model (0), $\hat{T}_B = \arg\max \lvert t_{\hat{\theta}} \rvert$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	43.66	29.78	22.22	4.44	0.74	0.08	0.00	0.00	0.00
$T_B - 4$	1.30	1.02	0.62	0.08	0.04	0.00	0.00	0.00	0.00
$T_B - 3$	1.04	0.98	0.68	0.12	0.06	0.00	0.00	0.00	0.00
$T_B - 2$	1.38	0.90	0.72	0.14	0.04	0.00	0.00	0.00	0.00
$T_B - 1$	1.52	0.82	0.84	0.18	0.00	0.00	0.00	0.00	0.00
T_B	1.30	24.12	40.08	86.08	97.00	99.84	100.00	100.00	100.00
$T_B + 1$	1.20	1.44	1.54	0.60	0.26	0.04	0.00	0.00	0.00
$T_B + 2$	0.96	1.46	1.32	0.48	0.16	0.00	0.00	0.00	0.00
$T_B + 3$	1.38	1.34	1.18	0.40	0.18	0.00	0.00	0.00	0.00
$T_B + 4$	1.23	1.46	1.40	0.54	0.10	0.00	0.00	0.00	0.00
$> T_B + 4$	45.14	36.68	29.40	6.94	1.42	0.04	0.00	0.00	0.00

Table 16: Power and adjusted power, $\rho = 0.8$, 5 percent significance level, IO-model (1)

	standard		modified	
$\hat{T}_B$	$\arg\max \lvert t_{\hat{\theta}} \rvert$		$\arg\max \lvert t_{\hat{\theta}} \rvert$	
θ	power	power$_{adj}$	power	power$_{adj}$
0	35.42	35.42	51.28	51.28
2	34.76	33.66	45.18	43.56
2.5	34.50	33.88	43.56	44.46
4	47.32	36.30	41.92	46.36
5	57.92	36.20	42.04	52.90
6	73.78	37.94	42.14	52.66
8	92.26	38.60	42.52	52.26
10	98.56	40.40	42.66	50.18
20	100.00	44.12	41.54	52.70

Table 17: Relative frequency of estimated break points, in percent

standard IO-model (1), $\hat{T}_B = \arg\max \lvert t_{\hat{\theta}} \rvert$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	44.40	36.86	33.18	19.28	11.58	4.84	1.08	0.14	0.00
$T_B - 4$	1.36	2.22	2.50	3.34	2.42	1.46	0.78	0.04	0.00
$T_B - 3$	1.22	2.92	3.58	4.16	3.84	2.92	1.50	0.32	0.00
$T_B - 2$	1.40	3.92	4.64	7.18	7.04	5.16	2.30	0.60	0.00
$T_B - 1$	1.34	13.98	23.46	50.06	68.00	82.42	94.04	98.86	100.00
T_B	1.80	1.46	0.72	0.10	0.00	0.00	0.00	0.00	0.00
$T_B + 1$	1.50	0.74	0.42	0.08	0.00	0.02	0.00	0.00	0.00
$T_B + 2$	1.54	1.02	0.82	0.28	0.08	0.02	0.00	0.00	0.00
$T_B + 3$	1.28	1.14	0.78	0.30	0.06	0.04	0.00	0.00	0.00
$T_B + 4$	1.32	0.98	0.92	0.32	0.18	0.00	0.00	0.00	0.00
$> T_B + 4$	42.84	34.76	28.98	14.90	6.80	3.12	0.30	0.04	0.00

Table 18: Relative frequency of estimated break points, in percent

modified IO-model (1), $\hat{T}_B = \arg\max \lvert t_{\hat{\theta}} \rvert$									
θ	0	2	2.5	4	5	6	8	10	20
$< T_B - 4$	44.48	33.46	25.70	6.06	1.16	0.08	0.00	0.00	0.00
$T_B - 4$	1.24	0.98	0.84	0.18	0.06	0.00	0.00	0.00	0.00
$T_B - 3$	1.18	0.96	0.74	0.08	0.04	0.00	0.00	0.00	0.00
$T_B - 2$	1.44	0.96	0.86	0.22	0.10	0.00	0.00	0.00	0.00
$T_B - 1$	1.58	0.92	0.94	0.28	0.00	0.02	0.00	0.00	0.00
T_B	1.36	23.78	39.28	85.44	96.96	99.82	100.00	100.00	100.00
$T_B + 1$	1.00	1.24	1.36	0.62	0.18	0.04	0.00	0.00	0.00
$T_B + 2$	1.04	1.42	1.34	0.40	0.10	0.00	0.00	0.00	0.00
$T_B + 3$	1.12	1.22	1.18	0.28	0.16	0.00	0.00	0.00	0.00
$T_B + 4$	1.12	1.26	1.32	0.40	0.08	0.00	0.00	0.00	0.00
$> T_B + 4$	44.44	33.80	26.44	6.04	1.16	0.04	0.00	0.00	0.00

