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Abstract

This paper analyzes the dynamic effects of anticipated and unanticipated foreign price increases of imported raw materials for a small two-country monetary union, which is simultaneously characterized by asymmetric wage adjustments and asymmetric interest rate sensitivities of private absorption. It is shown that both types of input price disturbances lead to temporary divergences in output developments across the monetary union. In the case of anticipated materials price increases the relative cyclical position of output effects is reversed in the course of the adjustment process. With anticipated input price increases complete stabilization of the output variables throughout the overall adjustment process requires restrictive monetary policy to be time-inconsistent from a quantitative but time-consistent from a qualitative point of view. That means that the central bank credibly announces a future reduction in the growth rate of nominal money stock but actually realizes a decrease in the monetary growth rate, which is less restrictive than the announcement.

JEL classification: E63, F41
Keywords: EMU, international policy transmission, oil price shock, time inconsistency

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1. Introduction

This paper analyzes the impact of foreign price shocks in an asymmetric monetary union and their implications for monetary policy. We concentrate on the dynamic effects of anticipated and unanticipated price increases of imported raw materials. The analysis is motivated by the establishment of the European Monetary Union (EMU) in 1999 and by the substantial increase in oil prices in 2000. The establishment of EMU introduced a common monetary policy by the European Central Bank (ECB) and brought about some convergence in the transmission patterns in the member countries. Nevertheless, important asymmetries in the macroeconomic structures remain.[1] While most studies focus on asymmetries in the financial sphere such as *Clausen* and *Wohltmann* (2002), there are also important asymmetries in goods and labor markets. These macroeconomic asymmetries imply that symmetric shocks such as a common oil price shock cause asymmetric developments in output and inflation within the monetary union.

The structure of the model is as follows: the monetary union consists of two countries. Our paper allows the monetary union to be simultaneously characterized by structural asymmetries on the demand and supply side. Our basic setup draws on the tradition of the overshooting model by *Dornbusch* (1976) where prices and wages are sticky and exchange rate expectations are rational. The model exhibits saddle path stability. The model can be viewed as an IS/LM model extended by a supply side with (1) a *Phillips* curve based on rational expectations of prices and exchange rates and (2) imported raw materials in domestic production and (3) a long-run supply function. We examine the short-run, medium-run and long-run effects of anticipated and unanticipated price increases in raw materials on both member countries. This serves as a background for the analysis of monetary policy. We assume that the primary goal of monetary policy is price stability and investigate to which extent the ECB may be able to dampen the differential cyclical developments within the monetary union caused by the common external price shock. To summarize, the novel features of our approach are that (1) we consider simultaneously in a dynamic model macroeconomic asymmetries on the demand and supply side (2) we introduce imported intermediate goods and (3) we investigate the implications for monetary policy. The implications of this paper go well beyond the role of imported raw materials in an open economy. In fact, it can also be applied to all intermediate goods produced abroad and used for domestic production. In view of the globalization of supply chains the importance of this factor has recently increased.

[1] See *Cecchetti* (1999), *Clausen* (2001 and 2002) and *Wohltmann* and *Clausen* (2001).

The paper is organized as follows: *Section 2* introduces our model. *Section 3* analyzes the dynamic effects of anticipated and unanticipated price increases in imported raw materials. *Section 4* discusses the appropriate response by monetary policy to these price shocks. *Section 5* introduces a modification to the model by moving from price rigidity to the assumption of price flexibility in the monetary union. *Section 6* summarizes our main results. The paper includes a mathematical appendix, which contains a detailed derivation of the analytical solution to our dynamic macroeconomic model.

2. A model of a monetary union

We consider a monetary union, which consists of two member countries U_1 and U_2. The member countries are identical in size and the monetary union as a whole is considered small relative to the rest of the world. Both member countries require imported raw materials in domestic production.

(1) $q_1 = (a_{01} + a_1 y_1 - a_{21}(i_1 - \dot{p}_1^c)) + g_1 + (b_0 - b_1 y_1 + b_2 y_2 + b_3 y^* - b_4(p_1 - p_2) - b_5 \tau_1)$

(2) $q_2 = (a_{02} + a_1 y_2 - a_{22}(i_2 - \dot{p}_2^c)) + g_2 + (b_0 - b_1 y_2 + b_2 y_1 + b_3 y^* - b_4(p_2 - p_1) - b_5 \tau_2)$

(3) $\tau_1 = p_1 - (p^* + e)$

(4) $\tau_2 = p_2 - (p^* + e)$

(5) $m = (p_1^c + l_0 + l_1 q_1 - l_2 i_1) + (p_2^c + l_0 + l_1 q_2 - l_2 i_2)$

(6) $i_1 = i_2 = i^* + \dot{e}$

(7) $y_1 = q_1 - \psi(p_R^* + e - p_1) - c_0$

(8) $y_2 = q_2 - \psi(p_R^* + e - p_2) - c_0$

(9) $\dot{p}_1 = \mu \dot{w}_1 + (1-\mu)(\dot{p}_R^* + \dot{e}) \qquad (0 < \mu < 1)$

(10) $\dot{p}_2 = \mu \dot{w}_2 + (1-\mu)(\dot{p}_R^* + \dot{e})$

(11) $\dot{w}_1 = \pi_1 + \delta(q_1 - \overline{q}_1) \qquad (\pi_1 = \frac{1}{2}\dot{m})$

(12) $\dot{w}_2 = \pi_2 + \delta(q_2 - \overline{q}_2) \qquad (\pi_2 = \dot{p}_2^c)$

(13) $p_1^c = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 (p^* + e) \qquad (\alpha_1 + \alpha_2 + \alpha_3 = 1)$

(14) $p_2^c = \alpha_1 p_2 + \alpha_2 p_1 + \alpha_3 (p^* + e)$

(15) $\overline{q}_1 = f_0 + f_1 \overline{\tau}_1 + f_2 (\overline{p_1 - p_2}) + f_3 (\overline{p_1 - (p_R^* + e)})$

(16) $\overline{q}_2 = f_0 + f_1 \overline{\tau}_2 + f_2 (\overline{p_2 - p_1}) + f_3 (\overline{p_2 - (p_R^* + e)})$

The notation is as follows:[2] q = real output, y = real income, i = nominal interest rate, $i - \dot{p}^c$ = real interest rate, g = real government expenditure, p = producer price level of the domestically produced good, τ = final goods terms of trade, e = nominal exchange rate of the union with respect to the rest of the world expressed in units of domestic currency for one unit of foreign currency, m = nominal money stock in the union[3], w = nominal wage, π = trend or core rate of inflation, p^c = consumer price index, $\overline{q}$ = steady state level of q (natural level of output), p_R^* = price of the imported raw material or intermediate product, $p_R^* + e - p$ = real raw materials price, $p - (p_R^* + e)$ = intermediate goods terms of trade.

The IS equations (1) and (2) describe aggregate demand in the two member countries of the monetary union. Goods market equilibrium in country U_j (j = 1, 2) requires domestic production (q_j) to equal the sum of real private absorption (first expression in brackets), real government expenditure (g_j) and the difference between the real ex- and import of final goods (second expression in brackets). Real private absorption is assumed to depend negatively on the real interest rate and international trade in final goods (trade balance excluding imports of raw materials) negatively with respect to the terms of trade. Equations (3) and (4) define the terms of trade with respect to final goods. The IS equations are assumed symmetric except for different semi-interest elasticities of private absorption. We assume private absorption in U_1 to respond more strongly to interest rates compared to U_2 ($a_{21} > a_{22}$).

The LM equation (5) reflects money market equilibrium in the monetary union. Money demand in country U_j is assumed to depend on domestic production q_j, which is considered a more appropriate measure of the volume of transactions than real income y_j. The nominal money stock m is deflated by the consumer price indices (13) and (14) to allow for the fact that in open economies money is also used for the purchase of imported goods. Perfect capital mobility within the monetary union as well as between the monetary union and the rest of the world implies uncovered interest parity (6). Interest rates within the union are equalized and

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[2] All variables – except for the interest rates i_1, i_2 und i^* - are logarithmized. The variables of member country U_1 (U_2) are labeled with the lower case index „1“ („2“). Variables with a „*“ describe foreign variables. A dot above a variable indicates the right-hand side derivative with respect to time; a bar indicates a steady state or long-run equilibrium value of the respective variable. All structural parameters in the equations (1)-(16) are taken to be positive. They can be interpreted as elasticities or semi-elasticities.

[3] The monetary aggregate m is derived from the underlying LM equation through log-linearization. Denoting the underlying levels of the aggregate money stock with M and of money demand with $P_j^c L_j (j = 1,2)$, money market equilibrium implies $M = P_1^c L_1 + P_2^c L_2$. Provided that the initial values of money demand are identical in both member countries and using the approximation $dM / M_0 \approx \ln M - \ln M_0$ it is possible to derive $m = 2(\ln M - \ln 2)$.

may deviate only by the rationally anticipated rate of depreciation $\dot{e}$ from the foreign interest rate i^*.

The equations (7) and (8) link domestic production q_j with real income or gross national product y_j. The difference originates from real intermediate imports. Real imports of raw materials (intermediate goods) can be expressed in non-logarithmized form as the product of the real price of raw materials ($P_R^* \cdot E / P_j$) and the physical import R_j.[4] Assuming for simplicity a proportional relationship between the quantity R_j and the (non-logarithmized) level of domestic production Q_j of the form $R_j = æ \cdot Q_j \quad (æ > 0)$,[5] we derive after a logarithmic-linear approximation the equations (7) and (8) where $\psi = æ / (1 - æ)$ provided that the initial value of the intermediate goods terms of trade $P_j / (P_R^* \cdot E)$ is normalized to unity [*Wohltmann* 1993]. [6]

The equations (9) and (10) describe price adjustment within the monetary union. The rates of inflation in U_j are determined by a weighted average of domestic nominal wage inflation and of the rate of change of the real price of raw materials expressed in domestic currency. The corresponding weights μ (1-μ) reflect the average shares of wage costs (raw materials costs) in the overall variable costs of a representative firm.[7] The equations (9) and (10) are dynamic versions of mark up-price setting [*Buiter* 1979].

The equations (11) and (12) describe the dynamics of wage adjustment in the monetary union. The rate of change of wages in U_j is determined by an expectations-augmented *Phillips*-curve where inflationary expectations are given by the trend or core rate of inflation (π_j).[8] We assume that wage setters in both member countries base their decisions on different time horizons. In U_1, the trade unions take a long-run perspective. Their rate of nominal wage increases is linked to the rate of growth of the money stock in the monetary union, which in turn determines the *long-run* rate of inflation. In U_2, wage setting is based on the rationally anticipated *short-run* development of consumer price inflation based on (14). Furthermore, both wage setting equations contain a component reflecting wage pressure, which is modeled according to *Okun's law* using the output gap, i.e., the difference between actual and potential

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4 In this paper, capital letters generally indicate non-logarithmized variables.

5 See for more details also *Findlay* and *Rodriguez* (1977) and *Buiter* (1978).

6 In production functions, which allow for factor substitution, the constant ψ depends on the elasticity of substitution between factors of production and from the share of imported inputs in aggregate production. See *Bhandari* and *Turnovsky* (1984).

7 The expression 1-μ can be interpreted as a measure of the degree of openness of the economy on the *supply* side (*Bhandari* and *Turnovsky* 1984). In contrast, the parameter α_3 in the price adjustment equations (13) and (14) reflect the openness of the economy with respect to the large foreign country on the *demand* side.

8 See for more details also *Buiter* und *Miller* (1982) and *van der Ploeg* (1990).

output.[9] Without this wage pressure component ($\delta = 0$), the trade unions show insider-outsider-behavior and the wage equation in U_1 corresponds with *nominal* wage rigidity while wage setting in U_2 can be interpreted as *real* wage rigidity based on the consumer price index.[10] Nominal wages are therefore more flexible in U_2 than in U_1. In U_1, nominal wages only respond to the output gap as long as the rate of monetary growth remains unchanged. In U_2, nominal wage changes $\dot{w}_2$ react via π_2 to anticipated changes in the consumer price index. Consumer price inflation includes the anticipated rate of depreciation $\dot{e}$, which, in turn, immediately responds to exogenous shocks. In consequence, the rate of change of nominal wages in U_2 responds directly to the anticipated rate of depreciation $\dot{e}$. In U_1, the rate of change is determined by the constant rate of growth of money stock. In contrast, producer price inflation $\dot{p}_1$ and $\dot{p}_2$ responds in both member countries directly to $\dot{e}$. This can be directly seen from the price adjustment equations (9) and (10).

Equations (15) and (16) describe long-run supply functions within the monetary union. In the long-run assuming labor market equilibrium, a neoclassical production function and a perfectly elastic supply of raw materials, goods supply depends positively on the final and intermediate goods terms of trade and on the internal price differential (internal terms of trade).[11]

In summary, our monetary union is assumed completely symmetric except for exactly one asymmetry in both, the demand and the supply side. On the demand side, U_1 is characterized by a relatively higher interest sensitivity of private absorption. On the supply side, U_2 is characterized by a relatively higher degree of nominal wage flexibility. We show in the following that as a result of these asymmetries foreign price shocks generate adjustment dynamics in both member countries, which display considerable quantitative as well as qualitative differences within the monetary union.

Using the solution method of *Aoki* (1981), the model is decomposed into an aggregate and a difference system. The aggregate system is derived by adding corresponding equations of U_1 and U_2. It describes the behavior of the monetary union as a whole. The difference system is derived by the subtraction of corresponding equations of U_1 and U_2. It describes the differential developments within the monetary union. The method by *Aoki*, which was predomi-

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9 See *Claassen* (1980).

10 The *OECD* (2000) finds that the EMU member countries differ considerably in their degrees of wage flexibility. For example, the degree of nominal wage flexibility in France is relatively small while being relatively large in Germany (see table 11, p. 108). In contrast, the impact of the rate of unemployment on real wages has a similar magnitude in both countries. As long as the monetary union is assumed to consist of France (U_1) and Germany (U_2) the parameter δ can be assumed to be identical in (11) and (12).

11 A more detailed theoretical derivation of the role of the terms of trade in aggregate supply is given in *Wohltmann* and *Bulthaupt* (1999) and *Devereux* and *Purvis* (1980). The supply equations (15) and (16) can also be derived by assuming static price and wage equations with full wage indexation in the long-run.

nantly applied to symmetric models, can only be used in our asymmetric model when we assume identical weights α_1 and α_2 in the consumer price definitions (13) and (14), i.e., that consumers in the monetary union are indifferent between domestically produced goods and imports from the respective partner country. The simplifying assumption $\alpha_1=\alpha_2$ may be justified for highly integrated monetary unions and allows us to solve the aggregate system independently from the difference system. With $\alpha_1=\alpha_2$, the price indices p_1^c and p_2^c are identical. In conjunction with the uncovered interest parity (6) it follows that real interest rates are equalized within the monetary union ($i_1 - \dot{p}_1^c = i_2 - \dot{p}_2^c$). In contrast, the producer price levels p_1 and p_2 and the corresponding rates of producer price inflation $\dot{p}_1$ and $\dot{p}_2$ differ across the monetary union. The arithmetic mean of the solutions to the aggregate and the difference system yields the solution paths for the output and price developments in the individual member countries. The detailed analytical derivations are provided in the mathematical appendix.

3. Effects of a Price Increase in Imported Raw Materials

This section analyzes the dynamic effects of anticipated and unanticipated increases in the price of imported raw materials ($\mathrm{d}p_R^* > 0$) on both member countries. In the case of an anticipated increase in p_R^*, we assume that the price increase is credibly announced in $t = 0$ and happens at the later date $T > 0$. An unanticipated price shock comes as a surprise to the private sector and happens already at $t = 0$. Both types of shocks have identical effects in the steady state but differ in their adjustment dynamics.

