Monetary and Fiscal Policy Dynamics in an Asymmetric Monetary Union

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Abstract

This paper investigates the dynamic effects of monetary and fiscal policy in a monetary union, which is characterized by asymmetric interest rate transmission. This asymmetry gives rise to intertemporal reversals in the relative effectiveness of policy on member country outputs. The direction and the number of these reversals depend on whether policies are unanticipated or anticipated. We also study the coordination between monetary and fiscal policy in a monetary union. Monetary policy may completely stabilize European output after unanticipated fiscal policy shocks. With anticipated fiscal policy shocks, complete stabilization throughout the overall adjustment process requires monetary policy to be time-inconsistent.

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Contents:

1. Introduction

Prior to the establishment of European Monetary Union (EMU) in 1999, the policy discussion focused almost exclusively on asymmetric shocks as a source of differential macroeconomic developments and adjustment problems within EMU. This emphasis implicitly assumed the macroeconomic structures in the member countries to be symmetric. In the presence of macroeconomic asymmetries, however, even symmetric shocks may be responsible for adjustment problems within EMU.[1] This also raises the question whether the common monetary policy by the European Central Bank (ECB) affects the EMU member countries in a symmetric fashion. While the ECB focuses on euro wide macroeconomic variables and is unable to respond to differential output developments within the EMU, it is nevertheless important to know the extent of comovement of macroeconomic variables such as output and inflation within the monetary union (*ECB,* 1999a and 1999b).

Differences in financial structure, in the degree of openness towards the rest of the world or in the nature of the supply side are potential sources of asymmetric monetary transmission in EMU.[2] To date, it is fair to conclude that most of the discussion has concentrated on asymmetries in financial structure and their implications for the impact of changes in interest rates on aggregate demand.[3]

This paper also focuses on asymmetries in interest rate transmission in EMU and investigates the implications of this asymmetry in a *dynamic* macroeconomic context. The theoretical approach analyzes over time the spillover effects within EMU as well as between EMU and the rest of the world arising from interest rate, real exchange rate and output developments.

Asymmetric interest rate transmission may originate from two sources: It may be rooted in a different pass-through behavior of banks in the transmission of policy impulses to the private sector. Asymmetries in the *strength* of pass-through behavior may stem from differences in legal structure of banking systems across the EMU member countries (*Cecchetti,* 1999). Asymmetries in the *speed* of pass-through behavior of banks also receive attention (*Cottarelli and Kourelis,* 1994 and *Mojon*, 2000). Assuming symmetric banking behavior, another source of asymmetric interest rate transmission may be different income and wealth effects of households and firms, which in turn originate in differences in financial structure across EMU member countries (*Bank for International Settlements,* 1994 and 1995 and *ECB*, 2000). In fact, recent empirical evidence supports the view that EMU member countries may respond differently to interest rate impulses of the ECB.[4]

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[1] *Clausen* (2001) and *De Grauwe* (2000) contrast the monetary policy implications of asymmetric shocks and asymmetric macroeconomic structures.

[2] *Clausen* (2001) and *Dornbusch et al.* (1998) survey and evaluate various sources of asymmetric monetary transmission in Europe.

[3] *Bank for International Settlements* (1994 and 1995), *Cecchetti* (1999), *Kakes* (2000) and *De Bondt* (2000).

[4] See for example *Ramaswamy and Sloek* (1998), *Dornbusch et al.* (1998), and *Ehrmann* (2000). A recent survey is provided by *Guiso et al.* (1999).

The purpose of this paper is to examine the implications of asymmetric interest rate transmission for the policy effects across the EMU member countries. Most of the early work on stabilization policy in a monetary union specifies static and symmetric models (e.g., *Läufer and Sundararajan* 1994). Somewhat later work introduces asymmetries into static models of a monetary union (e.g., *Daseking,* 1996). This paper is among the first to explore the implications of asymmetries in a *dynamic* model of a small monetary union.[5] The dynamic setup allows for the explicit computation of adjustment paths in response to economic policy. Furthermore, it will be possible to analyze the impact of anticipated policy changes.

We show that asymmetric interest rate transmission gives rise to *intertemporal reversals* in the relative effectiveness of policy on member country outputs. The direction and the number of these reversals depend on whether policies are unanticipated or anticipated. We also study the *coordination* between monetary and fiscal policy in a monetary union. Monetary policy may perfectly stabilize union output after unanticipated fiscal policy shocks. With anticipated fiscal policy shocks, however, complete stabilization throughout the overall adjustment process requires monetary policy to be time-inconsistent. Fiscal policies are often announced in advance, and we believe that this paper addresses for the first time in a dynamic context the specific challenges posed to a stability–oriented monetary policy by the ECB.

The paper is organized as follows: Section 2 presents a dynamic model of a monetary union. Two member countries are taken to be perfectly symmetric except for the semi-interest elasticity of aggregate demand. In the following it is assumed that aggregate demand in member country 1 is more strongly affected by the level of short-term real interest rates. The sections 3 and 4 contain a discussion of the implications of this particular asymmetry for the dynamic effects of monetary and fiscal policy. Section 5 evaluates the robustness of the results with respect to modifications of the model. Section 6 investigates the implications of a policy mix between monetary and fiscal policy. Monetary policy is assumed to take a contractionary stance in response to an expansionary fiscal shock, which may be unanticipated or anticipated. Section 7 concludes and discusses the implications for the design of monetary and fiscal policy in Europe. Section 8 contains the appendix with detailed derivations of the main analytical results.

2. The Model

The monetary union consists of two member countries of equal size. It is taken to be small relative to the rest of the world. Equations (1) to (6) model its demand side. The supply side is given by the equations (7) to (12).

(1)

$$y_1 = \left(a_{01} + a_1 y_1 - a_{21}\left(i_1 - E(\dot{p}_1^c)\right)\right) + g_1 + \left(b_0 - b_1 y_1 + b_2 y_2 + b_3 y^* - b_4\left(p_1 - p_2\right) - b_5 \tau_1\right)$$

(2)

$$y_2 = \left(a_{02} + a_1 y_2 - a_{22}\left(i_2 - E(\dot{p}_2^c)\right)\right) + g_2 + \left(b_0 - b_1 y_2 + b_2 y_1 + b_3 y^* - b_4\left(p_2 - p_1\right) - b_5 \tau_2\right)$$

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[5] See also *Wohltmann and Clausen* (2001).

(3) $\tau_1 = p_1 - \left(p^* + e\right)$

(4) $\tau_2 = p_2 - \left(p^* + e\right)$

(5) $m = \left(p_1^c + l_0 + l_1 y_1 - l_2 i_1\right) + \left(p_2^c + l_0 + l_1 y_2 - l_2 i_2\right)$

(6) $i_1 = i_2 = i^* + E(\dot{e})$

(7) $\dot{p}_1 = \dot{w}_1 = E(\dot{p}_1^c) + \delta\left(y_1 - \bar{y}_1\right)$

(8) $\dot{p}_2 = \dot{w}_2 = E(\dot{p}_2^c) + \delta\left(y_2 - \bar{y}_2\right)$

(9) $p_1^c = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3\left(p^* + e\right)$, $\alpha_1 + \alpha_2 + \alpha_3 = 1$

(10) $p_2^c = \alpha_1 p_2 + \alpha_2 p_1 + \alpha_3\left(p^* + e\right)$

(11) $\bar{y}_1 = f_0 + f_1 \bar{\tau}_1 + f_2\left(\overline{p_1 - p_2}\right)$

(12) $\bar{y}_2 = f_0 + f_1 \bar{\tau}_2 + f_2\left(\overline{p_2 - p_1}\right)$

The notation of the variables is as follows: y denotes real output, i the short-term nominal interest rate, g real government expenditure, p the price level of the domestically produced good, e the common exchange rate with respect to the US, which is expressed as the amount of euro for one dollar, τ the external terms of trade, m the nominal money stock, w the nominal wage, $\bar{y}$ the natural level of output and p^c the consumer price index. All variables are endogenous except for real government expenditure and the nominal money stock. A dot above a variable denotes the right hand side derivative with respect to time. A bar denotes the steady state value of the variable. E represents the expectations operator.

The variables of the member countries are indexed by 1 and 2. Variables without index refer to aggregate values for the monetary union. The variables of the rest of the world are taken as exogenous and marked with an asterisk (*). With the exception of interest rates all variables are expressed in logarithms.[6] Due to the log-linear structure of the model the behavioral parameters are to be interpreted as elasticities or semi-elasticities. All parameters are assumed to be positive.