Table 19: Power and adjusted power, $\rho = 0.8$, 5 percent significance level, IO-model (2)

		standard		modified			
	$\hat{T}_B$	$\arg\max \lvert t_{\hat{\gamma}} \rvert$		$\arg\max \lvert t_{\widehat{(\gamma+\theta)}} \rvert$		$\arg\max F_{\hat{\gamma}.\hat{\theta}}$	
θ	γ	power	power$_{adj}$	power	power$_{adj}$	power	power$_{adj}$
0	0	28.52	28.52	38.34	38.34	27.40	27.40
	0.5	35.30	27.78	15.38	20.30	21.52	23.00
	1	49.02	20.84	14.12	20.56	24.92	18.18
	2	92.78	10.04	16.02	19.58	31.78	10.46
	4	100.00	4.66	22.14	29.30	20.18	11.88
	10	100.00	4.72	33.00	34.74	10.14	36.48
2	0	23.86	22.00	33.92	33.10	24.14	24.90
	0.5	38.48	25.94	20.88	25.50	20.36	22.86
	1	52.82	20.44	23.12	27.26	20.08	19.08
	2	93.46	11.66	26.92	31.08	19.82	12.86
	4	99.94	5.48	30.32	32.70	14.72	31.72
	10	100.00	6.10	33.58	37.24	11.72	34.74
4	0	20.32	10.96	33.28	33.64	15.40	22.86
	0.5	53.30	25.34	32.02	34.88	15.20	26.94
	1	62.86	20.84	32.48	37.26	14.00	28.22
	2	93.92	15.60	32.94	36.84	13.34	29.94
	4	100.00	7.18	34.54	38.64	13.48	35.98
	10	100.00	7.60	34.94	37.40	11.98	35.18
6	0	22.54	8.70	34.18	35.82	10.92	36.60
	0.5	77.22	24.98	33.34	32.90	10.88	35.12
	1	81.28	22.72	33.26	36.48	10.16	35.60
	2	95.12	21.06	33.06	36.08	10.22	36.40
	4	100.00	9.42	34.20	36.20	10.60	35.34
	10	100.00	8.08	34.24	36.36	9.96	35.38
8	0	30.40	11.24	34.14	38.84	10.64	37.54
	0.5	94.62	22.00	34.54	39.30	10.42	37.02
	1	94.84	21.06	33.68	34.74	9.80	34.62
	2	97.54	22.16	34.28	35.76	9.66	36.60
	4	100.00	18.02	33.96	36.88	10.16	35.86
	10	100.00	9.32	34.06	36.34	9.90	35.30
10	0	38.76	13.44	34.16	36.06	9.06	35.94
	0.5	98.94	22.74	34.50	36.90	10.54	37.62
	1	99.00	20.76	34.04	36.36	9.70	35.38
	2	99.44	21.74	34.70	35.62	9.28	33.12
	4	100.00	21.06	34.12	38.66	9.80	36.72
	10	100.00	11.68	34.02	36.20	10.48	36.76