Effects on the Aggregate Monetary Union

The *long-run* or *steady state* effects of a price increase in imported raw materials result from the equilibrium condition $\dot{\tau} = 0 = (\dot{m} - \dot{p}) = \dot{p}_1 - \dot{p}_2$ for the dynamics of the aggregate and the difference system.[12] In long-run equilibrium, member country outputs q_1 and q_2 equal their long-run levels $\bar{q}_1$ and $\bar{q}_2$ according to the long-run supply functions (15) and (16). An increase in the price of imported raw materials leads in the monetary union to a decline in aggregate union output $\bar{q} = \bar{q}_1 + \bar{q}_2$, which is evenly distributed across the monetary union

[12] The long-run is considered as the imaginary period in which the dynamic adjustments of the aggregate and the difference system are completed. In the case of the aggregate variable we have τ and p: $\tau = \tau_1 + \tau_2$, $p = p_1 + p_2$.

($\mathrm{d}\overline{q}_1 = \mathrm{d}\overline{q}_2 < 0$).[13] It follows that asymmetries in wage setting or in the interest sensitivities of aggregate demand do not affect the ultimate impact on member country outputs $\overline{q}_1$ and $\overline{q}_2$. One reason is that in our model producer price inflation $\dot{p} = \dot{p}_1 + \dot{p}_2$, the rate of depreciation $\dot{e}$ and consumer price inflation $\dot{p}^c = \dot{p}_1{}^c + \dot{p}_2{}^c$ are ultimately only determined by the rate of monetary growth in the union ($\dot{m} = \dot{p} = 2\dot{e} = \dot{p}^c$). The second reason is that foreign price shocks of the form $\mathrm{d}p_R^* > 0$ leave nominal and real interest rates as well as the internal price differential p_1-p_2 ultimately unaffected. In contrast, the external terms of trade $\tau = \tau_1 + \tau_2$ change in the long run. Provided that

$$(17) \qquad b_5 > (a_1 - b_1 + b_2)\psi > f_3 \cdot \lambda \qquad (\lambda = 1 - a_1 + b_1 - b_2 > 0)$$

holds, the price increase in imported raw materials leads to a fall in the aggregate terms of trade ($\mathrm{d}\overline{\tau} / \mathrm{d}p_R^* < 0$).[14] The first inequality in (17) ensures that the *long-run* IS curve in $\overline{\tau} / \overline{q}$-diagram has a „normal“ negative slope[15], while the second inequality ensures that the long-run IS curve in *Figure 1* moves in response to $\mathrm{d}p_R^* > 0$ further to the left than the upward-sloping long-run aggregate supply curve (AS).[16]

The increase in the price of raw materials $\mathrm{d}p_R^* > 0$ normally leads to a permanent decline in the real money stock ($\mathrm{d}(\overline{m - p}) / \mathrm{d}p_R^* < 0$). Provided that the nominal money stock is con-

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[13] In contrast, an isolated increase in the price of imported final goods ($\mathrm{d}p^* > 0$) leads to an increase of $\overline{q}$. In this case, $\mathrm{d}\overline{q} / \mathrm{d}p^* = -\mathrm{d}\overline{q} / \mathrm{d}p_R^*$ holds.

[14] An increase in p^* leads to $\mathrm{d}\overline{\tau} / \mathrm{d}p^* = -\mathrm{d}\overline{\tau} / \mathrm{d}p_R^* > 0$ provided that (17) holds. The nominal exchange rate e can only achieve a long-run equilibrium as long as the nominal money stock in the union remains constant ($\dot{m} = 0$). In this case we find, given (17), $\mathrm{d}\overline{e} / \mathrm{d}p_R^* > 0$ and, respectively, $\mathrm{d}\overline{e} / \mathrm{d}p^* < 0$, i.e., an increase in the price of imported raw materials (final goods) leads to a real and a nominal depreciation (appreciation) of the common currency. Due to $\overline{y} = \overline{q} + \psi \cdot \overline{\tau} + 2\psi(p^* - p_R^*) - 2c_0$ we also find in the case $\mathrm{d}p_R^* > 0$ ($\mathrm{d}p^* > 0$) an overproportionate fall (rsp. increase) in real income $\overline{y} = \overline{y}_1 + \overline{y}_2$ relative to real output $\overline{q}$.

[15] The assumption of a „normal“ reaction of the trade balance to changes in the final goods terms of trade τ is insufficient because an increase in τ raises due to $y = q + \psi \cdot \tau + 2\psi(p^* - p_R^*) - 2c_0$ private absorption holding other factors constant.

[16] Using a CES production function, the parameter f_3 can be linked to the variable factors labor and imported raw materials as follows $f_3 = (\beta + \sigma)(1 - \mu) / \mu$ [*Wohltmann* und *Bulthaupt* (1999)]; where β indicates the real-wage elasticity of the labor supply and σ the elasticity of substitution. The second inequality in (17) holds provided that the sum of both elasticities is sufficiently small.

stant ($\dot{m} = 0$) this is equivalent to a permanent increase in the aggregate producer price level $p = p_1 + p_2$.[17]

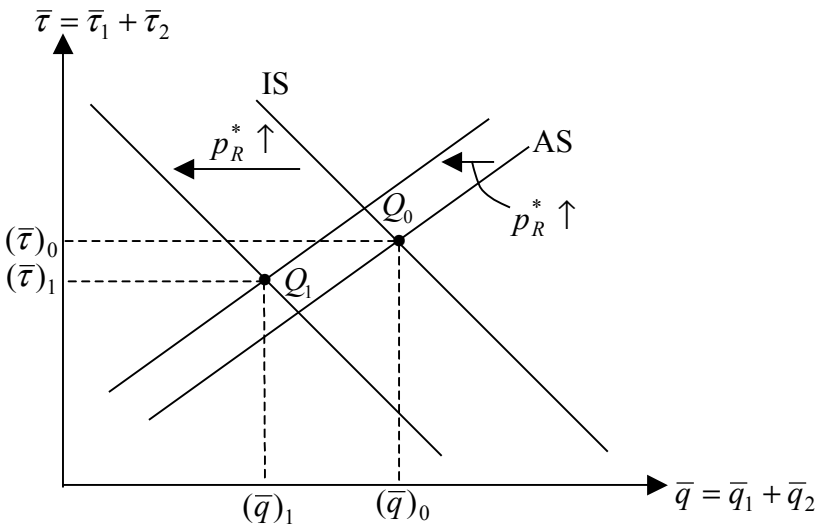

Figure 1: Long-Run Effects of a Price Increase in Raw Materials

The *dynamics of adjustment* of the aggregate monetary union to a price increase in imported raw materials can be illustrated in a phase diagram including the state variables τ and *m-p* (*Figure 2*).[18] The points Q_0 and Q_1 denote the initial and the final equilibrium of the aggregate system and S_0 and S_1 the corresponding convergent saddle paths. The dynamic adjustment of the state vector (τ , *m-p*) in response to an anticipated materials price increase shows on impact a discontinuous jump from Q_0 to *B* (or *B*'). It is followed by a movement along a trajectory, which converges asymptotically to the unstable branch I_0 belonging to the saddle point Q_0 (*BC* or *B'C*'). After the implementation in *T*, the system moves along the unique convergent saddle path S_1 to the new long-run equilibrium Q_1.[19]

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[17] We find $\mathrm{d}(\overline{m-p}) / \mathrm{d}p_R^* < 0$ if and only if $\alpha_3(a_1 - b_1 + b_2)\psi < (l_1 b_5 + \alpha_3 \lambda) f_3 + f_1 l_1 (a_1 - b_1 + b_2)\psi$. This condition holds if $\alpha_3 - f_1 l_1 < 0$ or b_5 is sufficiently large.

[18] We use the real variables τ and *m-p* as state variables in the phase diagram because steady state values of the nominal exchange rate *e* and the aggregate price level *p* do not exist in the presence of a positive rate of growth of the money stock. The terms of trade τ are taken as a jump variable as it includes the forward-looking variable *e*. As a result of price rigidity, the producer-oriented real money stock *m-p* is taken as predetermined. In contrast, the consumer-oriented real money stock $m\text{-}p^c$ is a non-predetermined variable because the aggregate consumer price index $p^c = p_1^c + p_2^c$ includes the jump variable *e*.

[19] It is assumed here that $\mathrm{d}\bar{\tau} / \mathrm{d}p_R^* < 0$ and $\mathrm{d}(\overline{m-p}) / \mathrm{d}p_R^* < 0$, which prevail as long as the weak condition (17) holds and provided that b_5 is sufficiently large. Moreover, it is assumed that Q_1 remains below the un-

The anticipation of a future increase in the price of imported raw materials leads on impact to a real depreciation of the common currency. It equals the nominal depreciation due to the assumption of short-run price rigidity. The *size* of the instantaneous fall in the terms of trade τ is inversely related to the time span T between the announcement and the actual implementation of the materials price increase. We find „undershooting“ of the final goods terms of trade in point B in the case of a long time span T and „overshooting“ in B' given a short span. In the extreme case of an unanticipated increase in p_R^* (T=0), the impact effect is given in B^* on the saddle path S_1 where the size of the impact depreciation reaches its maximum.

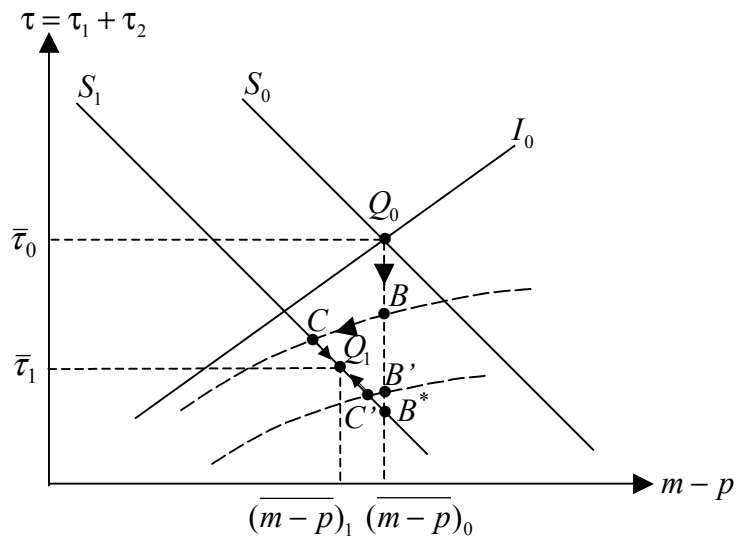

Figure 2: Effects of a Materials Price Increase on the Aggregate Monetary Union

After the immediate impact depreciation the real depreciation of the common currency continues further in the case of an anticipated price increase. This adjustment pattern differs from the case of an unanticipated price increase. Furthermore, in the adjustment process following the impact effect the real money stock *m-p* subsequently falls because the real depreciation ($\dot{\tau} < 0$) and the implicit nominal depreciation $\dot{e}$ cause nominal wage increases in U_2, which in turn raise the aggregate rate of producer inflation beyond the level determined by the initial rate of monetary growth ($\dot{p} > \dot{m}_0$). The parallel development of the terms of trade τ and of the real money stock *m-p* ends at the date of implementation T. In the case of a “large” T, i.e., that the time span between the anticipation and the implementation is relatively long,

stable branch I_0. The theoretical possibility that Q_1 is located above I_0, which arises for example with ex-

we find for $t>T$ a continuation of the real depreciation, which leads in conjunction with the discontinuous fall in the rate of inflation in T to an increase in the real money stock (adjustment path CQ_1). For "small" values of T (and, in particular, in the special case $T = 0$) we find after the implementation in T a real appreciation, which coincides with a further reduction in the real money stock (adjustment path $C'Q_1$). The adjustment dynamics ultimately arrive at the new final equilibrium Q_1 on the saddle path S_1 irrespective whether T is "small" or "large".

The aggregate real interest rate and the rate of real appreciation $\dot{\tau}$ behave inversely due to

$$(18) \qquad (i_1 - \dot{p}_1^c) + (i_2 - \dot{p}_2^c) = i - \dot{p}^c = 2i^* - (1-\alpha_3)\dot{\tau} \qquad (i = i_1 + i_2,\ \dot{p}^c = \dot{p}_1^c + \dot{p}_2^c)$$

and because the foreign interest rate i^* is considered exogenous. A real depreciation of the common currency ($\dot{\tau} < 0$) implies that real interest rates in Europe $i - \dot{p}^c$ are above their steady state level and vice versa. In *Figure 2*, the rate of real appreciation $\dot{\tau}$ is negative throughout the period up to T such that the aggregate real interest rate $i - \dot{p}^c$ exceeds its steady state level ($2i^*$).[20] The increase in $i - \dot{p}^c$ is accomplished despite the initial increase in the rate of inflation $\dot{p}^c$ by an even stronger increase in the short-term nominal interest rate i.[21] At the date of implementation T, the rate of appreciation $\dot{\tau}$ increases discontinuously, which implies a simultaneous fall in the aggregate real interest rate. For „small" T, the aggregate real interest rate remains for $t > T$ below its long run level $2i^*$, while remaining above this level for „large" T. In the long-run, the real interest rate returns to its initial steady state level ($2i^*$).

The effect on aggregate demand and output $q = q_1 + q_2$ in the period $0 < t < T$ is characterized by two opposing influences. The real depreciation of the common currency increasingly improves the unions trade balance and increases due to (17) aggregate demand. On the other hand, the increase in the aggregate real interest rate reduces real private absorption. Provided that the weak condition

$$(19) \qquad ((1+\alpha_3)\mu + 2(1-\mu))l_2(b_5 - (a_1 - b_1 + b_2)\psi) > (1-\alpha_3)\mu\, a_2\alpha_3$$

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tremely large values of b_5 or $a_2 = \frac{1}{2}(a_{21} + a_{22})$, is considered irrelevant on empirical grounds.

[20] After the impact effect, real interest rates increase continuously up to T.

[21] The rise in i follows from the uncovered interest parity (6) and the increase in the rate of depreciation $\dot{e}$.

holds, the impact effect of an anticipated price increase in imported raw materials on aggregate output $q = q_1 + q_2$ is on balance positive: $q(0+) > \bar{q}_0$.[22] Given (19), the anticipation of a future price increase in raw materials leads to an immediate expansion in aggregate output in the monetary union.[23]

The subsequent development of aggregate output up to *T* is theoretically ambiguous. On the one hand, the external trade balance successively improves following the discontinuous depreciation on impact and subsequent further reductions in the terms of trade τ .On the other hand, the real interest rate increases and lowers the interest-sensitive part of private absorption. With realistic empirical parameterizations, the contractionary real interest rate effect dominates the expansionary terms of trade effect such that aggregate output *q* falls after the initial expansion. The fall in aggregate output continues typically even below the initial equilibrium value $\bar{q}_0$ (*Figure 3*), except for very small values of *T*. In *T*, the implementation of the materials price increase leads notwithstanding the fall in real interest rates to a discontinuous fall in output, which can be explained using (7) and (8) by the fall in real income y in *T* and the implied fall in aggregate demand.[24] The *size* of the contraction in *T* does not depend on the time span between the anticipation and the implementation of the materials price increase.[25] For "small" values of *T*, union output experiences a convergence from above to the

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[22] Inequality (19) holds if the terms of trade elasticity of the trade balance (b_5) is large relative to the interest sensitivity of private absorption (a_2). In this case, the expansionary terms of trade effect dominates the contractionary real interest rate effect. Condition (19) also holds if the semi-interest elasticity of money demand (l_2) is sufficiently large. In the special case without imported raw materials ($\mu = 1, \psi = 0$) condition (19) simplifies to $(1+\alpha_3)l_2 b_5 > (1-\alpha_3)a_2\alpha_3$. This condition ensures that an anticipated increase in the rate of monetary growth $\dot{m}$ leads on impact to an increase of aggregate output [*Wohltmann* und *Clausen* (2001)]. Given a role for imported raw materials in domestic production, condition (19) ensures that an anticipated increase in $\dot{m}$ again leads to expansionary impact effect on output.