Equations (1) and (2) represent the IS-curves in both member countries. [7] In the short-run, output is demand-determined. The demand for domestic output in country 1 (y_1) depends on (output-related) private consumption, real-interest-sensitive investment demand, real government expenditure g_1, and the trade balance. As in *Turnovsky* (1986), the real interest rate is computed using expected inflation based on the consumer price index. We assume rational inflationary expectations $(E(\dot{p}_j^c) = \dot{p}_j^c; j = 1,\ 2)$. Imports from the second member country and from the US depend on domestic output ($b_1 y_1$). Exports to the second member country and to the US depend on the respective foreign outputs ($b_2 y_2$, $b_3 y^*$). The trade balance is also affected by the internal terms of trade with respect to the trading partner in the monetary union (p_1-p_2) and by the external terms of trade defined in (3) and (4). The

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[6] This implies that the aggregate output level $y = y_1 + y_2 = \ln Y_1 + \ln Y_2 = 2\ln(Y_1 \cdot Y_2)^{1/2}$ is equal to twice the *geometric mean* of the underlying (unlogarithmized) output variables Y_1 and Y_2.

[7] Logarithmic *Taylor* approximations of IS curves are discussed in more detail in *Bhandari* (1997) and *Argy* (1994).

IS equation for member country 2 has an analogous structure. Corresponding parameters are assumed to be identical except for the interest-sensitivity of aggregate demand, which is allowed to differ across the member countries of the monetary union ($a_{21} \neq a_{22}$). We assume in the following $a_{21} > a_{22}$.

Equation (5) represents money market equilibrium (LM) within the monetary union. The income elasticities and semi-interest elasticities of money demand are assumed to be identical. The aggregate European money stock *m* is exogenous due to the assumption of flexible external exchange rates. In contrast, the distribution of the money stock within the monetary union is endogenous and determined by money demand in the respective countries. As in *Turnovsky* (1986), the consumer price indices p_1^c and p_2^c are used as deflators of money demand.

Equation (6) reflects perfect capital mobility within EMU as well as between EMU and the US. Interest rates across EMU are identical ($i_1 = i_2$) since the nominal exchange rate is fixed within the union. In contrast, the exchange rate towards the rest of the world is allowed to be flexible. In this case, the level of interest rates in EMU will always equal the level in the US plus the expected rate of depreciation of the euro. Exchange rate expectations are assumed to be rational ($E(\dot{e}) = \dot{e}$). As in standard asset market approaches to the exchange rate we treat the nominal exchange rate as a jump variable and producer prices as sticky.[8]

Producer prices are set as a constant markup over wages such that the change in the producer price index equals the rate of change in nominal wages (eq. (7) and (8)). Nominal wages, in turn, are determined by expected inflation and by the current output gap. The responsiveness of nominal wages with respect to the current output gap is determined by the parameter δ. The inflationary expectations by wage setters are assumed rational and formed on the basis of the consumer price index $(E(\dot{p}_j^c) = \dot{p}_j^c; j = 1,2)$. Equations (9) and (10) contain the definitions of the consumer price indices in the two EMU member countries. The corresponding weights add up to unity.

Equations (11) and (12) model the long-run equilibrium level of output. Nominal wages are indexed to the consumer price index while firms take their employment decision on the basis of the producers' real wage, which uses the price level of the domestically produced good as a deflator of nominal wages. It follows that the equilibrium levels of employment and output are positive functions of the equilibrium internal and external terms of trade (*Wohltmann,* 1994a).

Long run equilibrium

We abstract from factors generating sustained economic growth (such as technological progress or changes in the capital stock) so that in long run equilibrium real output is

[8] On impact, interest rates in Europe, the terms of trade τ_1 and τ_2, the consumer prices p_1^c and p_2^c as well as the output levels y_1 and y_2 are also allowed to adjust instantaneously. Furthermore, all rates of change such as $\dot{e}$, $\dot{p}$ and $\dot{p}^c$ are allowed to react immediately to exogenous shocks.

assumed to be stationary. Formally, this implies $\overline{\dot{y}_j} = 0$ (j=1, 2). In contrast, nominal variables such as producer prices, consumer prices or the nominal exchange rate are allowed to be non-stationary when the rate of growth of the money stock is positive (i.e., $\dot{m} > 0$). Given a constant positive rate of monetary growth in the long run, ($\overline{\dot{m}} = \dot{m} = const > 0$), the rates of inflation and the rate of change of the exchange rate remain stationary ($\overline{\ddot{p}_j} = \overline{\ddot{p}_j^c} = \overline{\ddot{e}} = 0$). This implies in conjunction with the interest parity condition (6) that nominal and real interest rates in Europe are ultimately constant: $\overline{i_1} = \overline{i_2} = const_1$ and $\overline{i_1 - \dot{p}_1^c} = \overline{i_2 - \dot{p}_2^c} = const_2$. Total differentiation of the IS equations (1) and (2) and the LM equation (5) generates further steady state properties: $\overline{\dot{\tau}} = 0 = \overline{\dot{m}} - \overline{\dot{p}^c} = \overline{\dot{p}_1} - \overline{\dot{p}_2}$ where $\tau = \tau_1 + \tau_2$ and $p^c = p_1^c + p_2^c$. Due to $\tau = p - 2(p^* + e)$ and $p = p_1 + p_2 = p^c + \alpha_3 \tau$ we also find: $\overline{\dot{m}} = \overline{\dot{p}} = \overline{\dot{w}}$ and $\frac{1}{2}\overline{\dot{m}} = \overline{\dot{p}_1} = \overline{\dot{p}_2} = \overline{\dot{e}}$.[9] The price adjustment equations (7) and (8) imply that the output levels y_1 and y_2 ultimately reach their long run equilibrium levels $\overline{y}_1$ and $\overline{y}_2$, which in turn are endogenously determined by (11) and (12).

Initial equilibrium conditions

The initial equilibrium of the system is normalized such that corresponding *variables* in the two economies are identical. For example, the initial values of the output variables y_1 and y_2 are identical because both member countries of the monetary union are assumed to be of equal size. Furthermore, we assume that the initial rate of monetary growth equals zero ($\dot{m}_0 = 0$) and the level of the money stock is initially constant ($m_0 = const_3$). This implies constant and identical initial steady state values of the price levels at t = 0 ($\overline{p}_{10} = \overline{p}_{20} = const_4$). Some asymmetries in *parameters* need to be introduced. Due to the assumption $a_{21} > a_{22}$ we also have to assume without loss of generality that $a_{01} > a_{02}$.[10]

[9] It would be incorrect to work with $\overline{\dot{e}} = \overline{\dot{m}}$ as intuition might suggest. The monetary aggregate m is derived from the underlying LM equation through log-linearization. Denoting the underlying levels of the aggregate money stock with M and of money demand with $P_j^c L_j (j = 1,2)$, money market equilibrium implies $M = P_1^c L_1 + P_2^c L_2$. Provided that the initial values of money demand are identical in both member countries and using the approximation $dM / M_0 \approx \ln M - \ln M_0$ it is possible to derive m = 2(ln$M$-ln 2). It follows that $\frac{1}{2}\dot{m} = \frac{\dot{M}}{M}$, i.e., $\frac{1}{2}\dot{m}$ equals the growth rate of the underlying monetary aggregate and the steady state relationship is then given by $\frac{1}{2}\overline{\dot{m}} = \overline{\dot{e}}$.

[10] The difference of (1) and (2): $\mu \cdot y^d = \frac{1}{2}(a_{21} - a_{22})[(1-\alpha_3)\dot{\tau} - 2i^*] + a_{01} - a_{02} + g^d - (2b_4 + b_5)p^d$ with $y^d = y_1 - y_2$, $g^d = g_1 - g_2$, $p^d = p_1 - p_2$, $\mu = 1 - a_1 + b_1 + b_2$. The initial equilibrium conditions $\overline{y}_0^d = \overline{g}_0^d = \overline{p}_0^d = \overline{\dot{\tau}}_0 = 0$ then imply that $a_{01} - a_{02} = (a_{21} - a_{22})i^* > 0$.

Solution of the model

Models of a monetary union are commonly solved using the decomposition method by *Aoki* (1981). *Aoki* shows that if the underlying macroeconomic structure of the two EMU member countries is symmetric it is convenient to analyze two subsystems of the model. An aggregate system describes the aggregate behavior of EMU with respect to the rest of the world. A difference system models the behavior within the monetary union. The solutions from the two subsystems are then combined to generate the solutions for the variables in the model.