Table 20: Relative frequency of estimated break points, in percent

standard IO-model (2), $\hat{T}_B = \arg\max \lvert t_{\hat{\gamma}} \rvert$												
T_B		< -4	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$> +4$
θ	γ											
0	0	44.06	0.90	0.88	0.94	1.08	0.58	0.78	0.94	0.88	0.90	48.06
	0.5	69.00	7.34	5.34	3.24	1.90	2.24	2.08	1.94	1.58	1.16	4.18
	1	80.94	8.96	4.50	2.24	0.40	0.80	0.64	0.64	0.42	0.18	0.28
	2	88.46	7.72	3.22	0.48	0.04	0.02	0.00	0.06	0.00	0.00	0.00
	4	93.38	5.50	1.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	73.88	21.26	4.82	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0	42.52	0.64	0.90	0.96	2.84	1.28	0.64	0.94	0.78	0.98	47.52
	0.5	74.02	2.60	1.62	1.90	12.60	2.56	1.06	0.82	0.76	0.60	1.46
	1	89.92	3.18	1.04	0.30	3.96	0.52	0.44	0.34	0.10	0.10	0.10
	2	94.38	4.32	1.02	0.00	0.28	0.00	0.00	0.00	0.00	0.00	0.00
	4	95.38	4.04	0.50	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.06
	10	76.28	20.32	3.38	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0	36.82	0.38	0.36	1.18	10.40	2.66	0.80	0.66	1.02	1.14	44.58
	0.5	42.18	0.74	1.14	2.34	51.56	1.06	0.14	0.18	0.22	0.12	0.32
	1	67.00	0.76	0.26	0.42	31.16	0.20	0.12	0.06	0.00	0.00	0.02
	2	90.90	1.68	0.26	0.02	7.10	0.04	0.00	0.00	0.00	0.00	0.00
	4	96.26	3.34	0.30	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
	10	78.22	18.60	3.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0	31.22	0.26	0.32	0.68	19.30	2.56	0.92	1.10	1.16	1.50	40.98
	0.5	12.72	0.70	0.62	1.76	83.66	0.42	0.00	0.02	0.04	0.00	0.06
	1	24.90	0.14	0.06	0.40	74.48	0.02	0.00	0.00	0.00	0.00	0.00
	2	58.58	0.36	0.02	0.00	41.04	0.00	0.00	0.00	0.00	0.00	0.00
	4	93.40	2.48	0.12	0.00	4.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	79.16	18.14	2.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0	22.96	0.38	0.22	0.36	28.70	3.08	1.14	1.40	1.78	1.96	38.02
	0.5	2.14	0.20	0.20	0.38	97.04	0.04	0.00	0.00	0.00	0.00	0.00
	1	4.52	0.02	0.14	0.14	95.18	0.00	0.00	0.00	0.00	0.00	0.00
	2	19.10	0.00	0.00	0.00	80.90	0.00	0.00	0.00	0.00	0.00	0.00
	4	68.36	0.68	0.00	0.00	30.96	0.00	0.00	0.00	0.00	0.00	0.00
	10	79.58	17.80	2.52	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
10	0	15.50	0.20	0.16	0.06	37.86	2.62	1.34	2.10	2.08	2.88	35.20
	0.5	0.36	0.02	0.08	0.08	99.46	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.58	0.00	0.02	0.02	99.38	0.00	0.00	0.00	0.00	0.00	0.00
	2	2.94	0.00	0.00	0.00	97.06	0.00	0.00	0.00	0.00	0.00	0.00
	4	26.28	0.08	0.00	0.00	73.64	0.00	0.00	0.00	0.00	0.00	0.00
	10	78.78	16.82	1.48	0.00	2.92	0.00	0.00	0.00	0.00	0.00	0.00

Table 21: Relative frequency of estimated break points, in percent

	modified IO-model (2), $\hat{T}_B = \arg\max \lvert t_{\widehat{(\gamma+\theta)}} \rvert$											
T_B		< -4	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$> +4$
θ	γ											
0	0	43.90	1.10	1.18	1.32	1.30	1.12	1.56	1.42	1.00	1.14	44.96
	0.5	26.48	1.08	0.98	0.76	1.00	1.72	2.00	2.28	2.12	2.94	58.64
	1	18.16	0.66	0.70	0.74	0.52	3.46	5.10	7.26	7.52	7.40	48.48
	2	4.48	0.18	0.16	0.28	0.22	9.52	17.54	20.76	17.54	11.24	18.08
	4	0.06	0.00	0.04	0.00	0.00	29.98	41.74	22.82	4.56	0.46	0.34
	10	0.00	0.00	0.00	0.00	0.00	92.96	7.04	0.00	0.00	0.00	0.00
2	0	33.42	0.82	1.04	1.00	1.00	23.70	1.38	1.44	1.08	1.10	34.02
	0.5	17.18	0.42	0.44	0.50	0.68	34.52	2.36	2.72	2.90	2.84	35.44
	1	8.86	0.32	0.38	0.18	0.44	46.22	4.96	5.68	5.68	4.52	22.76
	2	0.16	0.00	0.12	0.04	0.02	64.72	10.08	9.26	6.84	3.26	4.06
	4	0.04	0.00	0.00	0.00	0.00	83.00	12.50	3.86	0.58	0.00	0.02
	10	0.00	0.00	0.00	0.00	0.00	99.82	0.18	0.00	0.00	0.00	0.00
4	0	5.60	0.12	0.32	0.24	0.30	85.64	0.58	0.34	0.36	0.46	6.04
	0.5	1.66	0.08	0.18	0.12	0.06	90.70	0.70	0.98	0.58	0.54	4.40
	1	0.72	0.04	0.04	0.06	0.00	94.36	0.90	0.90	0.64	0.72	1.62
	2	0.10	0.00	0.00	0.00	0.00	97.50	1.12	0.68	0.32	0.12	0.16
	4	0.00	0.00	0.00	0.00	0.00	99.38	0.56	0.06	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
6	0	0.08	0.02	0.00	0.02	0.00	99.70	0.02	0.04	0.00	0.02	0.10
	0.5	0.06	0.00	0.00	0.00	0.00	99.84	0.02	0.00	0.00	0.00	0.08
	1	0.00	0.00	0.00	0.00	0.00	99.92	0.04	0.00	0.02	0.00	0.02
	2	0.00	0.00	0.00	0.00	0.00	99.96	0.02	0.02	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
8	0	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	0.5	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
10	0	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	0.5	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00