[23] This result critically depends on the "normal" reaction of the trade balance with respect to changes in the terms of trade based on final goods, i.e., that the assumption $b_5 > 0$ already holds in the *very* short-run. This assumption was already made in the overshooting model by *Dornbusch* (1976) and also in *Buiter* and *Miller* (1982). The case $b_5 < 0$ (abnormal reaction of the trade balance) leads to a positive determinant of the dynamic aggregate system and to the loss of the saddle path property of the system. Moreover, we continue to assume that domestic production responds immediately and perfectly elastic to changes in aggregate demand. Assuming instead a Lundberg lag in domestic production where supply responds sluggishly to changes in aggregate demand, output remains predetermined on impact and increases only gradually over time in response to an anticipated increase in imported raw materials.

[24] For $y = y_1+y_2$, the link with domestic production is given by $y = q + \psi\tau + 2\psi(p^* - p_R^*) - 2c_0$. Contrary to *q*, *y* falls due to $\tau(0+) < \bar{\tau}_0$ already on impact ($y(0+) < \bar{y}_0$) and also in the subsequent adjustment process. As p_R^* jumps in *T*, *y* experiences another discontinuous contraction in *T*.

[25] This result holds generally and follows from the fact that the jump variable τ remains continuous in *T*. In contrast, the size of the initial jump depends on *T*. See *Turnovsky* (2000), pp. 187 ff.

new steady state value of q which is lower than the initial equilibrium value.[26] In contrast, for sufficiently „large“ values of T, q converges to the new steady state value $\overline{q}_1$ from below.

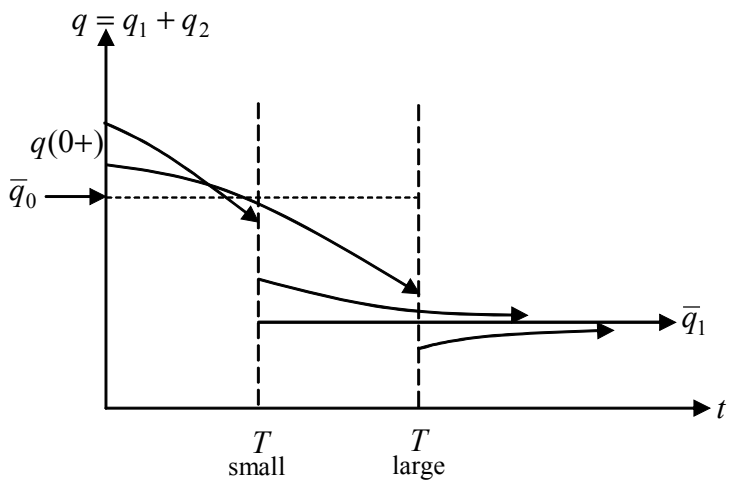

Figure 3: Aggregate output effects in the monetary union

Output effects in the individual member countries

As a consequence of the assumed asymmetries on the supply and the demand side, the cyclical developments in response to the symmetric foreign price shock $\mathrm{d}p_R^* > 0$ differ in the individual member countries. Differential output developments within the monetary union are captured by the output differential $q^d = q_1 - q_2$. The solution path of q^d in response to an anticipated price increase of imported raw materials typically looks as in *Figure 4*. Anticipating in t=0 the price increase in $T > 0$, both countries experience a discontinuous and (due to our assumption $\alpha_1 = \alpha_2$) identical increase in real interest rates. This leads in U_1 to a relatively stronger fall in aggregate demand than in U_2 because of the assumed asymmetry in the interest sensitivity of private absorption ($a_{21} > a_{22}$). The immediate real depreciation of the common currency increases the trade balance and aggregate demand symmetrically. In consequence, on impact and in the early period of adjustment output in U_2 is higher than in U_1, In other words, the output differential is on impact *negative*: $q^d(0+) < 0$. This initial cyclical differential favoring U_2 relative to U_1 changes quantitatively and even qualitatively over time in response to changes in the internal and external terms of trade. The internal terms of trade

[26] This statement also holds in the special case $T = 0$. The initial jump in q is ambiguous in sign because the shock $\mathrm{d}p_R^* > 0$ has a negative impact on y and therefore also on private absorption while on the other hand

$p^d = p_1 - p_2$ start to fall over time in response to differential inflation developments within the monetary union.[27] This improves the internal competitiveness of U_1 compared with U_2 and its bilateral trade balance within the union from the perspective of U_1. Furthermore, U_1 experiences a decline in the respective external (final goods) terms of trade τ_1 on impact and also in the subsequent adjustment process, which goes beyond the corresponding decline of τ_2 for U_2 such that the improvement of the trade balance of U_1 with respect to the large foreign country exceeds the one of U_2.[28] Viewed in isolation, the developments of the internal and external terms of trade following the impact effect result in a larger expansionary effect on q_1 than on q_2. It follows that the output differential $q^d = q_1 - q_2$ *increases* after the negative impact effect. Apart from that, the rise in real interest rates exerts a negative impact on both member countries. This effect is assumed to be stronger in U_1 than in U_2 and causes therefore a *decrease* in the output differential q^d. As long as the difference between the interest sensitivities of private absorption is sufficiently small, the terms of trade effects dominate the real interest rate effect on q^d such that on balance the output differential increases after the negative impact effect. The increase in q^d is rooted in the fall in the producer price differential p^d, whose development is driven by inflation differentials. The rate of producer price inflation in U_1 is relatively lower ($\dot{p}_1 < \dot{p}_2$) due to different wage dynamics in the anticipation period $0 < t < T$. This follows from two factors: First, the rate of consumer price inflation $\dot{p}^c$ exceeds on impact for $0 < t < T$ the constant rate of monetary growth $\dot{m}$ and increases even thereafter.[29] Second, the output gap in U_1 (i.e., the difference $q_1 - \overline{q}_1$) and the corresponding rate of wage inflation are higher than the corresponding values in U_2.

The country-specific developments in the internal and external terms of trade lead in the course of the adjustment process to a reversal in the relative cyclical impact of the price shock

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the terms of trade worsen and real interest rate falls. A realistic empirical parameterization suggests $q(0+) < \overline{q}_0$.

[27] The initial period of adjustment is characterized by $\pi_2 > \pi_1$ and $q_2 > q_1$ and following (11) and (12) $\dot{w}_2(0+) > \dot{w}_1(0+)$. The dynamic price adjustment equations (9) and (10) then imply $\dot{p}_2(0+) > \dot{p}_1(0+)$.

[28] In the period $0 < t < T$, the relationship $\tau_1 = 0{,}5(\tau + p^d)$ and the fall in τ and p^d imply that τ_1 also falls. For $\tau_2 = 0{,}5(\tau - p^d)$ we find $\tau_2 > \tau_1$ where after the initial discontinuous fall in $t = 0+$ it can not be ruled out theoretically that τ_2 subsequently increases.

[29] Not only $\dot{p}^c$ but also $\dot{p} = \dot{p}_1 + \dot{p}_2$ increases continuously in the interval $0 < t < T$. The effect on $\dot{p}$ is smaller (underproportional) than on $\dot{p}^c$ due to $\dot{p} - \dot{p}^c = \alpha_3 \dot{\tau} < 0$. In contrast, q generally falls after the (typically) positive impact effect such that as the result of the anticipation of a future price increase in imported raw materials the monetary union experiences *stagflation.*

across the monetary union. In other words, they lead to a sign change in the solution time path of the output differential, which occurs at the latest at the date of implementation T. More formally, we find a date of a *cyclical reversal* t^U with $0 < t^U \leq T$ such that $q^d < 0$ (or $q_1 < q_2$) holds for $t < t^U$ and $q^d > 0$ (or $q_1 > q_2$) holds for (at least sufficiently small) $t > t^U$. The reversal date t^U occurs before the date of implementation T, as long as the period between the anticipation and the implementation of the foreign price shock is sufficiently long. In the case that the period is very short, t^U coincides with T.[30] The special case $T = 0$ with an unanticipated price shock always leads $q^d > 0$.

At the date of implementation T, the output differential q^d increases discontinuously because the real interest rate rises and lowers by assumption private absorption in U_1 more strongly than in U_2. The rise in the output differential in T is associated with a discontinuous increase in the inflation differential $\dot{p}^d = \dot{p}_1 - \dot{p}_2$, which was negative up to T. The subsequent development of q^d depends on the time span between the anticipation and the implementation of the price increase. For "small" values of T, q^d increases somewhat even beyond T because the rate of inflation in U_2 remains in this case initially above U_1 ($\dot{p}_2 > \dot{p}_1$). The inflation differential $\dot{p}^d$ continues to be negative and the price differential $p^d = p_1 - p_2$ falls correspondingly. In the long run, the price differential p^d returns to its initial level $\overline{p}^d = 0$ such that p^d needs to increase for sufficiently large $t > T$, which requires a positive inflation differential ($\dot{p}^d = \dot{p}_1 - \dot{p}_2 > 0$). Simultaneously, the external terms of trade τ_1 of U_1 improve and U_1 experiences a decline in international competitiveness. The output differential q^d needs to fall for sufficiently large $t > T$ in order to return toward the end of the adjustment process to the initial value $\overline{q}^d = 0$.

For sufficiently "large" T, q^d starts to fall immediately after the initial increase in T because producer inflation in U_1 is higher than in U_2 ($\dot{p}_1 > \dot{p}_2$) and the price differential p^d increases correspondingly for $t > T$. In contrast with the case of a "small" T, the system may experience another date of reversal $t^* > T$ where the output differential again changes sign (it turns negative). Given T "large", the price differential may increase for sufficiently large t

[30] The solution path of q^d has a reversal date t^N in the interval $0 < t < T$, which is *independent* from T. For sufficiently "large" T, we find $t^N < T$ and therefore $t^U(=t^N) < T$. If, in contrast, T is sufficiently "small", $T < t^N$ holds such that a reversal date t^U does not exist in the interval $0 < t < T$. We then find $q^d < 0$ for all $t < T$ but $q^d > 0$ for $t > T$ (provided t is not too large), as q^d increases discontinuously in T, and because the jump in T is independent from T and always larger than $\left|q^d(t)\right|$ for any $t < T$. The real income differential $y^d = y_1 - y_2$ also

beyond the steady state level $\overline{p}^d = 0$ due to strong asymmetries in nominal wage developments across the monetary union. The member country U_1 experiences a loss in international competitiveness. At the reversal date t^*, the output differential q^d turns again negative and converges therefore for $t > t^*$ toward $\overline{q}^d = 0$ from below. This case is presented in *Figure 4*.[31]

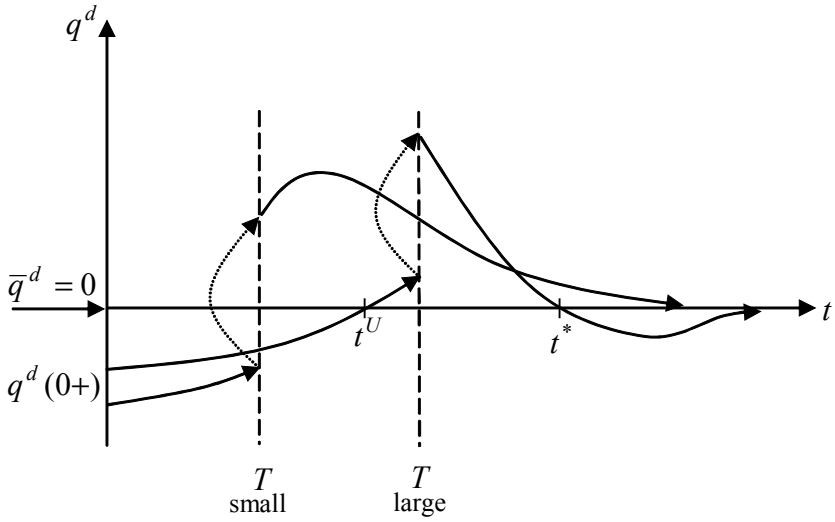

Figure 4: Development of the output differential

Output developments in the individual member countries are given in *Figure 5*.[32] They represent linear combinations of the solution paths for aggregate and differential output. We showed for empirically realistic parameterizations that aggregate output $q = q_1 + q_2$ experiences a discontinuous increase on impact and a monotonic decline in $0 < t < T$. The output variable q_2 therefore increases on impact and subsequently falls. In contrast with q_2, q_1 does *not* necessarily increase in the early phase of the adjustment process because the difference variable $q^d = q_1 - q_2$ falls. Furthermore, q_1 does not necessarily experience a contraction in the interval $0 < t < T$ due to the fall of the price differential p^d and of τ. In *Figure 5*, we present the most realistic case that q_1 falls on impact as well as in the subsequent adjustment process. However, the size of the subsequent contraction in U_1 is smaller than in U_2 such that after the date of reversal t^U, q_1 stays above q_2. At the date of implementation T, both countries experience a discontinuous contraction, which turns out to be larger for U_2 than for U_1. In the

experiences for $T > 0$ a reversal at $t^K \leq T$. Due to $y^d = q^d + \psi p^d$ and $p^d < 0$ for $0 < t < T$, we find $t^K > t^U$ if $t^U < T$ and $t^K = t^U$ if $t^U = T$.

31 The second date of reversal t^* only exists for sufficiently large values of T. For "intermediate" values of T, q^d converges monotonically for $t > T$ from above toward the steady state value $\overline{q}^d = 0$.

32 The qualitative developments of real output and real income y_1 and y_2 are identical.

case $t^U = T$, i.e., for T being „small“, both countries experience a monotonic convergence of their output levels from above to their respective new steady state levels, which are below the initial value. Output q_1 remains hereby consistently above q_2. In contrast, with $t^U < T$, i.e., T is sufficiently „large“, q_1 and q_2 experience divergent developments in the adjustment process following the discontinuous contraction in T. While q_1 continues to fall, q_2 experiences an expansionary process because the price differential $p^d = p_1 - p_2$ increases for $t > T$ while the terms of trade $\tau_2 = 0{,}5(\tau - p^d)$ simultaneously fall such that the international competitiveness of U_2 improves. For sufficiently large t, i.e., after the second date of reversal t^*, q_2 stays above q_1. In the long run, output levels in both member countries converge to the same equilibrium level.

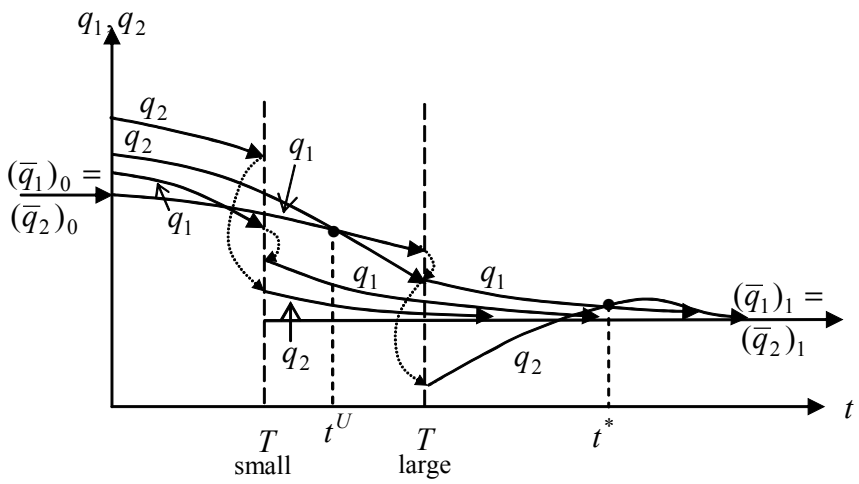

Figure 5: Output developments in individual member countries

4. Reaction of Monetary Policy to Price Increases in Raw Materials

This section investigates the ability of monetary policy to facilitate in an asymmetric monetary union the absorption of anticipated and unanticipated foreign price shocks. Related to EMU, the primary concern of the ECB is the achievement of price stability, which we interpret in our model as to minimize consumer price inflation. Furthermore, according to the Treaty of Maastricht and to the stability pact the ECB is expected to support the general economic policy of the member countries. Monetary policy actions are here considered conducive to this goal when they dampen business cycles at the aggregate level and also cyclical divergences within the monetary union caused by exogenous shocks.