In this model, both EMU member countries are assumed to be identical except for the semi-interest elasticity of aggregate demand. In order to continue to exploit the decomposition method introduced by *Aoki,* it is assumed that the weights of the internal producer prices in the consumer price index are identical across the EMU member countries ($\alpha_1 = \alpha_2$).[11] With this assumption, the adjustment dynamics can still be illustrated in phase diagrams. Moreover, it is possible to isolate the impact of different interest sensitivities of aggregate demand from the implications of different real interest rate developments across the union. With $\alpha_1 = \alpha_2$, the consumer price indices as well as the rates of inflation based on the consumer price index are identical in the two member countries $(\dot{p}_1^c = \dot{p}_2^c)$. In conjunction with the uniform short-term interest rate within EMU this implies that the short-term *real* interest rate is identical across the EMU member countries $(i_1 - \dot{p}_1^c = i_1 - \dot{p}_2^c)$. It follows that any differences in output are entirely driven by the asymmetric interest sensitivities of aggregate demand across EMU. [12]

Aggregate System

The aggregate system is derived by adding the relevant equations for the two countries. Its dynamic behavior can be described by a two-dimensional state space representation. As we allow for positive rates of monetary growth and therefore non-stationary values of prices and nominal exchange rates we use - following *Buiter* and *Miller* (1982) - the terms of trade $\tau = p - 2(p^* + e)$ as and the real money stock $m - p$ as state variables. The terms of trade are non-predetermined whereas the real money stock $m - p$ is taken as predetermined. [13] The aggregate system is described by the following dynamic system (see *Wohltmann,* 1994a and the appendix):

$$(13)\ \dot{\tau} = -\frac{\delta b_5}{\alpha_3 \lambda - \delta(1-\alpha_3) a_2}(\tau - \overline{\tau})$$

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[11] See *Wohltmann et al.* (1998). The assumption of identical preferences for the final goods produced within the two countries is standard in the “new open economy macroeconomics” (*Lane*, 1995), and especially in the redux model by *Obstfeld* and *Rogoff* (1995).

[12] On the other hand, this assumption may be considered unduly restrictive and somewhat counterfactual. We therefore consider in section 5 the implications of assuming $\alpha_1 > \alpha_2$ and evaluate the robustness of our results.

[13] In contrast, the consumption based real money stock $m - p^c$ responds discontinuously because p^c contains the jump variable *e*. Due to $m - p^c = m - p + \alpha_3 \tau$, we find the difference between both measures of the real money stock to be *proportional* with τ .

(14) $\dot{m} - \dot{p} = c(\tau - \bar{\tau}) + \frac{1}{l_2}\left((m-p) - \left(\overline{m-p}\right)\right)$

where $\tau = \tau_1 + \tau_2$, $p = p_1 + p_2$, $y = y_1 + y_2$, $\lambda = 1 - a_1 + b_1 - b_2 > 0$, $a_2 = \frac{1}{2}(a_{21} + a_{22})$ and $c = \frac{\delta + l_1\alpha_3/l_2}{\alpha_3\lambda - \delta(1-\alpha_3)a_2} b_5 + \frac{\alpha_3}{l_2} > 0$. Given $\alpha_3\lambda - \delta(1-\alpha_3)a_2 > 0$, the determinant of the system matrix is unambiguously negative so that we find one stable and one unstable root.[14] The aggregate system displays saddle path stability because the number of predetermined variables equals the number of stable characteristic roots. The characteristic roots can be read from the principal diagonal as: $r_1 = -\frac{\delta b_5}{\alpha_3\lambda - \delta(1-\alpha_3)a_2} < 0$ and $r_2 = {}^{1}\!/_{l_2} > 0$.

The dynamics of adjustment are illustrated in a phase diagram using the terms of trade and the real money stock as state variables. The implications for European output can be derived from the price adjustment equations (7) and (8) in conjunction with the definitions of the consumer price index and of the external terms of trade. It can be easily shown that the sign of the rate of change of the terms of trade provides unambiguous information on the level of European output relative to full employment output:

(15) $y = \bar{y} + \frac{\alpha_3}{\delta}\dot{\tau}$.

As long as the euro experiences a real appreciation (i.e., $\dot{\tau} > 0$), European output exceeds the full employment level. Put alternatively, output above capacity induces upward price adjustment and a real appreciation towards the rest of the world.

Difference System

Output developments across the EMU member countries are represented by the following relationship:

(16) $y_1 - y_2 = y^d = \bar{y}^d + \frac{1-\alpha_1+\alpha_2}{\delta}\dot{p}^d = \bar{y}^d + \frac{1}{\delta}\dot{p}^d \qquad (\alpha_1 = \alpha_2)$

where the inflation differentials across Europe are governed by:

(17) $\dot{p}^d = \dot{p}_1 - \dot{p}_2 = r_0(p^d - \bar{p}^d) - \frac{r_0\,\tilde{a}_2(1-\alpha_3)}{2b_4 + b_5}\dot{\tau}$

[14] See *Buiter and Miller* (1982) and *Wohltmann* (1993, 1994b) for a more detailed discussion of this class of saddle path models and an economic interpretation of the stability condition.

with $r_0 = -\frac{\delta(2b_4 + b_5)}{(1-\alpha_1+\alpha_2)\mu} = -\frac{\delta(2b_4+b_5)}{\mu} < 0$ and $\tilde{a}_2 = \frac{1}{2}(a_{21} - a_{22}) > 0$ since $a_{21} > a_{22}$.[15]

These equations illustrate the link between the aggregate system and the difference system. The rate of change of the terms of trade $\dot{\tau}$ drives inflation differentials within EMU (17), which in turn affect relative output developments (16). Note that in this model the rate of change of the terms of trade implicitly reflects the level of real interest rates in Europe relative to the steady state level:

(18) $i_1 - \dot{p}_1^c = i_2 - \dot{p}_2^c = i^* - \frac{1}{2}(1-\alpha_3)\dot{\tau}$.

As long as the euro experiences a real appreciation (i.e., $\dot{\tau} > 0$), real interest rates in Europe are necessarily below their steady state level i^*.[16] These real interest rate developments represent the mechanism by which the asymmetric output developments across EMU are incurred. The relevance of this source of asymmetric output developments across EMU increases with the size of the differences between the interest-sensitivities of aggregate demand ($\tilde{a}_2$ in (17)).

The overall solution for macroeconomic variables of the individual EMU member countries are derived on the basis of a linear combination of the solutions above. For example, member country outputs are gathered using the following formula:

(19) $y_{1/2} = \frac{1}{2}\left[(y_1 + y_2) \pm (y_1 - y_2)\right]$.

The model and the solution technique allow for the discussion of stabilization policy in a monetary union in the presence of asymmetric interest rate transmission across the EMU member countries.

3. Effects of Monetary Policy

The implications of ECB policy are discussed in the form of an increase in the growth rate of the European money stock. These policies may be unanticipated or anticipated. In the discussion of the policy effects, we start with the impact in the steady state and then turn to the adjustment dynamics.

An increase in the *growth rate* of the European money stock ($d\dot{m} > 0$) leaves in the steady state the external terms of trade τ unchanged (*Wohltmann,* 1994a). This implies a one-to-one increase in aggregate inflation and in nominal interest rates while the

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[15] Equation (17) illustrates that the dynamic behavior of the difference system reduces to a (stable) differential equation, which describes the price level differential in Europe p^d. For details see the appendix.

[16] Here we assume for simplicity that the foreign rate of inflation equals zero such that foreign nominal and real interest rates are identical.

level of real interest rates remains unchanged. Money demand depends on the *nominal* interest rate such that its rise in the steady state is accompanied by a fall in the real money stocks $m - p$ and $m - p^c$. In contrast, aggregate demand depends on the *real* interest rate, which remains unchanged. These implications of an increase in the growth rate of the European money stock are illustrated in *Figure 1*. The initial equilibrium is given by Q_0. The corresponding stable branch of the saddle path is denoted by S_0 while the unstable branch is horizontal and given by I_0. The increase in the growth rate of the money stock induces a leftward shift of the stable branch of the saddle path ($S_0 \rightarrow S_1$) while leaving the locus of the unstable path unchanged. The new steady state equilibrium is given by Q_1.

The dynamic adjustment towards the new steady state Q_1 depends on whether the increase in the growth rate of the money stock is unanticipated or anticipated. An *anticipated* increase in the growth rate of the money stock means that the ECB credibly announces at t=0 that at some future date T the growth rate of the money stock will be raised. This induces on impact a real depreciation of the euro ($Q_0 \rightarrow A$). Between the announcement and the implementation of the policy, prices in Europe gradually rise in anticipation of the expansionary monetary policy. These inflationary developments bring about a real appreciation and a fall in the real money stock ($A \rightarrow B$). In technical terms, the system moves along a trajectory, which still refers to the original equilibrium and gradually approaches the unstable branch I_0.

At the date of implementation T, the terms of trade and the real money stock remain unchanged since the policy was perfectly anticipated. After the implementation of the increase in the growth rate of the money stock, the rate of inflation exceeds the growth rate of the money stock and the rate of currency depreciation such that real money stock continues to fall and the terms of trade continue to rise.