Table 22: Relative frequency of estimated break points, in percent

		modifed IO-model (2), $\hat{T}_B = \arg\max F_{\hat{\gamma}.\hat{\theta}}$										
T_B		< -4	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$> +4$
θ	γ											
0	0	44.90	1.42	1.28	1.60	1.16	1.36	1.46	1.20	1.04	1.16	43.42
	0.5	43.54	4.44	3.94	4.32	4.94	4.48	4.14	4.20	4.40	3.72	17.88
	1	37.96	6.32	5.76	6.82	10.86	10.02	7.78	5.02	3.40	2.74	3.32
	2	20.40	8.30	6.70	8.32	22.84	23.30	7.00	2.22	0.62	0.24	0.06
	4	3.76	3.82	3.44	2.88	40.52	45.02	0.54	0.02	0.00	0.00	0.00
	10	0.00	0.00	0.02	0.10	50.88	49.00	0.00	0.00	0.00	0.00	0.00
2	0	33.98	1.66	2.02	2.78	4.24	18.58	1.04	1.02	1.04	0.98	32.66
	0.5	36.08	2.76	3.18	4.76	9.22	33.48	1.34	1.26	1.48	1.44	5.00
	1	23.02	3.14	3.48	4.78	14.12	47.04	1.24	1.40	0.76	0.32	0.70
	2	10.58	2.64	2.34	3.96	21.04	58.88	0.30	0.20	0.06	0.00	0.00
	4	1.34	0.76	0.74	0.74	26.54	69.88	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	28.68	71.32	0.00	0.00	0.00	0.00	0.00
4	0	7.74	0.94	1.12	1.64	4.12	76.74	0.04	0.10	0.20	0.20	7.16
	0.5	7.34	0.82	0.72	1.38	5.30	83.44	0.10	0.10	0.08	0.18	0.54
	1	3.90	0.34	0.54	1.36	5.88	87.86	0.02	0.04	0.06	0.00	0.00
	2	1.40	0.22	0.24	0.70	6.22	91.22	0.00	0.00	0.00	0.00	0.00
	4	0.08	0.12	0.02	0.12	7.32	92.34	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	7.42	92.58	0.00	0.00	0.00	0.00	0.00
6	0	0.30	0.06	0.08	0.26	1.12	98.04	0.00	0.00	0.00	0.02	0.12
	0.5	0.38	0.02	0.10	0.16	0.90	98.44	0.00	0.00	0.00	0.00	0.00
	1	0.24	0.02	0.08	0.18	1.10	98.38	0.00	0.00	0.00	0.00	0.00
	2	0.12	0.04	0.00	0.04	1.10	98.70	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	1.22	98.78	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	1.10	98.90	0.00	0.00	0.00	0.00	0.00
8	0	0.00	0.00	0.00	0.06	0.18	99.76	0.00	0.00	0.00	0.00	0.00
	0.5	0.02	0.00	0.00	0.00	0.22	99.76	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.02	0.00	0.40	99.58	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.24	99.76	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.28	99.72	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.18	99.82	0.00	0.00	0.00	0.00	0.00
10	0	0.00	0.00	0.00	0.00	0.02	99.98	0.00	0.00	0.00	0.00	0.00
	0.5	0.00	0.00	0.00	0.00	0.06	99.94	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.02	99.98	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.04	99.96	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.04	99.96	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.16	99.84	0.00	0.00	0.00	0.00	0.00