The analytical solutions to the aggregate and difference system determine the conditions to be met and the policies needed for stabilization (see the appendix). More formally, business

cycles at the aggregate level of euroland as well as asymmetric developments within EMU can be avoided if the constants A_1, A_2 and $\widetilde{A}_2$ in the solution for the state vector $(\tau, \quad (m-p))'$ are *all* set equal to zero. Setting $A_1 = A_2 = 0$ removes any anticipation effects, i.e., the overall system remains up to the date of implementation T in the initial steady state. Setting in addition $\widetilde{A}_2 = 0$, anticipated price increases in raw materials do *not* lead to further adjustment dynamics of the aggregate and difference variables, i.e., the system jumps in T immediately into the new steady state.

In the special case $T = 0$, i.e., an *unanticipated* increase in the price of raw materials, complete stabilization of the aggregate and the difference system is achieved as long as monetary policy sets $\widetilde{A}_2 = -\mathrm{d}(\overline{m-p}) = 0$. Taking the rate of growth of the aggregate money stock $\dot{m}$ as the monetary instrument, the common central bank has to change $\dot{m}$ such that the steady state value of the real money stock remains unchanged.[33] An increase in the price of imported raw materials leads in isolation to a permanent fall of the real money stock.[34] An increase in $\dot{m}$ by one unit leads ultimately to a *fall* in the real money stock where the size is determined by the semi-interest elasticity of money demand (l_2). The total differential

$$(20) \qquad \mathrm{d}(\overline{m-p}) = -l_2 \mathrm{d}\dot{m} + \frac{\partial(\overline{m-p})}{\partial p_R^*} \mathrm{d}p_R^* = 0$$

implies therefore the following reaction function for monetary policy

$$(21) \qquad \mathrm{d}\dot{m} = \frac{1}{l_2} \frac{\partial(\overline{m-p})}{\partial p_R^*} \mathrm{d}p_R^* < 0 .$$

This contractionary monetary policy implies that both member countries achieve *immediately* without any further dynamic adjustments the new (lower) steady state level of the outputs q_1 and q_2 *and* cyclical divergences within the monetary union are prevented because the

[33] In contrast to changes in the *growth rate* of the money stock $\dot{m}$, one-time changes in the *level* of the money stock m do *not* have any lasting effects on the real money stock. We therefore consider only changes in the *growth rate* in the discussion of monetary stabilization.

[34] This case prevails as long as b_5 is sufficiently large.

common real interest rate remains with this policy tied to its initial level.[35] The reduction in the rate of monetary growth in response to the exogenous foreign price shock does not only prevent temporary inflationary processes in the member countries but also a permanent reduction in the rates of inflation $\dot{p}_1^c$ and $\dot{p}_2^c$. The restrictive monetary policy (21) is therefore also consistent with the primary goal of ECB monetary policy of achieving price stability based on the consumer price index.

The stabilizing impact of the contractionary monetary policy (21) with respect to the aggregate system is illustrated in *Figure 6*. An unanticipated increase in the price of raw materials viewed in isolation leads to the adjustment process $Q_0B^*Q_1$ while the stabilizing monetary policy (21) creates the opposite development of the state variables τ and *m-p* (adjustment path $Q_0\hat{B}Q_1'$). With the simultaneous and unanticipated occurrence of both shocks ($\mathrm{d}p_R^* < 0$, $\mathrm{d}\dot{m} < 0$), the aggregate system moves instantaneously into the new steady state Q_2. Q_2 is vertically below the initial equilibrium Q_0. In comparison, Q_2 is characterized by a fall in the terms of trade τ at an unchanged equilibrium real money stock *m-p*.

Figure 6 also illustrates the combined effects of the monetary policy reaction to *anticipated* price increases in imported raw materials (T >0). In contrast with the case $T = 0$ monetary policy has to react *twice* in order to achieve stabilization of the complete system throughout the whole adjustment process. This is due to the fact that with anticipated shocks adjustment dynamics in the aggregate and difference system not only occur for $t>T$ but also *prior* to the implementation at T. Full system stabilization in the period $0 < t < T$ requires the credible announcement of a monetary policy, which exactly neutralizes the anticipation effects of the future price shock.[36] The anticipation of the future increase in p_R^* leads in the phase diagram (*Figure 6*) to the time path Q_0BC (for T "large") or $Q_0B'C'$ (for T "small"); the central bank designs its monetary policy such that the corresponding adjustment runs exactly opposite to Q_0BC (or $Q_0B'C'$), which is $Q_0\widetilde{B}\widetilde{C}$ (or $Q_0B''C''$). This requires at the date of anticipation t =

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[35] Setting $\widetilde{A}_2 = 0$ implies the following behavior of the aggregate $q = q_1 + q_2$ and the difference variable $q^d = q_1 - q_2$: $q \equiv \overline{q}_1 (< \overline{q}_0)$, $q^d \equiv \overline{q}^d = 0$ for all $t > 0$. Furthermore, $\dot{\tau} \equiv 0$ such that following (18) real interest rates also remain constant. This implicitly assumes that the *foreign* interest rate i^* remains unaffected by the price increase. An increase in the foreign interest rate i^* in response to the materials price increase leads in isolation to a permanent rise in the level of nominal and real interest rates in the union. This causes a permanent output differential between the member country with the lower interest sensitivity of private absorption ($\mathrm{d}\overline{q}^d < 0$ or, respectively, $\mathrm{d}\overline{q}_1 < \mathrm{d}\overline{q}_2$) (see *Wohltmann* and *Clausen* 2001).

[36] More formally, in the solution of the state vector $(\tau, \quad (m-p))'$, the constant A_2 (= $-A_1$) is set equal to zero.

0 the credible announcement or the expectation by the private sector of a *restrictive* monetary policy at T of the following form:[37]

$$(22) \qquad \mathrm{d}\dot{m}^{ann.} = -\frac{1}{h_{12}l_2}\mathrm{d}\overline{\tau} + \frac{1}{l_2}\frac{\partial(\overline{m-p})}{\partial p_R^*}\mathrm{d}p_R^* < 0$$

As long as this reaction of monetary policy in T is considered credible and therefore anticipated by the private sector, the aggregate as well as the difference system remain up to T in the initial steady state Q_0. The output variables $q = q_1 + q_2$ and $q^d = q_1 - q_2$ satisfy in the period $0 \le t < T$: $q = \overline{q}_0$ and $q^d = 0$. Moreover, inflation remains constant in this period ($\dot{p} = \dot{p}^c = \dot{m}_0$) such that the anticipation of the monetary policy reaction (22) avoids stagflationary developments within the monetary union, which are otherwise caused by the anticipation of the future increase in the price of imported raw materials.

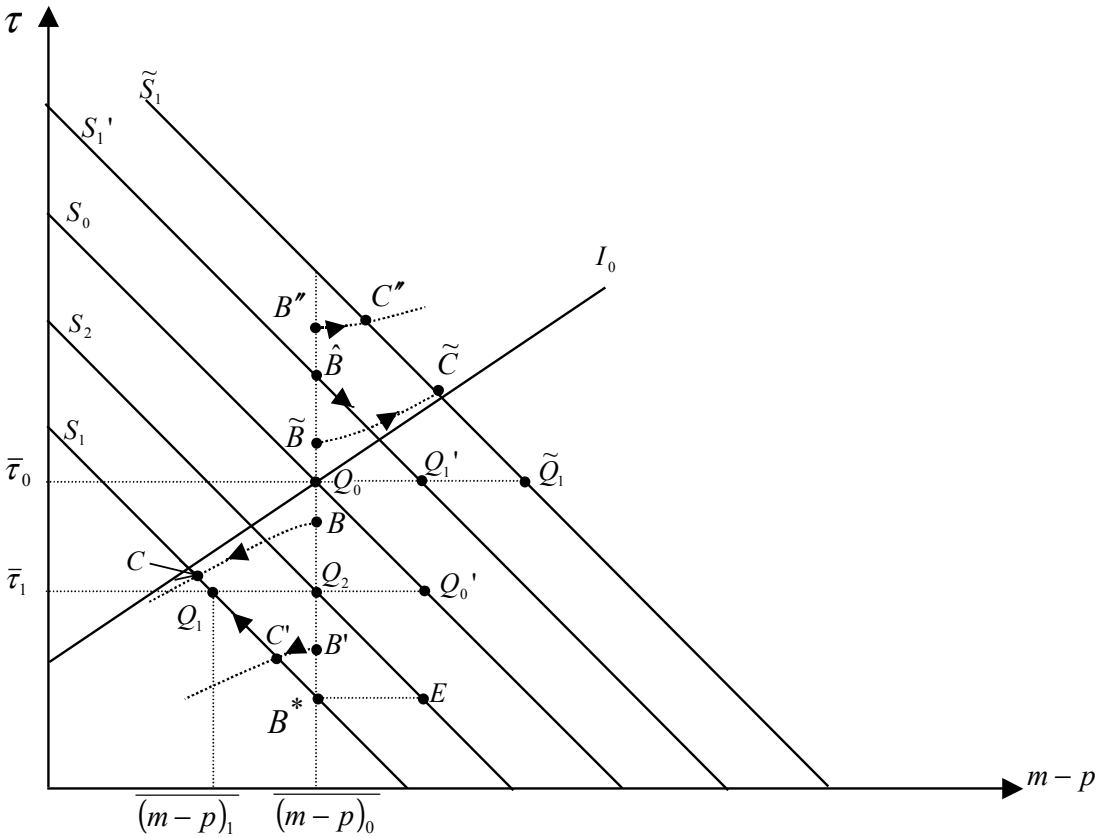

Figure 6: Monetary policy reactions to price increases in raw materials imports

[37] (22) is more contractionary than (21) because the element h_{12} of the corresponding eigenvector is negative

If the central bank *fully* implements the announced reduction in monetary growth and given that the price of raw materials increases in T, the system moves in the period $t > T$ continuously according to the state vector $(\tau, \quad (m-p))'$ along the initial saddle path S_0 belonging to Q_0 to the new equilibrium in Q_0'. In comparison with the new steady state Q_1 in the case of a passive monetary policy ($\mathrm{d}\dot{m} = 0$), we find here a permanent *increase* in the real money stock.[38] In contrast with the case $\mathrm{d}\dot{m} = 0$, the real interest rate now increases in T[39] such that aggregate output $q = q_1 + q_2$ falls on impact below the new steady state level $\overline{q}_1 (< \overline{q}_0)$ and converges subsequently to $\overline{q}_1$ from below. The output differential $q^d = q_1 - q_2$ immediately falls in T due to the interest rate increase and the relatively stronger response of private absorption in U_1.[40] In contrast with the case of a passive monetary policy where the negative output differential already occurs at the date of anticipation $t = 0$, it is shifted here to the date implementation T. A complete stabilization in the period $t > T$, i.e., the removal of any adjustment dynamics, will only be achieved if monetary policy does *not* fully reduce the growth rate of the money stock as previously announced but conducts instead in T a *less restrictive* monetary policy. The required reduction in $\dot{m}$ is determined by the condition that both impulses taken together – the price shock and the policy response - do not change the equilibrium real money stock. Drawing on (20) and (21), the size of the reduction in $\dot{m}$ equals the case $T = 0$.

The phase diagram (*Figure 6*) shows that the deviation from the previously announced and therefore anticipated contractionary monetary policy implies that the state vector $(\tau, \quad (m-p))'$ jumps in T in response to both, the materials price increase and the correction of expectations by the private sector, vertically onto the saddle path S_1 into point B^*. The unanticipated changeover to a less restrictive monetary policy in T causes another vertical jump

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and due to $\mathrm{d}\overline{\tau} = \dfrac{\partial \overline{\tau}}{\partial p_R^*} \mathrm{d}p_R^* < 0$.

38 Taken individually, the restrictive monetary policy announced in $t = 0$ and implemented in $T > 0$ generates the adjustment path $Q_0 \widetilde{B} \widetilde{C} \widetilde{Q}_1$ (for T large) or $Q_0 B'' C'' \widetilde{Q}_1$ (for T small). The necessary fall in the rate of growth of the money stock is *larger* than the policy response to an unanticipated shock (T=0). Correspondingly, the steady state-increase in the equilibrium money stock turns out to be higher ($\widetilde{Q}_1$ in comparison with Q_1').

39 The movement of the aggregate system from Q_0 to Q_0' along the saddle path S_0 implies a continuous decline in the terms of trade τ ($\dot{\tau} < 0$). Eq. (18) then implies that the real interest rate then exceeds its steady state level $2i^*$.

40 A passive monetary policy ($\mathrm{d}\dot{m} = 0$) would lead in T to a fall in interest rates and therefore to a discontinuous increase in q^d.

but now in opposite direction into Q_2. The aggregate variables remain constant afterwards and no further adjustment dynamics take place such that the point Q_2 – like in the special case $T = 0$ – represents the new final equilibrium of the system.[41] This two-stage monetary policy design also stabilizes the difference system completely, i.e., avoids any cyclical differences in output or price developments within the monetary union. Moreover, the rate of inflation within the monetary union is permanently reduced.

The announcement or anticipation of the contractionary monetary policy (22) and the actual implementation of a *less* restrictive monetary policy means that in *quantitative* terms the behavior of monetary policy is *time-inconsistent*. In contrast, monetary policy remains *time-consistent* in *qualitative* terms because the previously announced and the actual course of monetary policy still move in the same direction. The quantitative deviation of the actual policy from the announced reaction function (22) does not seriously undermine the *reputation* of monetary policy (this holds the more as price stability as the primary goal of monetary policy is still achieved). Finally, the changeover from the previously announced or expected monetary policy reaction (22) to the actual course followed (21) achieves complete stabilization. Business cycles at the aggregate level as well as cyclical differences within the monetary union resulting from anticipated increases in the price of imported raw materials can be *completely* avoided.

5. Robustness: Full Price Flexibility

This section modifies the previous setup by assuming instantaneous price adjustment or full price flexibility in the monetary union. The static relationships (23) and (24) replace the dynamic price adjustment equations (9) and (10)[42]

(23) $p_1 = \mu w_1 + (1-\mu)(p_R^* + e) + k_0 \qquad (0 < \mu < 1)$

(24) $p_2 = \mu w_2 + (1-\mu)(p_R^* + e) + k_0$

[41] Taken in isolation, the increase in the price of imported raw materials generates an adjustment path from B^* to Q_1 along the saddle path S_1. In contrast, the contractionary monetary policy (21) causes after the initial jump B^*Q_2 the mirror image adjustment process Q_2E along the somewhat higher located saddle path S_2. With both shocks occurring simultaneously, the aggregate system jumps immediately into the new steady state Q_2 and remains there thereafter.

[42] (23) and (24) are in logarithmized form and can be rationalized by static mark-up price setting. They can also be derived by assuming profit maximization subject to a CES-production function with labor and raw materials as variable inputs of production [see *Bhandari* and *Turnovsky* (1984) and *Wohltmann* (1993)].

The price equations imply that the producer prices instantaneously respond to changes in the nominal exchange rate e such that p_1 and p_2 can no longer be taken as predetermined variables.[43] In contrast, nominal wages w_1 and w_2 continue to be predetermined and respond over time according to the Phillips-curve-equations (11) and (12).