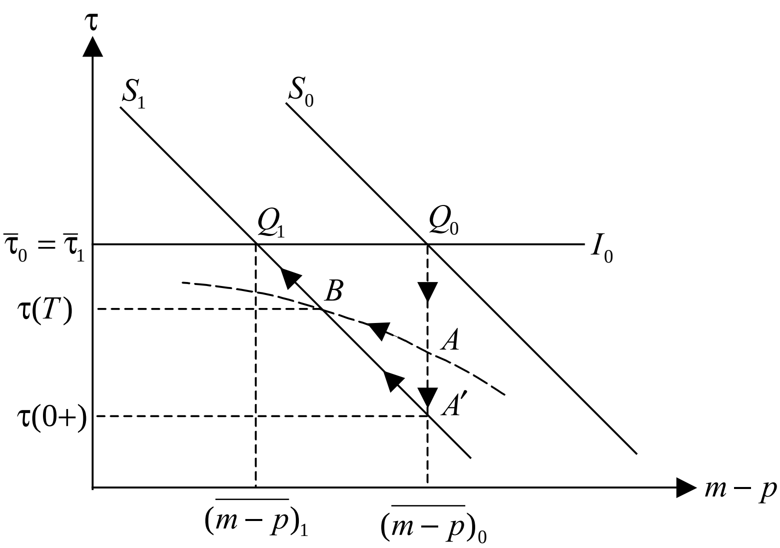

Figure 1: Aggregate Effects of an Increase in European Money Growth

An *unanticipated* increase in the growth rate of the money stock at time t=0 is a special case. It induces on impact a real depreciation towards the rest of the world. In contrast, the producer price level and the nominal money stock are initially given such that the real money stock remains unchanged. In consequence, *Figure 1* illustrates the impact effect as a vertical movement ($Q_0 \rightarrow A'$). The level of the terms of trade, which is attained immediately after the policy disturbance is denoted by $\tau(0+)$. During the subsequent adjustment process, the euro experiences a real appreciation until the initial terms of trade are reestablished. The real money stock declines throughout the adjustment period. This implies that the rate of inflation exceeds the new rate of growth of the money stock throughout the adjustment process.

Irrespective whether monetary policy is unanticipated or anticipated, the rate of change of the terms of trade $\dot{\tau}$ is positive throughout the subsequent adjustment process. This implies according to (18) that the level of *real* interest rates in Europe necessarily falls in response to the expansionary monetary policy by the ECB.[17] Stimulated by the real depreciation of the euro and by the fall in real interest rates, European output $y = y_1 + y_2$ exceeds the equilibrium level of output throughout the subsequent adjustment process (*Figure 2*). After the initial jump, aggregate output converges monotonically towards the unchanged steady state level $\bar{y}$. This qualitative pattern of output adjustment holds for both types of monetary policy. In quantitative terms, the impact of an unanticipated monetary policy is relatively stronger.[18]

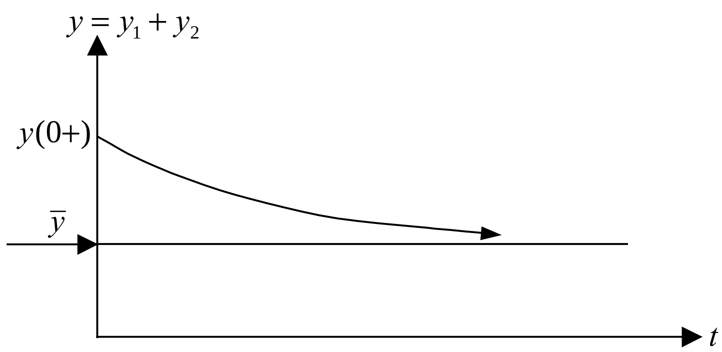

Figure 2: Aggregate Output Effects of an Increase in European Money Growth

We now turn to the differential output effects within EMU, which are represented in *Figure 3*. The increase in the growth rate of the money stock, whether unanticipated or anticipated, was demonstrated to lower on impact European real interest rates. Given that country 1 has a relatively higher real interest sensitivity of aggregate demand it will initially experience stronger output effects and inflationary pressure. The difference between the two member country outputs $y^d = y_1 - y_2$ is therefore initially positive ($y^d(0+) > 0$). In the course of the adjustment process, real interest rates as

[17] In contrast, *nominal* interest rates rise. This follows from the money market equation. The nominal money stock remains in both cases initially unchanged. Following the depreciation of the euro, import and consumer prices rise which reduces the real money supply. However, money demand increases with the rise in output. The excess demand for money drives up nominal interest rates.

[18] The size of the initial jump of τ, $\dot{\tau}$ and, consequently, in y falls with increasing *T*. This is a well-known result, which also holds in other models in this class (see *Turnovsky* (2000), p. 191).

well as the terms of trade begin to rise towards their initial steady state values. This dampens the initial expansionary impact on aggregate demand leading over time to a fall in output in both member countries. This dampening effect will be comparatively stronger in country 1 where the real interest sensitivity of aggregate demand is higher. Furthermore, from the perspective of country 1, the internal and external terms of trade start to increase as a consequence of the stronger initial stimulus leading to a deterioration of the overall trade balance of country 1.

It follows that the fall in output over time is stronger in country 1 than in country 2. The difference variable $y^d = y_1 - y_2$ begins to fall and at some stage during the adjustment process, which is labeled t^*, a reversal in the relative effectiveness of ECB policy with respect to output in the EMU member countries takes place. As long as $t < t^*$, monetary policy is more effective with respect to output in country 1 with the larger interest sensitivity of aggregate demand. For $t > t^*$, the opposite holds.[19]

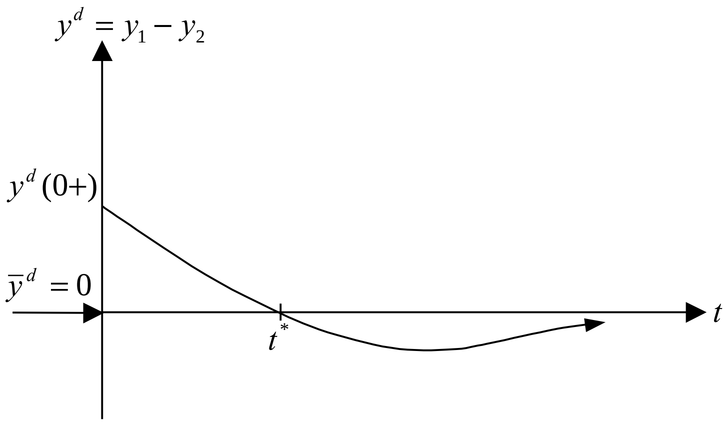

Figure 3: Differential Output Effects of an Increase in European Money Growth

The timing of the policy effectiveness reversal t^* depends on the characteristic roots of the system:[20]

$$(20)\quad t^* = \frac{1}{r_0 - r_1}\ln\left(\frac{r_1}{r_0}\right) \text{ where } r_0 = -\frac{\delta(2b_4 + b_5)}{(1-\alpha_1+\alpha_2)\mu} \text{ and } r_1 = -\frac{\delta b_5}{\alpha_3\lambda - \delta(1-\alpha_3)a_2}.$$

With both $r_0 < 0$ and $r_1 < 0$, t^* is necessarily positive. Note that the timing of the policy effectiveness reversal does *not* depend on whether monetary policy is unanticipated or anticipated. Furthermore, in the case of anticipated policy, t^* is also independent from the time span T between the announcement and the implementation of policy. This means that the policy effectiveness reversal may occur before or after the announced date of implementation T.

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[19] Formally, the variable y^d changes sign at t^*, being positive for $t < t^*$ and negative for $t > t^*$.

[20] This formula is derived in the appendix (see also *Turnovsky* (1986), Footnote 8.). In addition, the appendix shows that t^* is independent from T.

The expressions for the characteristic roots provide information on the determinants of the timing of the policy effectiveness reversal. For example, a larger sensitivity of the trade balance with respect to changes in the terms of trade as measured by the parameters b_4 and b_5 is associated with an earlier reversal in the relative policy effectiveness. Country 1 is initially more strongly affected by monetary policy and experiences stronger inflationary pressure. This leads to a worsening of competitiveness towards the other member country and towards the rest of the world, which dampens the initial expansionary effect. Simulations based on plausible parameter values confirm that this dampening effect increases in the parameters b_4 and b_5 and speeds up the policy effectiveness reversal.[21]

Furthermore, t^* is affected by the average interest sensitivity of aggregate demand in Europe (a_2) but invariant with respect to the degree of asymmetry in interest rate transmission across the EMU member countries ($\tilde{a}_2$). A larger spread in the interest sensitivity of aggregate demand will be accompanied by a larger initial cyclical divergence across the EMU member countries (i.e., a rise in $y^d(0+)$), but this difference will also be narrowed more rapidly. These two effects exactly balance each other with respect to the timing of the policy effectiveness reversal.

Despite the fact that the formula for t^* does not include $\tilde{a}_2$ it has to be noted that the policy effectiveness reversal as such requires *some* asymmetry in interest rate transmission. In the extreme case of symmetric interest rate transmission, i.e., $a_{21} = a_{22}$ or $\tilde{a}_2 = 0$, and given a symmetric shock such as $d\dot{m} > 0$ all time paths in the difference system equal zero throughout the overall adjustment process and a policy effectiveness reversal no longer exists.

Provided that the stability condition continues to hold, an increase in the average interest sensitivity of European aggregate demand a_2 leads to an earlier reversal. A large value of a_2 is accompanied by a large absolute value of r_1, which implies that the external terms of trade converge more rapidly to their original steady state level. Equation (18) then states that real interest rates also converge more quickly to their initial levels. This leads to an earlier reversal in the relative effectiveness of ECB policy across the EMU member countries.[22]

The output paths in the two member countries represent the mean of the aggregate European output and the differential output developments across EMU. The solutions are presented in *Figure 4*. The equilibrium levels of output are unaffected by a change in the growth rate of the European money stock. Member country outputs remain above the respective full employment levels throughout the adjustment period.