The previous state variables in the aggregate system $\tau = p - 2(p^* + e)$ and *m-p* need to be replaced because both state variables now include the price level as a jump variable. In contrast, the aggregate nominal wage $w = w_1 + w_2$ continues to be a predetermined variable. We therefore replace the previous state variables τ and *m-p* by the variables $\tau_w = w - 2(p^* + e)$ and *m-w*.[44] The dynamic state equations for these newly defined state variables are derived by a transformation of the original dynamic equations for the aggregate system.[45]

The adjustment processes resulting from an anticipated or unanticipated increase in the price of imported raw materials are illustrated in *Figure 7*. The phase diagram uses the redefined state variables τ_w and *m-w*. The time path of the state vector (τ_w, *m-w*)′ moves up to the date of implementation *T* like the corresponding vector (τ, *m-p*)′ of the original model (cf. *Figure 2*). In contrast with the original model, the new long-run equilibrium Q_1 is typically located to the *south-east* of the initial equilibrium Q_0. It follows that after the increase in the price of imported raw materials $\mathrm{d}p_R^* > 0$ in *T*, the system experiences an *increase* in the real money stock expressed in wage units and, irrespective of the size of *T*, a continuation of the real depreciation (again measured in wage units). In the long-run, we assume the following effects on the terms of trade τ_w and on the real money stock *m-w*:

$$(25)\quad \frac{\mathrm{d}\overline{\tau}_w}{\mathrm{d}p_R^*} = \frac{1}{\mu}\frac{\mathrm{d}\overline{\tau}}{\mathrm{d}p_R^*} - 2\frac{1-\mu}{\mu} \quad < 0$$

$$(26)\quad \frac{\mathrm{d}(\overline{m-w})}{\mathrm{d}p_R^*} = \frac{\mathrm{d}(\overline{m-p})}{\mathrm{d}p_R^*} + (1-\mu)(2 - \frac{\mathrm{d}\overline{\tau}_w}{\mathrm{d}p_R^*}) \quad > 0$$

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43 See *Buiter* and *Miller* (1982), p. 96.

44 This specification draws on *Buiter* and *Miller* (1982), p. 96. In a „classic“ interpretation of the modified model where next to the price level *p* also nominal wages *w* are considered as a jump variable our transformed state variables τ_w and *m-w* are both jump variables. Given *one* unstable and one stable characteristic root, the system has an infinite number (instead of one unique one) of convergent solution paths for the state vector (τ_w, *m-w*)′. See *Turnovsky* (2000), p. 147 and *Buiter* (1984).

45 See the mathematical appendix.

Due to $\mu < 1$ it follows with $\mathrm{d}p_R^* > 0$ that $\bar{\tau}_w$ falls more strongly than the external terms of trade $\bar{\tau}$ ($\mathrm{d}\bar{\tau}_w < \mathrm{d}\bar{\tau} < 0$). In the long-run, the two real money stocks *m-w* and *m-p* typically move in opposite directions, i.e., if p_R^* increases ($\mathrm{d}(\overline{m-w}) > 0 > \mathrm{d}(\overline{m-p})$).[46] Assuming the money stock to be constant ($\dot{m} = 0$), the increase in the price of imported raw materials leads in the long-run to an increase in the domestic producer price level and to a fall in the nominal wage.[47]

Further differences from the previous model arise from the fact that the aggregate price level $p = p_1 + p_2$ reacts already on impact to the change in the nominal exchange rate. The rise in the production price level caused by anticipated price increase in imported raw materials remains weaker than the change in the nominal exchange rate as long as $\mu > 0.5$. This means that the terms of trade τ continue to fall in the early part of the adjustment process (but less strongly than with sluggish price adjustment) such that the real interest rate again increases.[48] As in the basic model, the reaction of output in the short-run is theoretically ambiguous. Empirically, however, the expansionary terms of trade-effect dominates the contractionary real interest rate effect such that in the presence of sluggish price adjustment output increases on impact in response to an anticipated future increase in the price of imported raw materials.[49] The subsequent behavior of *q* runs parallel to the results from the basic model. In

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[46] The price adjustment equations (23) and (24) imply $m - w = m - p - \frac{1-\mu}{\mu}(\tau + 2(p^* - p_R^*)) + 2k_0$. The LM equation and the long run aggregate supply function yield in the case $\mathrm{d}p_R^* > 0$ $\mathrm{d}(\overline{m-w}) = (l_1(f_1 + f_3) - (\alpha_3 + \frac{1-\mu}{\mu}))\mathrm{d}\bar{\tau} + 2(\frac{1-\mu}{\mu} - f_3 l_1)\mathrm{d}p_R^*$. Assuming $\mathrm{d}\bar{\tau} / \mathrm{d}p_R^* < 0$ gives $\mathrm{d}(\overline{m-w}) / \mathrm{d}p_R^* > 0$, if $\alpha_3 + \frac{1-\mu}{\mu} > l_1(f_1 + f_3)$ and $\frac{1-\mu}{\mu} > f_3 l_1$. The microeconomic foundations imply that $f_3 = (\beta + \sigma)\frac{1-\mu}{\mu}$ and $f_1 = \beta\alpha_3$ where β denotes the elasticity of the labor supply with respect to the real wage and σ the elasticity of substitution between the variable factors labor and imported raw materials using a CES-production technology (see *Wohltmann* and *Bulthaupt* 1999). A *sufficient* (but not necessary) condition for $\mathrm{d}(\overline{m-w}) / \mathrm{d}p_R^* > 0$ is $(\beta + \sigma)l_1 < 1$. The parameter l_1 denotes the income elasticity of money demand, which is often found empirically to be close to unity. As σ is considerably smaller than unity, a sufficiently inelastic labor supply ensures $\mathrm{d}(\overline{m-w}) / \mathrm{d}p_R^* > 0$.

[47] The real wage $\overline{w-p}$ then declines while the real price of raw materials $\overline{p_R^* + e - p}$ increases; factor substitution then leads to a rise in the demand for labor and the level of production $\bar{q}$ falls.

[48] We find $\dot{\tau}(0+) = \mu \cdot \dot{\tau}_w(0+) < 0$ such that $i - \dot{p}^c$ increases immediately. Furthermore, nominal interest rates again increase on impact.

[49] The condition $((1+\alpha_3)\mu + 2(1-\mu))l_2(b_5 - (a_1 - b_1 + b_2)\psi) + \frac{1}{2}\frac{(1-\mu)^2}{\mu}\frac{\lambda}{\delta} > (1-\alpha_3)a_2(\mu\alpha_3 + 1 - \mu)$ is necessary and sufficient for $q(0+) > \bar{q}_0$. It is fulfilled as long as b_5 is sufficiently large relative to a_2. It is also fulfilled if the semi-interest elasticity of money demand is sufficiently large. Then, the fall in real money demand due to the increase in the nominal interest rate exceeds the fall in the consumer oriented real money stock $m - p^c$ such

T, aggregate output q again declines discontinuously. The size of the decline is larger than with lagged price adjustment. The reason is that the raw materials price increase induces a jump in the producer price level amounting to $p(T+) - p(T-) = 2(1-\mu)\mathrm{d}p_R^*$, which in turn causes an identical increase in τ and, consequently, a fall in aggregate demand. After T, output always converges toward the new steady state-level $\bar{q}_1$ from below irrespective of the size of T.

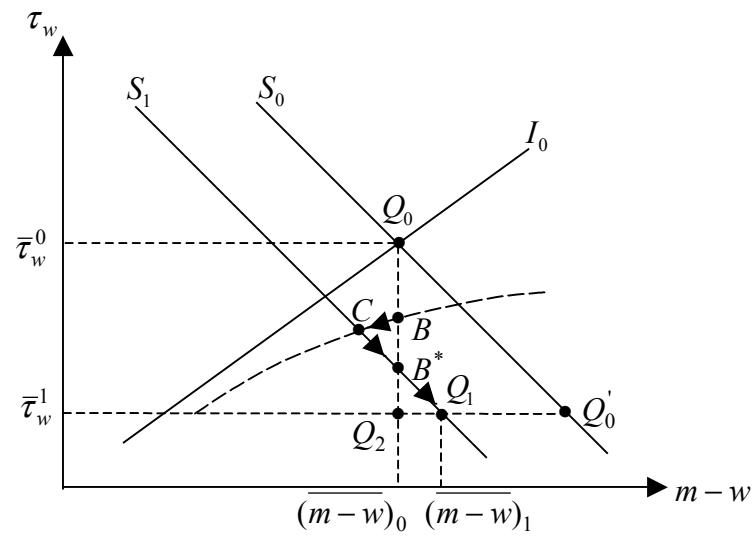

Figure 7: Phase diagram of the aggregate system with full price flexibility

The output differential $q^d = q_1 - q_2$ behaves in the anticipation period $0 < t < T$ qualitatively as in the case with sluggish price adjustment. On impact, the rise in real interest rates generates in the view of U_1 a negative cyclical differential ($q^d(0+) < 0$), which is reduced and ultimately reversed by changes in the internal price differential p^d. [50]

Differences in the behavior of the output differential under sluggish versus immediate price adjustment prevail in the case of unanticipated shocks ($T = 0$). With price rigidity, the terms of trade immediately start to improve ($\dot{\tau}(0+) > 0$) after the negative impact effect. This implies a fall in the real interest rate and a positive output differential ($q^d(0+) > 0$) from the perspective of U_1. With price flexibility, the real depreciation continues ($\dot{\tau}(0+) = \mu \cdot \dot{\tau}_w(0+) < 0$). This is equivalent to a rise in real interest rates and a negative out-

that the transactions demand for money being related to q has to increase in order to reestablish money market equilibrium in the monetary union.

[50] The inflation rate differential satisfies $\dot{p}^d = \dot{p}_1 - \dot{p}_2$: $\dot{p}^d(0+) < 0$ also with immediate price adjustment such that p^d falls gradually.

put differential ($q^d(0+) < 0$), which does *not* experience a reversal in the course of the adjustment process.

The optimal response of monetary policy to an anticipated increase in the price of imported raw materials critically depends on the nature of price adjustment. In the case $T = 0$ and *immediate* price adjustment, the condition for the simultaneous stabilization of the aggregate and the difference system is $\widetilde{A}_2 = -\mathrm{d}(\overline{m-w}) = 0$. This implies the following decision rule for monetary policy concerning the necessary change in the rate of growth of the money stock:

$$(27)\quad \mathrm{d}\dot{m} = \frac{1}{l_2}\frac{\partial(\overline{m-w})}{\partial p_r^*}\mathrm{d}p_R^* > 0 .$$

It requires in the case $\mathrm{d}p_R^* > 0$ and $\partial(\overline{m-w})/\partial p_R^* > 0$ an increase in the rate of monetary growth.[51] With sluggish price adjustment, however, (21) requires a fall in $\dot{m}$ in order to prevent disequilibrium dynamics in the aggregate and the difference system. In contrast with the contractionary monetary policy in (21), the policy rule (27) implies an expansionary stance. This raises the rate of inflation permanently and is therefore inconsistent with the achievement of price stability as the primary policy objective.

In the case $T > 0$, i.e., an anticipated increase in the price of imported raw materials, complete stabilization requires monetary policy to react twice, even in the presence of immediate price adjustment. The neutralization of anticipation effects in the period $0 < t < T$ requires the credible announcement of a specific monetary policy, which generates in the phase diagram (*Figure 7*) an opposite adjustment or a mirror image to the adjustment path Q_0BC of the state vector $(\tau_w, m-w)'$. Formally, this leads to the following decision rule for monetary policy[52]

$$(28)\quad \mathrm{d}\dot{m}^{ann.} = -\frac{1}{\widetilde{h}_{12}l_2}\mathrm{d}\overline{\tau}_w + \frac{1}{l_2}\frac{\partial(\overline{m-w})}{\partial p_R^*}\mathrm{d}p_R^* ,$$

[51] In the phase diagram (*Figure 7*) the import price shock $\mathrm{d}p_R^* > 0$ leads in conjunction with the monetary reaction (27) to an immediate jump of the aggregate system into the new steady state Q_2, which is below Q_0.

[52] This relationship is derived by setting in the solution form of the state vector $(\tau_w, m-w)'$ the constant A_2 equal to zero. In (28), the corresponding component of the eigenvector $\widetilde{h}_{12}$ is negative.

which typically requires – as in the case of the rule (22) with sluggish price adjustment – in the case $\mathrm{d}p_R^* > 0$ a contractionary stance.[53] The anticipation of (28) by the private sector leads in conjunction with the anticipation in $t = 0$ of the future price increase in imported raw materials that the aggregate as well as the difference system remain up to the date of implementation T in the initial equilibrium. At the date of the actual price increase in T, adjustment dynamics in the aggregate and the difference system can only be avoided, if the central bank does *not* follow the previously announced contractionary stance required by (28) but conducts instead in T a monetary *expansion* according to the policy reaction (27).[54] In contrast with the case of sluggish price adjustment we find here a change in the *direction* of monetary policy compared with the previously announced rule (28), i.e., monetary policy is *time-inconsistent* not only in quantitative but also in *qualitative* terms. This undermines the credibility of monetary policy and the reputation of the central bank. This holds the more as the actual implementation of the *expansionary* monetary policy (27) leads to a permanent rise in the rate of inflation in the monetary union.

The derivation of the policy decision rules (21), (22) and (27), (28) assumed that the increase in the price of imported raw materials leads to price changes within the monetary union but leaves the foreign price level p^* unchanged. This assumption is not realistic as, for example, the oil price hikes initiated by the OPEC countries caused increased inflation worldwide. Viewed in isolation, an anticipated and an unanticipated increase in the price of imported final goods ($\mathrm{d}p^* > 0$) incur adjustment dynamics at the aggregate level of the monetary union, which run opposite to previous effects of an increase in the price of imported raw materials.[55] The *steady state* effects on real variables and relative prices in the system are

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[53] The multiplier $\partial(\overline{m-w})/\partial p_R^*$ is always positive for realistic parameter values but $\frac{1}{\tilde{h}_{12}}\mathrm{d}\bar{\tau}_w = \frac{1}{\tilde{h}_{12}}\frac{\partial \bar{\tau}_w}{\partial p_R^*}\mathrm{d}p_R^*$ turns out to be relatively larger.

[54] The implementation of the previously announced restrictive monetary policy (28) leads in conjunction with the price increase in imported raw materials in *Figure 7* to a movement of the aggregate system along the saddle path S_0 from Q_0 to the new steady state Q_0', which is characterized in comparison with Q_1 by an even stronger rise in the real money stock $m - w$. The unanticipated implementation of the *expansionary* monetary policy (27) implies in the phase diagram that the aggregate system moves at the date of implementation discontinuously into the new steady state Q_2, which leaves the real money stock $m - w$ unchanged at its initial value in the original equilibrium.

[55] In contrast with the previous case $\mathrm{d}p_R^* > 0$, the constants A_1, A_2 and $\tilde{A}_2$ now change their sign in the solution form of the state vector. The absolute values of the constants are *not* equal to those in the case $\mathrm{d}p_R^* > 0$. It follows that the adjustment dynamics in response to $\mathrm{d}p^* > 0$ are *not* an exact mirror image to the effects caused by an identical increase in the price of imported raw materials.

quantitatively identical but opposite in sign provided that $dp^* = dp_R^*$.[56] In contrast to the case of raw materials, a price increase in imported final goods leads to jumps in the external terms of trade τ not only at the anticipation date but also at the date of implementation *T*. This stems from the fact that an increase in p^* per se leads due to $\tau = p - 2(p^* + e)$ to a discontinuous fall in τ.