[21] Note, however, that $\partial t^*/\partial b_j < 0$ (j = 4, 5) does *not* hold for all theoretically admissible parameter values.

[22] Again, $\partial t^*/\partial a_2 < 0$ seems realistic but does *not* hold for all admissible parameter values.

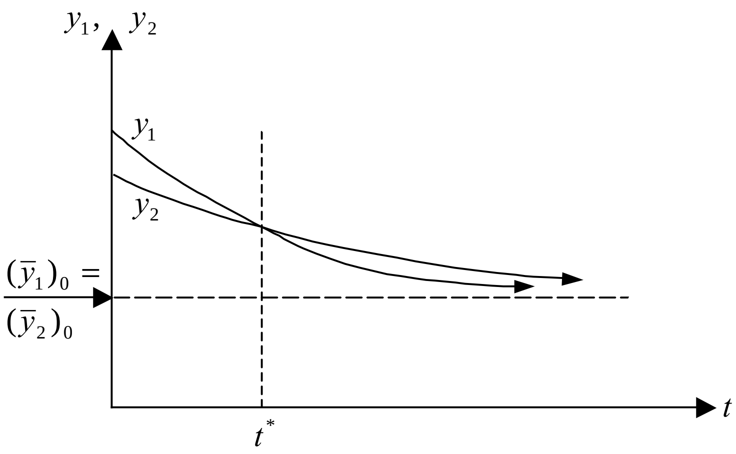

Figure 4: Member Country Output Effects of an Increase in European Money Growth

In summary, we find the following effects of a monetary expansion.[23] On impact, the level of short-term *real* interest rates in Europe unambiguously falls. Monetary policy is initially always more effective in the member country with the relatively larger interest sensitivity of aggregate demand. The presence of asymmetric interest rate transmission in Europe introduces exactly one reversal in the relative effectiveness of ECB policy on output in the EMU member countries.[24] The timing of the policy effectiveness reversal does not depend on whether monetary policy is unanticipated or anticipated.

4. Effects of Fiscal Policy

An expansionary fiscal policy is assumed to take the form of a symmetric increase in real government expenditure in all EMU member countries ($dg_1 = dg_2 > 0$).[25] This policy has lasting real effects. In the long run, it will induce a real appreciation of the euro and a rise in the equilibrium level of output in Europe (*Wohltmann*, 1994a). The money market equilibrium condition (5) implies that the expansionary fiscal policy is necessarily associated with a *fall* in the consumer price level in Europe. In contrast, the impact on the equilibrium level of producer prices is ambiguous. Using the producer price index as a deflator of money demand, the steady state equilibrium condition in the money market can be reformulated as:

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[23] The output effects of an increase in the *level* of the European money stock $(dm > 0)$ are very similar to the above results. The qualitative behavior of output in Europe as well as in the individual EMU member countries and the timing of the policy effectiveness reversal are identical. However, the relative quantitative impact of the two types of monetary policy on European output differs and depends on the semi-interest elasticity of money demand.

[24] If additional macroeconomic asymmetries are introduced there may be more than one policy effectiveness reversal in response to the common monetary policy by the ECB. See for a simultaneous introduction of supply side asymmetries and multiple reversals in relative policy effectiveness *Wohltmann* and *Clausen* (2001).

[25] In this section, we assume for the ease of interpretation that the level of the money stock is fixed at $m = m_0 = const_3$. This implies $\dot{m} = 0$, long-run price stability $\overline{\dot{p}} = 0$ and a stationary steady state not only in real but also in nominal variables.

(21) $\overline{m-p}=-\alpha_3\overline{\tau}+2l_0+l_1\overline{y}-2l_2i^*$

Two factors are at work: the real appreciation towards the rest of the world lowers import prices, which taken by itself requires a rise in the producer price level to equilibrate the money market. On the other hand, the rise in equilibrium output and real money demand requires a rise in real money balances and therefore producer prices to fall. It is plausible to assume that the first effect dominates the latter such that the fiscal expansion in Europe is associated with a rise in the producer price level in the long run.[26]

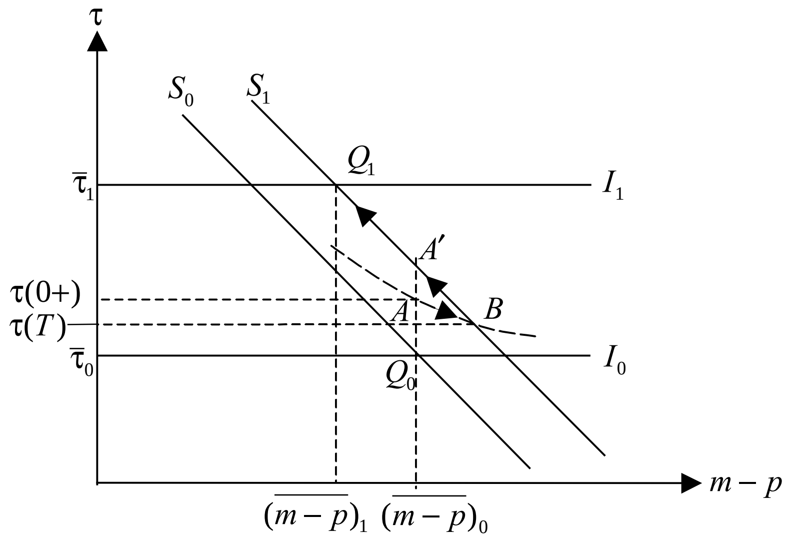

Figure 5: Aggregate Effects of an Increase in European Government Expenditure

The implications of a symmetric increase in government expenditure in Europe are illustrated in *Figure 5*. The initial equilibrium is given by Q_0. The increase in government expenditure in Europe results in a rightward shift of the stable branch of the saddle path ($S_0 \rightarrow S_1$). The locus of the unstable path shifts upwards ($I_0 \rightarrow I_1$). The new steady state equilibrium is given by Q_1 with an improvement in the terms of trade and a fall in the real money stock. The dynamics of adjustment again depend on whether the expansionary fiscal policy is unanticipated or anticipated.

An *anticipated* symmetric increase in government expenditure in Europe means that the European governments credibly announce at t=0 that at some future date T government expenditure will be raised. On impact, this induces a discontinuous real appreciation of the euro ($Q_0 \rightarrow A$). Between the announcement and the implementation of the expansionary fiscal policy, prices in Europe gradually fall, which brings about a real depreciation and a rise in the real money stock ($A \rightarrow B$). At

[26] The appendix shows that $\partial(\overline{m-p})/\partial(g_1+g_2)=-(\alpha_3-l_1f_1)/(b_5+\lambda f_1)$ so this requires $\alpha_3 > l_1f_1$, which is likely to be met. The income elasticity of money demand l_1 is often found to be unity. Note that the parameter f_1 in (11) and (12) is linked to α_3 by $f_1 = \alpha_3\beta$ where the parameter β reflects the elasticity of the labor supply with respect to the consumers' real wage (*Wohltmann* and *Bulthaupt*, 1999). Empirically, this elasticity is smaller than unity.

the date of implementation T, the terms of trade and the real money stock remain unchanged. After the implementation of the increase in government expenditure, the rate of inflation turns positive.[27] The real money stock falls and the terms of trade rise until the new equilibrium in Q_1 is reached ($B \rightarrow Q_1$).

The special case $T = 0$, i.e., an *unanticipated* increase in real government expenditure, leads on impact to a real appreciation of the euro ($Q_0 \rightarrow A'$). The euro appreciates further during the subsequent adjustment period until the new equilibrium terms of trade are attained ($A' \rightarrow Q_1$). Throughout the adjustment process, the rate of inflation is positive and the real money stock declines.[28]

As the euro appreciates during the transition period, it follows from (15) that European output overreacts to the fiscal expansion. It increases on impact beyond the new higher equilibrium level of output. Furthermore, $\dot{\tau} > 0$ implies in conjunction with (18) that the fiscal expansion in Europe is associated with a *fall* of *real* interest rates in Europe. An unanticipated expansionary fiscal policy is on impact accompanied by a crowding-*in* of private investment. This result follows from the fact that the rise in the rate of inflation necessarily exceeds in this model the increase in short-term nominal interest rates.[29] It follows that the country with the larger interest sensitivity of aggregate demand will also experience a relatively stronger boost in output in response to an expansionary fiscal policy in Europe.[30]

In contrast to the case of monetary policy, it matters whether fiscal policy is unanticipated or anticipated. The unanticipated fiscal expansion initiates a boom in European output. The announcement of a future increase in European government expenditure produces on impact a real appreciation and a rise in real interest rates, which incur a recession in Europe. In the period between the announcement and the implementation - the anticipation period - European output remains below the *initial* full employment level of output. In the period after implementation, the rate of change of the terms of trade is positive throughout the subsequent adjustment process. This implies that the level of *real* interest rates in Europe necessarily falls below the steady state level. Stimulated by the increase in government expenditure and by the fall in

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[27] Formally, the rate of inflation satisfies $\dot{p} < \overline{\dot{p}}$ for $t < T$ and $\dot{p} > \overline{\dot{p}}$ for $t > T$. We assume above $\overline{\dot{p}} = 0$.

[28] For completeness, we also discuss the case where the unanticipated fiscal expansion leads to a fall in the producer price level: The real money stock in the new steady state will be *higher* than the real money stock in the original equilibrium and the stable branch of the saddle path moves further to the northeast. The euro still appreciates on impact but this goes beyond the improvement in the equilibrium terms of trade. The transition period is characterized by a real depreciation of the euro with the corresponding implications for European output and real interest rates.

[29] In the case $T = 0$ we found $\dot{\tau}(0+) > 0$ (cf. *Figure 5*). Using (18), the aggregate *real* interest rate in Europe falls on impact. Under fairly weak conditions we find $\dot{p}^c(0+) = (\dot{p}_1^c + \dot{p}_2^c)(0+) > \dot{m}_0$ and $i(0+) > \overline{i_0}$. While therefore both, nominal interest rates and consumer price inflation increase we know from $\dot{\tau}(0+) > 0$ and (18) that the rise in inflation exceeds the increase in nominal interest rates. Furthermore, on impact, $\dot{p}^c$ increases less than $\dot{p}$ because $\dot{p} - \dot{p}^c = \alpha_3 \dot{\tau}$ and $\dot{\tau}(0+) > 0$. See also the last Figure in section 6.2.

[30] For the sake of brevity, the adjustment patterns following an unanticipated fiscal policy are not shown graphically.

real interest rates in *T*, European output exceeds the *new* equilibrium level of output throughout the subsequent adjustment process (*Figure 6*).

Figure 6 illustrates that anticipated fiscal policy may be responsible for a substantial variance of output around the equilibrium levels. Before *T,* European output stays below the original equilibrium level. After *T,* European output stays above the new equilibrium level. At the date of implementation *T*, European output overreacts to the fiscal expansion since the actual increase in European output exceeds the rise in equilibrium output ($\bar{y}_1 - \bar{y}_0$). The size of this jump in European output in *T* does not depend on the length of the anticipation period.[31] On the other hand, the size of the impact effect of anticipated fiscal policy on European output falls with *T,* i.e., falls with the length between the announcement and the implementation of policy. The qualitative adjustment of European output does not depend on the date of implementation.

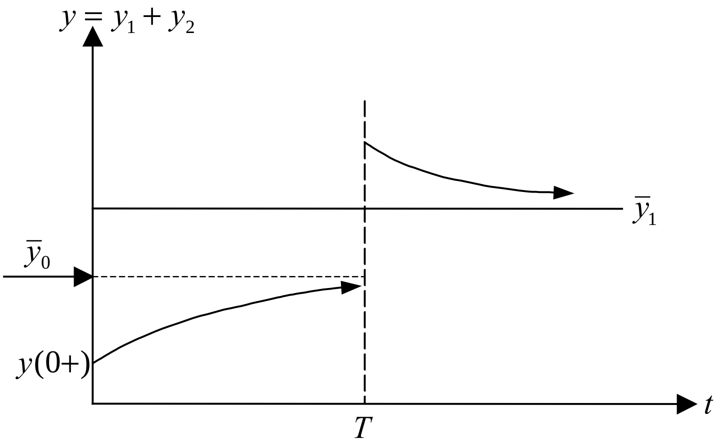

Figure 6: Aggregate Output Effects of an Anticipated Increase in Government Expenditure

We now turn to the differential output effects across the EMU (*Figure 7*). As shown above, the anticipated expansionary fiscal policy raises on impact European real interest rates. Correspondingly, country 1 initially experiences a stronger recession and more deflationary pressure than country 2. The difference between the two member country outputs $y^d = y_1 - y_2$ is initially negative ($y^d(0+) < 0$). As a consequence of the relatively stronger recession in country 1, internal and external terms of trade start to fall. Competitiveness improves and output developments are more favorable in country 1 than in country 2. At some stage during the adjustment process a reversal in the relative effectiveness of anticipated fiscal policy with respect to output in the EMU member countries takes place and the variable y^d turns positive. It can be shown that if the point of reversal occurs before the implementation date *T* it

[31] It is shown in the appendix that the size of the jump in *T*, $y(T+) - y(T-)$, is independent from *T*. This is a general result. Only the size of the initial jump depends on *T*. The reason for this difference in behavior is that the terms of trade jump at the time of the announcement, but stay continuous at the date of implementation.

must be identical to the one found in the cases of monetary policy and of unanticipated fiscal policy.[32]

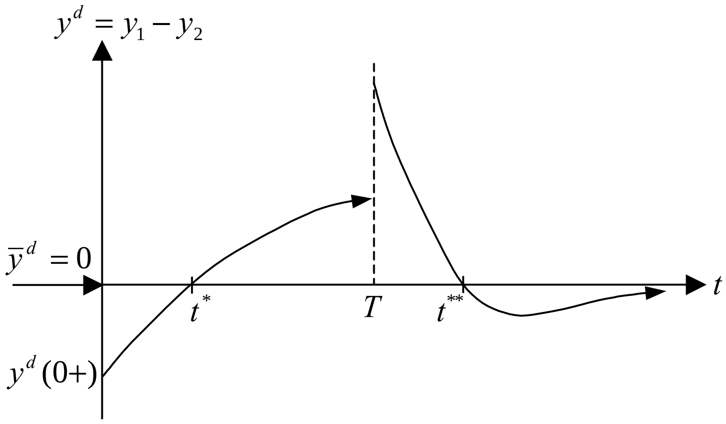

Figure 7: Differential Output Effects of an Anticipated Increase in Government Expenditure

Whereas in the cases of monetary policy and of unanticipated fiscal policy, only one point of reversal can be identified, anticipated fiscal policy is always associated with two reversals in relative policy effectiveness. One point of reversal is given in $t^* \leq T$. A second point of reversal t^{**} necessarily occurs after the implementation of the expansionary fiscal policy. The increase in government expenditure in T leads to a fall in European real interest rates and stimulates country 1 more strongly. Prices in country 1 rise relative to those in country 2. The internal price differential $p^d = p_1 - p_2$ increases for $t > T$ and rises beyond the unchanged equilibrium price differential $\bar{p}^d$. Towards the end of the adjustment process, the price differential falls back to the unchanged equilibrium level ($\dot{p}^d < 0$). Using (16), this implies that the output differential y^d approaches its long run equilibrium from a situation in which $y_1 - y_2 < 0$. The formula for the timing of the second reversal in policy effectiveness t^{**} is provided in the appendix. In the special case of an unanticipated increase in government expenditure, t^{**} coincides with t^*, which is given by (20).

[32] The timing of the first policy effectiveness reversal t^* is either before T or exactly at T. *Figures 7* and *8* illustrate the case $t^* < T$. If the date of policy implementation T is not too distant in the future, the first reversal occurs at T. More formally, we find in the case $T < t^*$ where t^* is defined by (20): $y^d(T-) < 0 < y^d(T+)$, i.e., the variable y^d experiences at the date of implementation T a (discontinuous) sign change.

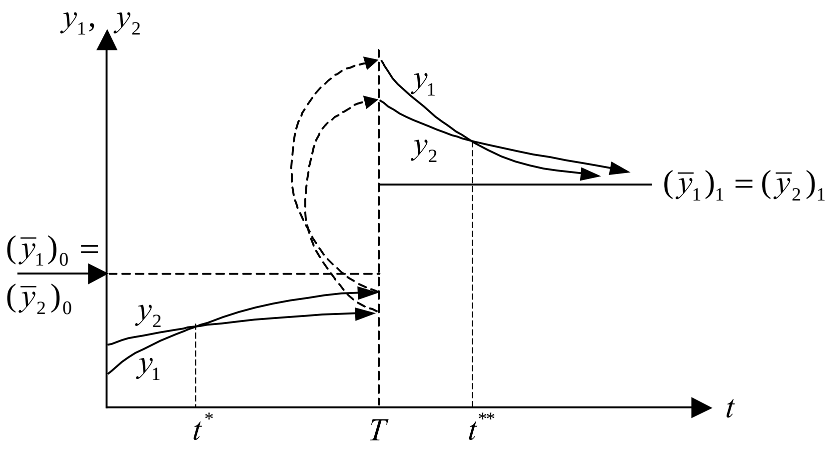

Figure 8: Member Country Output Effects of an Increase in Government Expenditure

Figure 8 shows the effects of the anticipated symmetric increase in European government expenditure on output in the individual EMU member countries. There are two reversals in relative policy effectiveness, which occur in t^* and in t^{**}. At the time of implementation, there is no reversal in the relative policy effectiveness since this policy is more effective in country 1 than in country 2 before and after T. Overall, it can be concluded that anticipated fiscal policy may initiate considerable cyclical asymmetries in the EMU.