Monetary stabilization in the *simultaneous* presence of price increases in imported raw materials and final goods requires the following: given the realistic assumptions that the increase in the price of raw materials leads to an *underproportional* increase in the foreign price level ($\mathrm{d}p^* / \mathrm{d}p_R^* < 1$ or, respectively, $0 < \mathrm{d}p^* < \mathrm{d}p_R^*$) and lagged adjustment in the domestic price level, unanticipated combined foreign price shocks can be perfectly neutralized at the aggregate and the difference level by an adequate reduction in the rate of growth of the money stock. The policy reaction (21) is modified by replacing the isolated price shock $\mathrm{d}p_R^* > 0$ by the (quantitatively smaller) combined shock $\mathrm{d}p_R^* - \mathrm{d}p^* (> 0)$.[57] The required fall in the rate of growth of the money stock in response an unanticipated combined foreign price shock is *smaller* than with an isolated increase in the price of imported raw materials.[58] With anticipated combined foreign price shocks ($T > 0$) the system can be stabilized in the period between the anticipation and the implementation of the foreign price increases by the credible announcement in $t = 0$ of a policy implemented in $T > 0$:[59]

$$(29) \qquad \mathrm{d}\dot{m}^{ann.} = -\frac{1}{h_{12} l_2}\left(\frac{\partial \bar{\tau}}{\partial p_R^*}(\mathrm{d}p_R^* - \mathrm{d}p^*) + 2\mathrm{d}p^*\right) + \frac{1}{l_2}\frac{\partial(\overline{m-p})}{\partial p_R^*}(\mathrm{d}p_R^* - \mathrm{d}p^*)$$

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[56] In the case of sluggish price adjustment it follows that $\frac{\partial \bar{q}}{\partial p^*} = -\frac{\partial \bar{q}}{\partial p_R^*} > 0$, $\frac{\partial \bar{\tau}}{\partial p^*} = -\frac{\partial \bar{\tau}}{\partial p_R^*} > 0$ and $\partial(\overline{m-p}) / \partial p^* = -\partial(\overline{m-p}) / \partial p_R^* > 0$. Analogous results can be derived for the case of immediate price adjustment for the state variables τ_w and *m-w*.

[57] The rule (21) is replaced by $\mathrm{d}\dot{m} = \frac{1}{l_2}\left(\frac{\partial(\overline{m-p})}{\partial p_R^*}\mathrm{d}p_R^* + \frac{\partial(\overline{m-p})}{\partial p^*}\mathrm{d}p^*\right) = \frac{1}{l_2}\frac{\partial(\overline{m-p})}{\partial p_R^*}(\mathrm{d}p_R^* - \mathrm{d}p^*) < 0,$ if $\mathrm{d}p_R^* > \mathrm{d}p^*$.

[58] This is due to the fact that the adjustment dynamics following a foreign price increase in final goods $\mathrm{d}p^* > 0$ mirror those arising from $\mathrm{d}p_R^* > 0$, and, furthermore, due to our assumption $\mathrm{d}p_R^* > \mathrm{d}p^*$.

[59] (29) simplifies with $\mathrm{d}p^* = 0$ to (22). The first expression in brackets in (29) can be rearranged to $\mathrm{d}\bar{\tau} + 2\mathrm{d}p^* = \left(\frac{\partial(\overline{p-2e})}{\partial p_R^*}\mathrm{d}p_R^* + \frac{\partial(\overline{p-2e})}{\partial p^*}\mathrm{d}p^*\right)$. The two multipliers $\frac{\partial(\overline{p-2e})}{\partial p_R^*}$ and $\frac{\partial(\overline{p-2e})}{\partial p^*}$ are quantitatively *not* identical.

This policy rule for the rate of growth of the money stock implies a negative stance ($\mathrm{d}\dot{m}^{ann.} < 0$) when the foreign price increase in imported raw materials has a relatively *small* impact on the foreign price level p^*, i.e., as long as $\mathrm{d}p^* / \mathrm{d}p_R^*$ is sufficiently small. In this case, both terms in (29) are negative[60] and the monetary policy rule is *time-consistent* in *qualitative* terms. If monetary policy pursues in T in order to stabilize the system *after* T a *less* contractionary stance than previously announced[61], the direction of monetary policy remains *unchanged*. In contrast, if $\mathrm{d}p^*$ is large relative to $\mathrm{d}p_R^*$ or the difference $\mathrm{d}p_R^* - \mathrm{d}p^*$ sufficiently small, we find $\mathrm{d}\dot{m}^{ann.} > 0$. The stabilization of the complete system requires the announcement of an expansionary but the later conduct of a contractionary monetary policy. Policy is time-inconsistent in both, in quantitative and qualitative terms. This undermines the credibility of monetary policy and causes the central bank to lose reputation.

6. Summary

This paper investigates the dynamic effects of an anticipated price increase in imported raw materials in an asymmetric two-country model of a small monetary union. The union is characterized on the supply side by different wage setting behavior of the trade unions and on the demand side by different interest sensitivities of private absorption. Wage setting behavior in U_1 is supposed to consider the growth of money supply in the union, whereas the rate of wage increase in U_2 is linked to the short-run variation of the consumer price index. Private consumption in member country U_1 is supposed to respond more strongly to the interest rate as in country U_2. It is shown that a stagflationary process for the whole union starts even before the price increase of raw materials takes place. Furthermore, due to macroeconomic asymmetries cyclical divergences occur within the monetary union throughout the adjustment process. The direction of the cyclical divergence depends on the short-run reaction of the real rate of interest. On impact, real interest rates increase and from the perspective of U_1 we find a negative cyclical differential within the monetary union.[62] In the subsequent adjustment proc-

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[60] The first expression in brackets in (29) is only negative, if $\mathrm{d}p^*$ is sufficiently small or, respectively, $\mathrm{d}p_R^* - \mathrm{d}p^*$ is sufficiently large.

[61] The monetary policy actually conducted in T is identical to the case $T = 0$ and equals the second term in (29).

[62] It can be shown that on impact the direction of the real interest rate change and, consequently, the direction of the cyclical divergence within the monetary union crucially depends on the asymmetry on the supply side. Given a symmetric setup in which wage setting in both countries is linked to consumer price inflation we find that the anticipation of a future price increase in imported raw materials leads on impact to a *fall* in the real interest rate and therefore in view of U_1 a positive output differential.

ess, the improvements in the internal and external terms of trade of U_1 improve its international competitiveness and its relative cyclical position within the monetary union. At some stage during the adjustment process, even a complete reversal of the cyclical differential takes place. It happens at the latest when the price increase is implemented.

In the special case of an unanticipated price increase, the monetary union does *not* experience a cyclical reversal between the member countries. In the case of sluggish price adjustment, the output level q_1 in U_1 remains due to a relatively larger interest sensitivity of aggregate demand and to the fall in the level of real interest rates consistently *above* the output level q_2 in U_2. In the case of full price flexibility, real interest rates increase and q_1 remains *below* q_2 throughout the *entire* adjustment process.

It is shown that the adjustment dynamics following of foreign price increase (in particular, the stagflationary consequences) as well as cyclical divergences within the monetary union can be avoided by a countercyclical monetary policy. In the case of an *unanticipated* increase in the price of raw materials, sluggish price adjustment and the condition that the price level of imported final goods increases less than proportionately it is necessary to reduce the rate of monetary growth in the union. This policy response ensures that the output variable and the external terms of trade immediately attain their new (lower) long-run equilibrium values and a permanently lower rate of inflation in the union.

In the case of an *anticipated* increase in the price of raw materials, complete stabilization in both periods, i.e., the period between the anticipation and the implementation as well as the period after the implementation, requires a *qualitatively time-consistent* monetary policy, which on impact credibly announces for the date of implementation of the price increase a contractionary monetary policy taking the form of a reduction in the rate of monetary growth but reduces at the actual implementation date the monetary growth by a relatively *smaller* amount than the initial announcement. The *direction* of the actual policy is consistent with the announced change such that the credibility of monetary policy remains intact. However, monetary policy encounters a time-inconsistency problem in the presence of anticipated price increases if the union is characterized by full price flexibility *or* if the increase in the price of raw materials is accompanied by a sufficiently large increase in the price of imported final goods. Moreover, in the presence of full price flexibility, monetary policy has to turn *expansionary* in order to prevent cyclical divergences within the monetary union, which conflicts with the achievement of primary goal of price stability.

7. Appendix

Aggregate and difference system

Using the decomposition method by *Aoki* (1981) the model (1)-(16) is transformed into the following set of equations using either aggregate or difference variables:

(A1) $\lambda \cdot q = g - 2a_2 i^* + 2b_3 y^* + a_2(1-\alpha_3)\dot{\tau} - (b_5 - (a_1 - b_1 + b_2)\psi)\tau$

$$- 2(a_1 - b_1 + b_2)\psi(p_R^* - p^*) + k$$

(A2) $y = q + \psi \cdot \tau + 2\psi(p^* - p_R^*) - 2c_0$

(A3) $m - p^c = 2l_0 + l_1 q - 2l_2(i^* + \dot{e})$

(A4) $p^c = p - \alpha_3 \cdot \tau$

(A5) $\dot{p} = \frac{1}{2}\mu(\dot{m} + \dot{p}^c) + \mu\delta(q - \overline{q}) + 2(1-\mu)(\dot{p}_R^* + \dot{e})$

(A6) $\overline{q} = 2f_0 + f_1\overline{\tau} + f_3(\overline{p - 2(p_R^* + e)})$

(A7) $æ_1\, q^d = \tilde{a}_2(1-\alpha_3)\dot{\tau} - 2\tilde{a}_2 i^* + (g_1 - g_2) + (a_{01} - a_{02})$

$$-\big((2b_4 + b_5) - (a_1 - b_1 - b_2)\psi\big)p^d$$

(A8) $y^d = q^d + \psi \cdot p^d$

(A9) $\dot{p}^d = \frac{1}{2}\mu(\dot{m} - \dot{p}^c) + \mu\delta(q^d - \overline{q}^d)$

(A10) $\overline{q}^d = (f_1 + 2f_2 + f_3)\overline{p}^d$

with

$\lambda = (1 - a_1 + b_1 - b_2)$, $æ_1 = (1 - a_1 + b_1 + b_2)$, $a_2 = \frac{1}{2}(a_{21} + a_{22})$,

$\tilde{a}_2 = \frac{1}{2}(a_{21} - a_{22}) > 0$, $k = a_{01} + a_{02} + 2b_0 - 2(a_1 - b_1 + b_2)c_0$, $\alpha_1 = \alpha_2$,

$q = q_1 + q_2$, $y = y_1 + y_2$, $\tau = \tau_1 + \tau_2$, $g = g_1 + g_2$, $p^c = p_1^c + p_2^c$,

$p_1^c = p_2^c$, $p = p_1 + p_2$, $q^d = q_1 - q_2$, $y^d = y_1 - y_2$, $p^d = p_1 - p_2$

The equations (A1)-(A6) describe the aggregate system while the equations (A7)-(A10) represent the difference system.

Model reduction

The price adjustment equation (A5) yields in conjunction with (A4) the output equation

$$\text{(A11)}\quad q-\overline{q}=\frac{1}{\mu\delta}\left(\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right)\dot{\tau}-\frac{1}{2}\mu(\dot{m}-\dot{p})+2(1-\mu)(\dot{p}^*-\dot{p}_R^*)\right)$$

The LM equation (A3) can be reformulated as

$$\text{(A12)}\quad (m-p)-\overline{(m-p)}+\alpha_3(\tau-\overline{\tau})=l_2\dot{\tau}+l_2(\dot{m}-\dot{p})+l_1(q-\overline{q})$$

Using (A1), (A11) and (A12) we find the following state equations for the aggregate system

$$\text{(A13)}\quad \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} \dot{\tau} \\ \dot{m}-\dot{p} \end{pmatrix}=\begin{pmatrix} -\mu\tilde{b}_5 & 0 \\ \alpha_3 & 1 \end{pmatrix}\begin{pmatrix} \tau-\overline{\tau} \\ (m-p)-\overline{(m-p)} \end{pmatrix}$$

where

$$b_{11}=\frac{\lambda}{\delta}\left(\frac{1}{2}\mu\alpha_3+(1-\mu)-\frac{\mu\delta}{\lambda}a_2(1-\alpha_3)\right),$$

$$b_{12}=-\frac{1}{2}\mu\frac{\lambda}{\delta},\qquad b_{21}=l_2+\frac{l_1}{\mu\delta}\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right),$$

$$b_{22}=l_2-\frac{1}{2}\frac{l_1}{\delta},\qquad \tilde{b}_5=b_5-(a_1-b_1+b_2)\psi$$

The determinant of (A13) $\Delta=b_{11}b_{22}-b_{12}b_{21}$ is

$$\text{(A14)}\quad \Delta=\frac{1}{2}\frac{\lambda}{\delta}\mu l_2(1+\alpha_3)+(1-\alpha_3)\mu a_2\left(\frac{1}{2}\frac{l_1}{\delta}-l_2\right)+\frac{\lambda}{\delta}(1-\mu)l_2$$

The explicit form of the state equations for the aggregate system (A13) is given by

$$\text{(A15)}\quad \begin{pmatrix} \dot{\tau} \\ \dot{m}-\dot{p} \end{pmatrix}=\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}\begin{pmatrix} \tau-\overline{\tau} \\ (m-p)-\overline{(m-p)} \end{pmatrix}$$

with

$$d_{11}=\frac{\mu}{\Delta}\left(\left(\frac{1}{2}\frac{l_1}{\delta}-l_2\right)\tilde{b}_5+\frac{1}{2}\frac{\lambda}{\delta}\alpha_3\right),$$

$$d_{12} = \frac{1}{2}\frac{\mu}{\Delta}\frac{\lambda}{\delta} > 0,$$

$$d_{21} = \frac{1}{\Delta}\left(\left(\mu l_2 + \frac{l_1}{\delta}\left(\frac{1}{2}\mu\alpha_3 + (1-\mu)\right)\right)\tilde{b}_5\right.$$

$$\left. + \alpha_3\mu\left(\frac{1}{2}\frac{\lambda}{\delta}\alpha_3 - a_2(1-\alpha_3)\right) + \frac{\alpha_3\lambda}{\delta}(1-\mu)\right),$$

$$d_{22} = \frac{1}{\Delta}\left(\mu\left(\frac{1}{2}\frac{\lambda}{\delta}\alpha_3 - a_2(1-\alpha_3)\right) + \frac{\lambda}{\delta}(1-\mu)\right)$$

The determinant $\Delta_1 = d_{11}d_{22} - d_{12}d_{21}$ of (A15) reduces to $\Delta_1 = -\mu\tilde{b}_5 / \Delta$. In the case $\tilde{b}_5 > 0$ and $\Delta > 0$, we obtain a negative determinant Δ_1 and the system (A15) displays the saddle path property. The real eigenvalues r_1 and r_2 belonging to the matrix (d_{ij}) follow from the well-known formula

(A16) $$r_{1,2} = \frac{1}{2}(d_{11} + d_{22}) \pm \sqrt{\frac{1}{4}(d_{11} + d_{22})^2 - \Delta_1}$$

Assuming $\Delta_1 < 0$ implies one unstable and one stable root, which we define as $r_1 > 0 > r_2$. The corresponding eigenvectors h_1 and h_2 are normalized as

(A17) $$h_1 = \begin{pmatrix} h_{11} \\ 1 \end{pmatrix}, \qquad h_2 = \begin{pmatrix} h_{12} \\ 1 \end{pmatrix}$$

with

$$h_{11} = \frac{d_{12}}{r_1 - d_{11}}, h_{12} = \frac{d_{12}}{r_2 - d_{11}}$$

We assume that $h_{11} > 0$ and $h_{12} < 0$.

An increase in the price of imported raw materials/intermediate goods ($\mathrm{d}p_R^* > 0$) or in imported final goods ($\mathrm{d}p^* > 0$), which is announced in $t = 0$ and implemented in $T > 0$, results in the following state vector $(\tau,\ m-p)'$, which describes the unique convergent time path from the initial equilibrium $(\bar{\tau}_0,\ \overline{(m-p)}_0)$ to the new long-run equilibrium $(\bar{\tau}_1,\ \overline{(m-p)}_1)$:[63]

[63] See *Wohltmann* and *Clausen* (2001). The expression $e^{r_j t}$ in (A18) and (A19) represents $\exp(r_j t)$ $(j = 1,2)$.