5. Modifications and Robustness

This section introduces two modifications to the previous framework: At first, we relax the assumption $\alpha_1 = \alpha_2$ in the price index definitions p_1^c and p_2^c and assume instead a preference for the domestically produced good ($\alpha_1 > \alpha_2$). Secondly, we assume lagged instead of instantaneous output adjustment on the supply side. We show how these extensions modify the solution of the model and investigate the robustness of our previous results about the dynamic effects of monetary and fiscal policy.

5.1 The case $\alpha_1 > \alpha_2$

Assuming $\alpha_1 > \alpha_2$ implies that exogenous shocks lead to differential price index and inflation developments within the union. Given identical nominal interest rates, real interest rates $i_1 - \dot{p}_1^c$ and $i_2 - \dot{p}_2^c$ now differ across the union. The previous decomposition is no longer applicable because the aggregate system now depends on the difference system. Our previous system (13), (14) and (17) is replaced by the now interdependent system (22):[33]

[33] See the appendix. We use the abbreviations $\lambda = 1 - a_1 + b_1 - b_2$, $\mu = 1 - a_1 + b_1 + b_2$, $a_2 = \frac{1}{2}(a_{21} + a_{22})$, $\tilde{a}_2 = \frac{1}{2}(a_{21} - a_{22})$, where $a_{21} > a_{22}$.

$$(22)\ \begin{pmatrix} \frac{\lambda\alpha_3 - \delta(1-\alpha_3)a_2}{\delta} & -(\alpha_1-\alpha_2)\tilde{a}_2 & 0 \\ -\delta(1-\alpha_3)\tilde{a}_2 & (1-\alpha_1+\alpha_2)\mu - \delta(\alpha_1-\alpha_2)a_2 & 0 \\ \frac{l_2\delta + l_1\alpha_3}{\delta} & 0 & l_2 \end{pmatrix} \cdot \begin{pmatrix} \dot{\tau} \\ \dot{p}^d \\ \dot{m}-\dot{p} \end{pmatrix}$$

$$= \begin{pmatrix} -b_5 & 0 & 0 \\ 0 & -\delta(2b_4+b_5) & 0 \\ \alpha_3 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \tau - \overline{\tau} \\ p^d - \overline{p}^d \\ (m-p) - \overline{(m-p)} \end{pmatrix}$$

System (22) has the saddle path property with one unstable $(r_2 = 1/l_2 > 0)$ and two stable roots (r_0, r_1), where $r_1 < r_0 < 0$. In contrast with the case $\alpha_1 = \alpha_2$, the aggregate variables now depend on the characteristic root and vector belonging to the difference variable $p^d = p_1 - p_2$. In principle, this introduces the possibility of non-monotonous behavior of the aggregate variables in the interval $t > T$. An *anticipated increase in money growth* has qualitatively the same implications for aggregate output and the output differential as in the case $\alpha_1 = \alpha_2$ (*Figure 2* and *3*). On impact, a real depreciation of the common currency boosts aggregate union output. This initial stimulus fades over time as the terms of trade τ continuously increase and as real interest rates rise to the initial level. The output differential $y^d = y_1 - y_2$ increases on impact as a result of two factors. With $\alpha_1 > \alpha_2$, real interest rates fall more strongly in U_1 than in U_2[34] and member country U_1 is characterized by a relatively higher interest sensitivity of aggregate demand. The differential real interest rate development results from the difference in consumer price inflation, which is initially positive $((\dot{p}_1^c - \dot{p}_2^c)(0+) > 0)$. The relationship between consumer price and producer price inflation is given by

$$(23)\ \dot{p}_1^c - \dot{p}_2^c = (\alpha_1 - \alpha_2)(\dot{p}_1 - \dot{p}_2)$$

such that with $\alpha_1 > \alpha_2$, both inflation differentials respond in the same direction whereby the response of consumer price inflation differentials is relatively weaker. As a mirror image to inflation, the real interest rate differential can be expressed as

$$(24)\ (i_1 - \dot{p}_1^c) - (i_2 - \dot{p}_2^c) = -(\dot{p}_1^c - \dot{p}_2^c) = -(\alpha_1 - \alpha_2)\dot{p}^d$$

On impact, $\dot{p}^d(0+) > 0$ and the real interest rate differential falls. Over time, the differences in producer price inflation lead to a gradual increase in the price level differential $p^d = p_1 - p_2$, which trigger a fall in the output differential. As with $\alpha_1 = \alpha_2$, we find a reversal point t^* independent from *T,* after which the output

[34] With $\alpha_1 = \alpha_2$, real interest rates behave identically. With $\alpha_1 > \alpha_2$, U_2 may actually experience an *increase* in real interest rates because $i_2 - \dot{p}_2^c$ is equal to $i^* - \frac{1}{2}(1-\alpha_3)\dot{\tau} + \frac{1}{2}(\alpha_1 - \alpha_2)\dot{p}^d$ and both, $\dot{\tau}$ and $\dot{p}^d$, increase on impact. The initial (positive) output differential is definitely higher than in the case $\alpha_1 = \alpha_2$.

differential turns negative, i.e., where output y_2 exceeds y_1.[35] The output differential is proportional to the producer price inflation differential $\dot{p}^d$

(25) $$y^d - \bar{y}^d = \frac{1-\alpha_1+\alpha_2}{\delta}\dot{p}^d$$

It follows that the inflation differentials $\dot{p}^d$ and $\dot{p}_1^c - \dot{p}_2^c$ as well as the output and real interest differentials change sign at the same date t^* (*Figure 9*).[36]

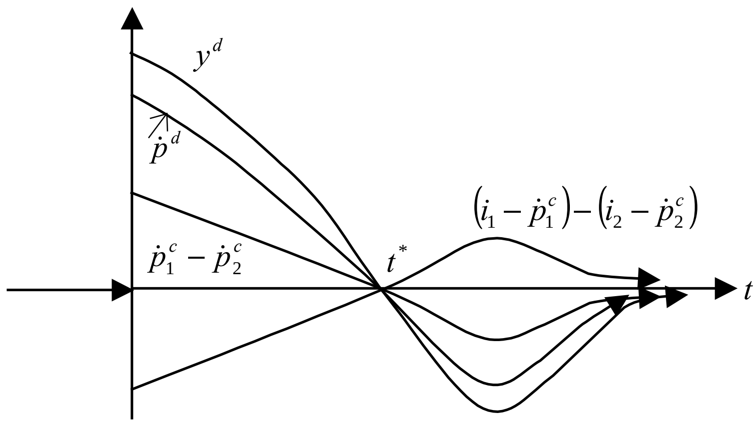

Figure 9: Response of difference variables to $d\dot{m} > 0$ with $\alpha_1 > \alpha_2$

Figure 10 shows the response of the difference variables to an *anticipated symmetric fiscal expansion* in the union.[37] In contrast to the case of a monetary expansion, we find on impact a fall in the output differential, which results from the fact that the real interest rate in U_1 increases more strongly than in U_2[38] and that demand in U_1 responds more sensitively to interest rates than in U_2. It follows that $\dot{p}_1$ falls relative to $\dot{p}_2$, which leads over time to a gradual fall in the producer price differential p^d. The real interest rate differential falls correspondingly. There again exists a point in

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[35] As shown in the appendix, t^* again satisfies (20), where r_0 and r_1 now denote the stable roots of system (22).

[36] *Figure 9* also includes the special case $T = 0$. The size of the effect in the case $T = 0$ exceeds the one for $T > 0$.

[37] The aggregate variables behave qualitatively as in the case $\alpha_1 = \alpha_2$ (cf. *Figures* 5 and 6). In the special case $T = 0$, the solution to the difference system is given by the adjustment paths to the right of T in *Figure 10*.