(A18) $$\begin{pmatrix} \tau \\ m-p \end{pmatrix} = \begin{pmatrix} \bar{\tau}_0 \\ \overline{(m-p)}_0 \end{pmatrix} + A_1 h_1 e^{r_1 t} + A_2 h_2 e^{r_2 t} \qquad \text{for } 0 < t < T,$$

(A19) $$\begin{pmatrix} \tau \\ m-p \end{pmatrix} = \begin{pmatrix} \bar{\tau}_1 \\ \overline{(m-p)}_1 \end{pmatrix} + \tilde{A}_2 h_2 e^{r_2 t} \qquad \text{for } t > T$$

with

$$A_2 = \frac{1}{\chi} e^{r_2 T} \left(-(\mathrm{d}\bar{\tau} + 2\mathrm{d}p^*) + h_{12} \mathrm{d}\overline{(m-p)} \right) = -A_1$$

(A20) $$\tilde{A}_2 = A_2 + \frac{1}{\chi} e^{r_1 T} \left((\mathrm{d}\bar{\tau} + 2\mathrm{d}p^*) - h_{11} \mathrm{d}\overline{(m-p)} \right)$$

$$\chi = (h_{11} - h_{12}) e^{(r_1 + r_2)T} > 0$$

The constants A_1, A_2 and $\tilde{A}_2$ are determined by three conditions: assuming price rigidity, the variable $m-p$ is predetermined and therefore required to remain continuous at $t = 0$ and $t = T$ while the jump variable τ satisfies at the date of implementation T the condition $\tau(T+) - \tau(T-) = -2\mathrm{d}p^*$.[64] In the special case $T = 0$, i.e., with unanticipated foreign price shocks, the solution time path is only given by (A19) where $\tilde{A}_2 = -\mathrm{d}\overline{(m-p)}$. The steady state solution to the system (A15) is determined by $\dot{\tau} = 0 = \dot{m} - \dot{p}$. The steady state responses $\mathrm{d}\overline{(m-p)} = \overline{(m-p)}_1 - \overline{(m-p)}_0$ and $\mathrm{d}\bar{\tau} = \bar{\tau}_1 - \bar{\tau}_0$ to foreign price increases ($\mathrm{d}p_R^* > 0$, $\mathrm{d}p^* > 0$) can be derived from the total differential of the IS equation, the long-run supply function and the LM equation:

(A21) $$\begin{aligned} \lambda \mathrm{d}\bar{q} &= -\tilde{b}_5 \mathrm{d}\bar{\tau} - 2(a_1 - b_1 + b_2)\psi(\mathrm{d}p_R^* - \mathrm{d}p^*) \\ \mathrm{d}\bar{q} &= (f_1 + f_3)\mathrm{d}\bar{\tau} + 2f_3(\mathrm{d}p^* - \mathrm{d}p_R^*) \\ \mathrm{d}\overline{(m-p)} &= -\alpha_3 \mathrm{d}\bar{\tau} + l_1 \mathrm{d}\bar{q} \end{aligned}$$

An isolated price increase in imported raw materials ($\mathrm{d}p_R^* > 0$, $\mathrm{d}p^* = 0$) leads in the steady state to $\mathrm{d}\bar{\tau} < 0$ and $\mathrm{d}\overline{(m-p)} < 0$. In this case we find $A_2 > 0$ while $\tilde{A}_2$ turns out to be posi-

[64] The nominal exchange rate e is a component of τ and responds discontinuously only at the time of announcement $t = 0$. In the case $\mathrm{d}p^* = 0$, τ remains continuous in T.

tive (negative) for sufficiently small (large) values of T. In contrast, an isolated price increase in imported final goods ($\mathrm{d}p_R^* = 0$, $\mathrm{d}p^* > 0$) ultimately leads to $\mathrm{d}\bar{\tau} > 0$ and $\mathrm{d}\overline{(m-p)} > 0$. Correspondingly, $A_2 < 0$ and $\widetilde{A}_2 < 0$ ($\widetilde{A}_2 > 0$) for T sufficiently small (large).

Using (A19), the unique convergent saddle path for the aggregate system S follows as:

(A22) $\tau - \bar{\tau} = h_{12}\left((m-p) - \overline{(m-p)}\right)$

Due to $h_{12} < 0$ the saddle path has a negative slope in the phase diagram.

The difference system (A7)-(A10) can be reduced to the following dynamic equation governing the predetermined variable $p^d = p_1 - p_2$:

(A23) $\dot{p}^d = r_0(p^d - \bar{p}^d) + \frac{1}{2}\mu(\dot{m} - \dot{p}) + \mu\gamma_1\dot{\tau}$

with

$$r_0 = -\frac{\mu\delta}{æ_1}(2b_4 + b_5 - (a_1 - b_1 - b_2)\psi) < 0$$

$$\gamma_1 = \frac{1}{2}\alpha_3 + \frac{\delta}{æ_1}\widetilde{a}_2(1-\alpha_3) > 0$$

The state equation (A23) has the following solution:

(A24) $$p^d = \bar{p}_0^d + \mu(\gamma_1 h_{11} + \frac{1}{2})r_1 A_1 \frac{1}{r_1 - r_0}(e^{r_1 t} - e^{r_0 t})$$
$$+ \mu(\gamma_1 h_{12} + \frac{1}{2})r_2 A_2 \frac{1}{r_2 - r_0}(e^{r_2 t} - e^{r_0 t}) \qquad \text{for } 0 < t < T$$

(A25) $$p^d = \bar{p}_0^d e^{r_0(t-T)} + \bar{p}_1^d(1 - e^{r_0(t-T)}) + \mu(\gamma_1 h_{11} + \frac{1}{2})r_1 A_1 \frac{1}{r_1 - r_0} e^{r_0 t}(e^{(r_1 - r_0)T} - 1)$$
$$+ \mu(\gamma_1 h_{12} + \frac{1}{2})r_2 A_2 \frac{1}{r_2 - r_0} e^{r_0 t}(e^{(r_2 - r_0)T} - 1)$$
$$+ \mu(\gamma_1 h_{12} + \frac{1}{2})r_2 \widetilde{A}_2 \frac{1}{r_2 - r_0} e^{r_0 t}(e^{(r_2 - r_0)t} - e^{(r_2 - r_0)T}) \qquad \text{for } t > T$$

where $\overline{p}_1^d = \overline{p}_0^d + \mathrm{d}\overline{p}^d$. This solution path is continuous everywhere.[65] In the case $\mathrm{d}p_R^* > 0$ or $\mathrm{d}p^* > 0$ we have $\mathrm{d}\overline{p}^d = 0$ because foreign price changes affect both member countries symmetrically in the long run.[66] We assume the initial values of corresponding variables in both member countries to be identical such that $\overline{p}_0{}^d = \overline{y}_0{}^d = 0$. As a result of asymmetric parameters in interest rate transmission ($\widetilde{a}_2 > 0$) the difference of IS equations (A7) requires in conjunction with $\overline{y}_0{}^d = 0$ the additional assumption $a_{01} > a_{02}$.

Effects of an increase in the price of imported raw materials

Assume that the price of imported raw materials increases by one unit while keeping the foreign producer price level constant ($\mathrm{d}p_R^* = 1, \mathrm{d}p^* = 0$). Due to $\mathrm{d}\overline{\tau} = (\partial\overline{\tau}/\partial p_R^*)\mathrm{d}p_R^* < 0$ and $\mathrm{d}\overline{(m-p)} = (\partial\overline{(m-p)}/\mathrm{d}p_R^*)\mathrm{d}p_R^* < 0$ we derive the following immediate anticipation effect with respect to a future increase of p_R^* in T

$$\text{(A26)}\quad \tau(0+) = \overline{\tau}_0 + A_1 h_{11} + A_2 h_{12}$$
$$= \overline{\tau}_0 - e^{-r_1 T}\underbrace{(-\mathrm{d}\overline{\tau} + h_{12}\mathrm{d}\overline{(m-p)})}_{>0} < \overline{\tau}_0$$

On impact, we find a real depreciation. Its size varies inversely with T. Differentiating (A18) with respect to t yields

$$\text{(A27)}\quad \dot{\tau}(0+) = r_1 h_{11} A_1 + r_2 h_{12} A_2$$
$$= A_1(r_1 h_{11} - r_2 h_{12}) = A_1 d_{11}(h_{11} - h_{12})$$ [67]

With $d_{11} > 0$, (A27) gives $\dot{\tau}(0+) < 0$ such that the real depreciation continues in the subsequent adjustment process. Furthermore, (18) implies that real interest rates increase instantaneously. At the date of implementation τ remains continuous while the corresponding rate of change $\dot{\tau}$ adjusts discontinuously in T:

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[65] See *Wohltmann* and *Clausen* (2001) for the case without imported raw materials ($\mu = 1, \psi = 0$).

[66] In contrast, an increase in the level of foreign interest rates ($\mathrm{d}i^* > 0$) leads due to $\widetilde{a}_2 > 0$ and following (A7) and (A10) to $\mathrm{d}\overline{p}^d < 0$ und $\mathrm{d}\overline{q}^d < 0$.

[67] The last equality holds because the equation for the eigenvectors $D \cdot h_j = r_j h_j$ ($j = 1,2$; $D = (d_{ij})$) implies $d_{11} h_{1j} + d_{12} = r_j h_{1j}$ ($j = 1,2$).

$$\text{(A28)}\quad \dot{\tau}(T+)-\dot{\tau}(T-)=(\widetilde{A}_2-A_2)h_{12}r_2e^{r_2T}-A_1h_{11}r_1e^{r_1T}$$

$$=\frac{1}{h_{11}-h_{12}}\Big((-\mathrm{d}\overline{\tau})d_{11}(h_{11}-h_{12})-h_{11}h_{12}(r_2-r_1)\mathrm{d}\overline{(m-p)}\Big)$$

$$=-d_{11}\mathrm{d}\overline{\tau}-d_{12}\mathrm{d}\overline{(m-p)}>0$$ [68]

This implies real interest rates also fall discontinuously in T. The aggregate output level $q=q_1+q_2$ behaves according to (A11) as follows

$$\text{(A29)}\quad q=\overline{q}_0+\frac{1}{\mu\delta}\Bigg(r_1A_1\left[\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right)h_{11}-\frac{1}{2}\mu\right]e^{r_1t}$$

$$+r_2A_2\left[\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right)h_{12}-\frac{1}{2}\mu\right]e^{r_2t}\Bigg)\qquad \text{for } 0<t<T$$

$$\text{(A30)}\quad q=\overline{q}_1+\frac{1}{\mu\delta}r_2\widetilde{A}_2\left[\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right)h_{12}-\frac{1}{2}\mu\right]e^{r_2t}\qquad \text{for } t>T$$

with $\overline{q}_1=\overline{q}_0+\mathrm{d}\overline{q}$ and $\mathrm{d}\overline{q}<0$ in the case $\mathrm{d}p_R^*>0$. The reaction of output at the announcement date is

$$\text{(A31)}\quad q(0+)-\overline{q}_0=\frac{1}{\mu\delta}\Bigg(r_1A_1\left[\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right)h_{11}-\frac{1}{2}\mu\right]$$

$$+r_2A_2\left[\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right)h_{12}-\frac{1}{2}\mu\right]\Bigg)$$

$$=\frac{1}{\mu\delta}A_2\left(\frac{1}{2}\mu(r_1-r_2)+\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right)(r_2h_{12}-r_1h_{11})\right)$$

Due to $A_2>0$ and $r_2h_{12}-r_1h_{11}=d_{11}(h_{12}-h_{11})$ we find in conjunction with (A16) and (A17)

$$\text{(A32)}\quad q(0+)-\overline{q}_0>0\Leftrightarrow\frac{1}{2}\mu(r_1-r_2)+\left(\frac{1}{2}\mu\alpha_3+(1-\mu)\right)d_{11}(h_{12}-h_{11})>0$$

$$\Leftrightarrow d_{21}>\left(\alpha_3+\frac{2(1-\mu)}{\mu}\right)d_{11}$$

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[68] The last expression follows from $r_2-r_1=\frac{d_{12}}{h_{11}h_{12}}(h_{11}-h_{12})$.

$$\Leftrightarrow \big((1+\alpha_3)\mu + 2(1-\mu)\big) l_2 \tilde{b}_5 > (1-\alpha_3)\mu a_2 \alpha_3$$

This inequality holds empirically. The anticipation of a future increase in the price of imported raw materials has initially an expansionary impact on union output. At the date of implementation *T*, we find due to $\mathrm{d}\overline{q} < 0$ a contraction in output:

$$\text{(A33)} \quad q(T+) - q(T-) = \mathrm{d}\overline{q} + \frac{1}{\mu\delta}\left\{\left(\frac{1}{2}\mu\alpha_3 + (1-\mu)\right)\big(\dot{\tau}(T+) - \dot{\tau}(T-)\big) - \frac{1}{2}\mu\big((\dot{m}-\dot{p})(T+) - (\dot{m}-\dot{p})(T-)\big)\right\} < 0$$

The size of the contraction in *T* does *not* depend on the time span between the announcement and the implementation because the difference $\dot{\tau}(T+) - \dot{\tau}(T-)$ does not depend on *T* according to (A28) and also the jump in the growth rate of the real money stock in *T* is independent from *T*:

$$\begin{aligned}
\text{(A34)} \quad (\dot{m}-\dot{p})(T+) - (\dot{m}-\dot{p})(T-) &= (\tilde{A}_2 - A_2) r_2 e^{r_2 T} - A_1 r_1 e^{r_1 T} \\
&= \frac{1}{h_{11} - h_{12}}\Big((r_2 - r_1)\mathrm{d}\overline{\tau} + (r_1 h_{12} - r_2 h_{11})\mathrm{d}\overline{(m-p)}\Big) \\
&= \frac{d_{12}}{h_{11} h_{12}}\mathrm{d}\overline{\tau} + (d_{11} - (r_1 + r_2))\mathrm{d}\overline{(m-p)} \\
&= \frac{d_{12}}{h_{11} h_{12}}\mathrm{d}\overline{\tau} - d_{22}\mathrm{d}\overline{(m-p)}
\end{aligned}$$

In the special case $T = 0$ we find the following contraction in output in *T* due to $\tilde{A}_2 = -\mathrm{d}\overline{(m-p)} > 0$:

$$\begin{aligned}
\text{(A35)} \quad q(0+) - \overline{q}_0 &= \mathrm{d}\overline{q} + \frac{1}{\mu\delta}\left\{\left(\frac{1}{2}\mu\alpha_3 + (1-\mu)\right)\dot{\tau}(0+) - \frac{1}{2}\mu(\dot{m}-\dot{p})(0+)\right\} \\
&= \mathrm{d}\overline{q} + \frac{1}{\mu\delta} r_2 \tilde{A}_2 \left[\left(\frac{1}{2}\mu\alpha_3 + (1-\mu)\right) h_{12} - \frac{1}{2}\mu\right] > \mathrm{d}\overline{q}
\end{aligned}$$

i.e., the decline in steady state output exceeds the short-run effect. The consumer-oriented real money stock using the consumer price level as a deflator $m - p^c$ falls on impact in contrast to the producer-oriented real money stock $m - p$ due to $m - p^c = m - p + \alpha_3 \tau$. In the case $T > 0$

we find $\dot{m} - \dot{p}^c < 0$ for $0 < t < T$ as long as the weak condition $1 + \alpha_3 h_{12} > 0$ holds. This means that the decline of $m - p^c$ continues up to T. As long as T is sufficiently small, $m - p^c$ continues to decline even for $t > T$ while $m - p^c$ increases for $t > T$ as long as T is sufficiently large. This can be inferred from the solution to $\dot{m} - \dot{p}^c$:

(A36) $\dot{m} - \dot{p}^c = \widetilde{A}_2 r_2 (1 + \alpha_3 h_{12}) e^{r_2 t}$ for $t > T$

As $\widetilde{A}_2 > 0$ (< 0) for T sufficiently small (large), we find $\dot{m} - \dot{p}^c < 0$ (> 0) for T small (large).[69] In the steady state, $m - p^c$ declines more strongly than $m - p$ due to $\mathrm{d}\overline{\tau} / \mathrm{d}p_R^* < 0$.

Cyclical differences within the union are described by the output differential $q^d = q_1 - q_2$. It follows from (A7) and $\dot{\tau}(0+) < 0$ (given $T > 0$) so that q^d falls on impact discontinuously:[70]

(A37) $q^d(0+) = \dfrac{\widetilde{a}_2 (1 - \alpha_3)}{æ_1} \dot{\tau}(0+) < 0$

The subsequent decline in the price level differential $p^d = p_1 - p_2$ generates a continuous increase in q^d.[71] The decline of the internal price differential p^d can be inferred from (A23):

(A38) $\dot{p}^d(0+) = \dfrac{1}{2} \mu (\dot{m} - \dot{p})(0+) + \mu \gamma_1 \dot{\tau}(0+) < 0$

Eq. (A7) implies $q^d - \overline{q}^d = 0$ as long as the following equation holds in $t = t^U$:

(A39) $\widetilde{a}_2 (1 - \alpha_3) \dot{\tau} = \left((2b_4 + b_5) - (a_1 - b_1 - b_2) \psi \right) \left(p^d - \overline{p}^d \right)$

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69 Exploiting the relationship $\dot{p} - \dot{p}^c = \alpha_3 \dot{\tau}$ in conjunction with the behavior of $\dot{\tau}$ we find for $t > T$ $\dot{p} > \dot{p}^c > \dot{m}$ ($\dot{p} < \dot{p}^c < \dot{m}$) given T sufficiently small (large). In the period $0 < t < T$ we have $\dot{p}^c > \dot{p} > \dot{m}$. At the date of implementation T both rates of inflation $\dot{p}^c$ and $\dot{p}$ decline discontinuously.

70 In the special case $T = 0$ we find $q^d(0+) > 0$ as a result of $\dot{\tau}(0+) > 0$.

71 In the case that the difference in the semi-interest elasticities a_{21} and a_{22} is very large, q^d may even decline somewhat further after the initial discontinuous fall on impact. This case, however, is empirically very unrealistic and therefore considered irrelevant.

Note that $\overline{p}^d = \overline{q}^d = 0$ holds in the case $\mathrm{d}p_R^* > 0$. If a reversal point t^U in q^d exists in the interval $0 < t < T$, it satisfies the equation[72]

$$\text{(A40)}\quad \tilde{a}_2(1-\alpha_3)\left(-r_1 h_{11} e^{r_1 t} + r_2 h_{12} e^{r_2 t}\right) =$$

$$\left((2b_4 + b_5) - (a_1 - b_1 - b_2)\psi\right)\mu\left\{-\left(\gamma_1 h_{11} + \frac{1}{2}\right) r_1 \cdot \frac{1}{r_1 - r_0}\left(e^{r_1 t} - e^{r_0 t}\right)\right.$$

$$\left. + \left(\gamma_1 h_{12} + \frac{1}{2}\right) r_2 \frac{1}{r_2 - r_0}\left(e^{r_2 t} - e^{r_0 t}\right)\right\}$$

The solution to this equation does not depend on T and is identical for all symmetric shocks.[73] At the date of implementation, T another jump in q^d occurs, which due to (A28) turns out to be positive:

$$\text{(A41)}\quad q^d(T+) - q^d(T-) = \frac{1}{æ_1}\tilde{a}_2(1-\alpha_3)\left(\dot{\tau}(T+) - \dot{\tau}(T-)\right) > 0$$

Furthermore, we find in T an increase in the inflation differential:

$$\text{(A42)}\quad \dot{p}^d(T+) - \dot{p}^d(T-) = \frac{1}{2}\mu\left((\dot{m} - \dot{p})(T+) - (\dot{m} - \dot{p})(T-)\right) + \mu\gamma_1\left(\dot{\tau}(T+) - \dot{\tau}(T-)\right) > 0$$

For sufficiently large T, we find after the date of implementation a further increase in the price level differential p^d, which corresponds with a fall in the output differential q^d. In the case of a sufficiently small T, we find a decline in p^d beyond T, which turns out to be weaker than in the period between the date of anticipation and the date of implementation.

Robustness: Full Price Flexibility

In the presence of price flexibility, the previous state variables $\tau = p - 2(p^* + e)$ and $m - p$, where $p = p_1 + p_2$ have to be replaced by the redefined state variables $\tau_w = w - 2(p^* + e)$

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[72] (A40) follows from (A39) by inserting in the interval $0 < t < T$ the solution paths for $\dot{\tau}$ and p^d. The constant A_2 cancels hereby out.

[73] It is *not* possible to derive a closed-form solution for t^U because (A40) contains three different exponentials $\exp(r_j t)$ $(j = 0,1,2)$. For $t > T$, it is possible to solve (A39) for t because the solution paths for $\dot{\tau}$ and p^d contain in this case only the exponentials with the stable eigenvalues r_0 and r_2. For a sufficiently large T, a second reversal point t^* in q^d may exist with $t^* > T$. In contrast with the case $t^U < T$, t^* does depend here on T.

and $m-w$ where $w = w_1 + w_2$. The variable τ_w is a non-predetermined variable and $m-w$ is predetermined. The relationships between the original and the redefined state variables are given by:

$$\text{(A43)} \qquad \tau = \mu \cdot \tau_w + 2(1-\mu)(p_R^* - p^*)$$

$$(m-p) = (m-w) + (1-\mu)\tau_w + 2(1-\mu)(p^* - p_R^*)$$

It follows that

$$\text{(A44)} \qquad \tau - \bar{\tau} = \mu(\tau_w - \bar{\tau}_w)$$

$$\dot{\tau} = \mu \dot{\tau}_w$$

$$(m-p) - \overline{(m-p)} = (m-w) - \overline{(m-w)} + (1-\mu)(\tau_w - \bar{\tau}_w)$$

$$(\dot{m} - \dot{p}) = (\dot{m} - \dot{w}) + (1-\mu)\dot{\tau}_w$$

Defining the auxiliary matrices

$$\text{(A45)} \quad M = \begin{pmatrix} \mu & 0 \\ 1-\mu & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$$

the original aggregate system (A15) can be transformed as:

$$\text{(A46)} \quad \begin{pmatrix} \dot{\tau}_w \\ \dot{m} - \dot{w} \end{pmatrix} = M^{-1} D M \begin{pmatrix} \tau_w - \bar{\tau}_w \\ (m-w) - \overline{(m-w)} \end{pmatrix}$$

Using the definition $\tilde{D} = M^{-1} D M = (\tilde{d}_{ij})$ we have $\det \tilde{D} = \det D = \Delta_1 < 0$, i.e., the transformed system (A46) also exhibits the saddle path property. The coefficients $\tilde{d}_{ij}$ in the matrix $\tilde{D}$ are given by:

$$\text{(A47)} \quad \tilde{d}_{11} = d_{11} + d_{12} \frac{1-\mu}{\mu}$$

$$\tilde{d}_{12} = \frac{1}{\mu} d_{12}$$

$$\tilde{d}_{21} = \frac{\mu - 1}{\mu}(d_{11}\mu + d_{12}(1-\mu)) + d_{21}\mu + d_{22}(1-\mu)$$

$$\tilde{d}_{22} = \frac{\mu - 1}{\mu} d_{12} + d_{22}$$

Due to $\tilde{d}_{11} + \tilde{d}_{22} = d_{11} + d_{22}$, we find the eigenvalues of $\tilde{D}$ and D to be identical and of the form (A16). The corresponding eigenvectors $\tilde{h}_1$ and $\tilde{h}_2$ of $\tilde{D}$ are defined analogously to (A17):

(A48) $\tilde{h}_1 = \begin{pmatrix} \tilde{h}_{11} \\ 1 \end{pmatrix}, \qquad \tilde{h}_2 = \begin{pmatrix} \tilde{h}_{12} \\ 1 \end{pmatrix}$

with

$$\tilde{h}_{11} = \frac{\tilde{d}_{12}}{r_1 - \tilde{d}_{11}} = \frac{\frac{1}{\mu} d_{12}}{r_1 - d_{11} - d_{12}\frac{1-\mu}{\mu}}$$

$$\tilde{h}_{12} = \frac{\tilde{d}_{12}}{r_2 - \tilde{d}_{11}} = \frac{\frac{1}{\mu} d_{12}}{r_2 - d_{11} - d_{12}\frac{1-\mu}{\mu}}$$

The relationship between the eigenvectors $\tilde{h}_j$ and h_j $(j = 1,2)$ is as follows: Define $\tilde{h}_j$ to be the eigenvector of the matrix $\tilde{D} = M^{-1}DM$ belonging to the eigenvalue r_j we have

(A49) $M^{-1}DM \cdot \tilde{h}_j = r_j \cdot \tilde{h}_j$ or, respectively,

$$D(M\tilde{h}_j) = r_j (M\tilde{h}_j)$$

It follows that $h_j^* := M\tilde{h}_j$ is the eigenvector of the matrix D belonging to the same eigenvalue r_j, similarly also $\tilde{c}_j \cdot h_j^*$ with $\tilde{c}_j \in R \setminus \{0\}$ arbitrary. Assuming in particular

(A50) $\tilde{c}_j = \dfrac{r_j - d_{11} - d_{12}\frac{1-\mu}{\mu}}{r_j - d_{11}}$

we find $\tilde{c}_j \cdot h_j^* = h_j$ or, respectively,

(A51) $\tilde{h}_j = \frac{1}{\tilde{c}_j} M^{-1} \cdot h_j \qquad (j = 1,2)$

In the special case without imported raw materials $\mu = 1$, $\tilde{h}_j$ and h_j are identical.

The solutions paths of the transformed state variables τ_w and $m - w$ are analogous to (A18) and (A19) whereby it is assumed that $\tilde{h}_{11} > 0 > \tilde{h}_{12}$. In the expressions for A_2 and $\tilde{A}_2$ (cf. (A20)) it is necessary to replace $\mathrm{d}\bar{\tau}$ by $\mathrm{d}\bar{\tau}_w$, $\mathrm{d}\overline{(m-p)}$ by $\mathrm{d}\overline{(m-w)}$ and h_{ij} by $\tilde{h}_{ij}$. The same holds for the saddle path (A22). In the case of an isolated increase in imported raw materials $\mathrm{d}p_R^* > 0$, we again assume $A_2 > 0$. Furthermore, we now have $\tilde{A}_2 < 0$ for all $T \geq 0$ because it can be assumed that $\mathrm{d}\left(\overline{m-w}\right) > 0$. The state vector $(\tau_w, \quad m - w)'$ looks in the phase diagram like the one in *Figure 7*. Now we find $\dot{\tau}_w(0+) < 0$ and using (A44) $\dot{\tau}(0+) = \mu\dot{\tau}_w(0+) < 0$ for $T > 0$ as well as for $T = 0$ such that we yield $q^d(0+) < 0$ for all $T \geq 0$.

The dynamics of the internal price level differential $p^d = p_1 - p_2$ follow from (A23) in conjunction with (A44) as:

(A52) $\dot{p}^d = r_0(p^d - \bar{p}^d) + \frac{1}{2}\mu(\dot{m} - \dot{w}) + \mu\tilde{\gamma}_1\dot{\tau}_w$

with $\tilde{\gamma}_1 = \frac{1}{2}(1-\mu) + \mu\gamma_1 > 0$.

Due to $(\dot{m} - \dot{w})(0+) < 0$ and $\dot{\tau}_w(0+) < 0$ we find $\dot{p}^d(0+) < 0$ such after the negative impact effect p^d subsequently falls and q^d subsequently rises. We again find a sign change of q^d, which occurs for sufficiently large T before the date of implementation.[74]

[74] The point of reversal t^U is determined by (A39) where $\dot{\tau}$ has to be replaced by $\mu\dot{\tau}_w$.

References

Aoki, *M.* (1981), Dynamic Analysis of Open Economies. New York, London, Toronto.

Bhandari, *J. S.*, *S. J. Turnovsky* (1984), Materials Price Increases and Aggregate Adjustment in an Open Economy. A Stochastic Approach. European Economic Review 25, 151-182.

Buiter, *W. H.* (1978), Short-Run and Long-Run Effects of External Disturbances Under a Floating Exchange Rate. Economica 45, 251-272.

Buiter, *W. H.* (1979), Unemployment-Inflation Trade-Offs With Rational Expectations in an Open Economy. Journal of Economic Dynamics and Control 1, 117-141.

Buiter, *W. H.* (1984), Saddlepoint Problems in Continuous Time Rational Expectations Models: A General Method and Some Macroeconomic Examples. Econometrica 52, 665-680.

Buiter, *W. H.*, *M. Miller* (1982), Real Exchange Rate Overshooting and the Output Cost of Bringing down Inflation. European Economic Review 18, 85-123.

Cecchetti, *S. G.* (1999), Legal Structure, Financial Structure, and the Monetary Policy Transmission Mechanism. Economic Policy Review, Federal Reserve Bank of New York, 9-28.

Claassen, E.-M. (1980), Grundlagen der makroökonomischen Theorie. München.

Clausen, *V.* (2001), Asymmetric Monetary Transmission in Europe. Berlin, Heidelberg, New York.

Clausen, *V.* (2002), Unterschiedliche makroökonomische Strukturen, wirtschaftliche Integration und einheitliche Geldpolitik in Europa. Jahrbücher für Nationalökonomie und Statistik 222, 1-21.

Clausen, *V., H.-W. Wohltmann* (2002), Monetary and Fiscal Policy Dynamics in an Asymmetric Monetary Union, University of Essen Discussion Paper No. 119, Second revision, September 2002, Essen.

Devereux, *M. B., D. D. Purvis* (1990), Fiscal Policy and the Real Exchange Rate. European Economic Review 34, 1201-1211.

Dornbusch, *R.* (1976), Expectations and Exchange Rate Dynamics. Journal of Political Economy 84, 1161-1176.

Findlay, R., C. A. Rodriguez (1977), Intermediate Imports and Macroeconomic Policy under Flexible Exchange Rates. Canadian Journal of Economics 10, 208-217.

Jarchow, *H.-J.* (1998), Theorie und Politik des Geldes I. 10., überarbeitete und wesentlich erweiterte Auflage. Göttingen.

OECD (2000), EMU – One Year on. Paris.

Turnovsky, *S. J.* (2000), Methods of Macroeconomic Dynamics. Second Edition. Cambridge.

van der Ploeg, *F.* (1990), International Interdependence and Policy Coordination in Economies with Real and Nominal Wage Rigidity. In: *Courakis, A. S.*, *M. P. Taylor* (eds.), Private Behaviour and Government Policy in Interdependent Economies. Oxford, 403-450.

Wohltmann, *H.-W.* (1993), Die Auswirkungen inländischer Geld- und Fiskalpolitik sowie ausländischer Preisschocks bei importierten Rohstoffen. Zeitschrift für Wirtschafts- und Sozialwissenschaften 113, 557-577.

Wohltmann, *H.-W.*, *F. Bulthaupt* (1999), Intertemporale Output- und Beschäftigungseffekte angebotsorientierter Politikmaßnahmen in einer Währungsunion. Jahrbücher für Nationalökonomie und Statistik 218, 1-22.

Wohltmann, *H.-W.*, *V. Clausen* (2001), Geldpolitik, Fiskalpolitik und ausländische Zinsschocks in einer asymmetrischen Währungsunion. Kredit und Kapital 34, 149-177.