[38] On impact, we find in the case $dg > 0$ and $\alpha_1 > \alpha_2$ that $\dot{\tau}(0+) < 0$ and $\dot{p}^d(0+) < 0$, which implies that $i_1 - \dot{p}_1^c = i^* - \frac{1}{2}(1-\alpha_3)\dot{\tau} - \frac{1}{2}(\alpha_1 - \alpha_2)\dot{p}^d$ rises in $t = 0+$, while the increase in $i_2 - \dot{p}_2^c = i^* - \frac{1}{2}(1-\alpha_3)\dot{\tau} + \frac{1}{2}(\alpha_1 - \alpha_2)\dot{p}^d$ turns out to be unambiguously smaller. There is even a theoretical possibility that $i_2 - \dot{p}_2^c$ actually *falls* on impact. The decline in the output differential y^d is stronger than in the case $\alpha_1 = \alpha_2$.

time $t^* \le T$, at which the variable y^d experiences a sign change and the relative cyclical position in the union is reversed.[39]

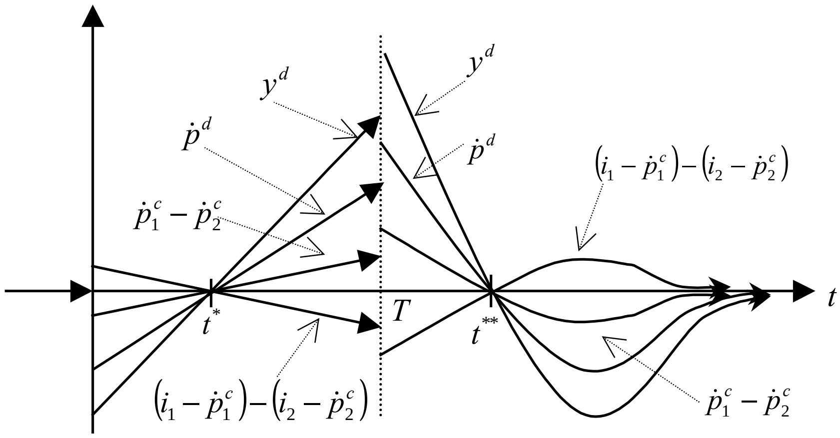

Figure 10: Response of difference variables to $dg_1 = dg_2 > 0$ with $\alpha_1 > \alpha_2$

At the date of implementation *T,* the system experiences a discontinuous adjustment in the difference variables. The output differential further increases in *T* at which the real interest differential falls discontinuously. Towards the end of the adjustment process the development of the producer price differential gradually pulls back the output differential. In contrast to the case of monetary policy, we find a second sign change in the difference variable, i.e., a second cyclical reversal point t^{**}, so that for $t > T$ output in U_1 (y_1) is initially above and for sufficiently large t $(t > t^{**})$ below y_2.

5.2 Gradual output adjustment in goods markets

We now replace the assumption of instantaneous output supply adjustment in goods markets and introduce gradual adjustment as follows:

(26) $\dot{y}_i = \beta(y_i^{AD} - y_i) \qquad (i = 1, 2;\ 0 < \beta < \infty)$.

y_i^{AD} represents aggregate demand in U_i and is defined by the right hand side of the IS-equations (1) and (2). The speed of supply adjustment β is assumed identical in both countries. The system dynamics are then determined by the following five dimensional differential equation system with the state variables $x = (y, y^d, \tau, p^d, m - p)'$:

[39] In the case $t^* < T$, the reversal point t^* is identical with the one derived for monetary policy. If *T* is sufficiently small $(T < t^*)$, the reversal takes place discontinuously in *T.*

$$(27)\ \dot{x} = \begin{pmatrix} d_{11} & d_{12} & d_{13} & 0 & 0 \\ d_{21} & d_{22} & 0 & d_{24} & 0 \\ d_{31} & 0 & 0 & 0 & 0 \\ 0 & d_{42} & 0 & 0 & 0 \\ d_{51} & 0 & d_{53} & 0 & d_{55} \end{pmatrix} \cdot (x - \bar{x}),$$

where

$$d_{11} = \beta\left((1-\alpha_3)\frac{a_2\delta}{\alpha_3} - \lambda\right),$$

$$d_{12} = \beta(\alpha_1 - \alpha_2)\frac{\tilde{a}_2\delta}{1-\alpha_1+\alpha_2},$$

$$d_{13} = -\beta b_5$$

$$d_{21} = \beta(1-\alpha_3)\frac{\tilde{a}_2\delta}{1-\alpha_1+\alpha_2},$$

$$d_{22} = \beta\left((\alpha_1-\alpha_2)\frac{a_2\delta}{1-\alpha_1+\alpha_2} - \mu\right),$$

$$d_{24} = -\beta(2b_4 + b_5),$$

$$d_{31} = \frac{\delta}{\alpha_3}$$

$$d_{42} = \frac{\delta}{1-\alpha_1+\alpha_2}$$

$$d_{51} = -\left(l_1 + \frac{\delta}{\alpha_3}\right)$$

$$d_{53} = \frac{\alpha_3}{l_2}$$

$$d_{55} = \frac{1}{l_2}$$

Lagged output adjustment in goods markets transforms the output variables $y = y_1 + y_2$ and $y^d = y_1 - y_2$ into predetermined variables. Thus, the state vector x now contains four predetermined variables $(y, y^d, p^d, m-p)$ and one non-predetermined variable (τ).

The system matrix $D = (d_{ij})$ has a positive determinant and generally four stable roots $(r_1,...,r_4)$ and one unstable root $(r_5 = d_{55} = 1/l_2)$, so the system continues to have the saddle path property (see appendix). Monetary and fiscal policy no longer incur an instantaneous jump in output. In consequence, y and y^d are on impact predetermined such that $y(0+) = \bar{y}_0$ and $y^d(0+) = \bar{y}_0^d = 0$. In conjunction with the relationships

$$(28)\ \alpha_3 \cdot \dot{\tau} = \delta(y - \bar{y})$$

$$(29)\ (1-\alpha_1+\alpha_2)\dot{p}^d = \delta(y^d - \bar{y}^d)$$

it implies that, on impact, $\dot{\tau} = \dot{p}^d = 0$. As a result of $\dot{\tau}=0$, real interest rates also remain constant. An announced *increase in the rate of monetary growth* leads on impact merely to a jump depreciation in *e*, which triggers an increase in demand for union output. Following (26), the union experiences a gradual increase in aggregate output (*Figure 11*). Using (28), this triggers a real appreciation, which in turn implies a fall in real interest rates. These conflicting effects on aggregate demand result in a hump-shaped dynamic adjustment of aggregate output *y*.

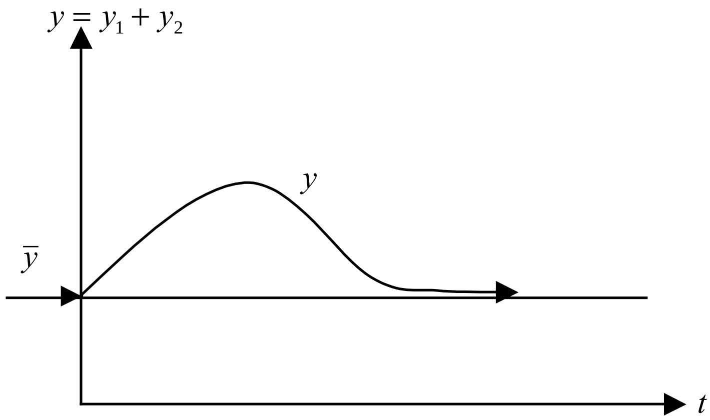

Figure 11: Response of aggregate output to $d\dot{m} > 0$ with lagged output adjustment

The decline in real interest rates is stronger in U_1 than in U_2, which leads to a rise in the output differential y^d. This leads over time to an increase in the price differential p^d, which in turn works towards a fall of y^d. The output differential also experiences a hump shaped adjustment, which – as in the case with instantaneous output reaction $(\beta = \infty)$ – shows a cyclical reversal (*Figure 12*). As shown in the appendix, this reversal point t^* is again independent from T.

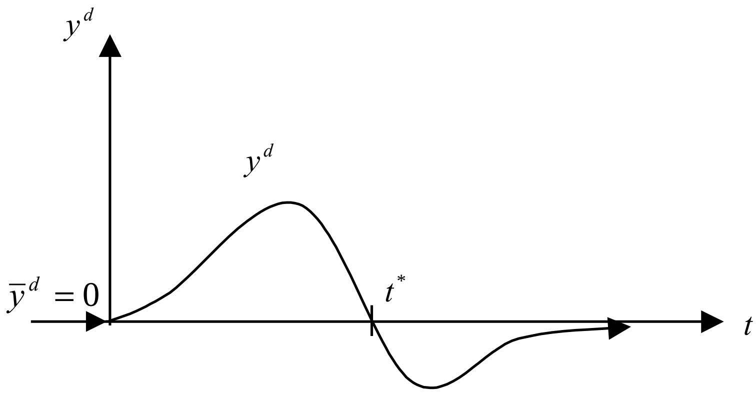

Figure 12: Response of output differential to $d\dot{m} > 0$ with lagged output adjustment

Analogous adjustment patterns with the same reversal point in t^* follow for the inflation differentials $\dot{p}^d$ and $\dot{p}_1^c - \dot{p}_2^c$ and - as a mirror image - for the real interest differential (*Figure 9*).

An *anticipated symmetric fiscal expansion* $(dg_1 = dg_2 > 0)$ leads to an immediate increase in the terms of trade τ and a lagged asymmetric increase in real interest rates, which generate a subsequent fall in the output variables y and y^d. In the period between the announcement and the implementation, y and y^d only start to increase as long as T is sufficiently large. This case is presented in the *Figures 13* and *14*, where the adjustment of y and y^d in the interval $0 \le t \le T$ is hump-shaped. For y^d, we may have a reversal point t^* before T.[40] In contrast with the case of instantaneous output adjustment, y and y^d are continuous even in t=0 and T. After T, the model generates adjustment patterns for y and y^d, which are qualitatively the same as with $\beta = \infty$. The output differential experiences a second reversal point t^{**}.[41]

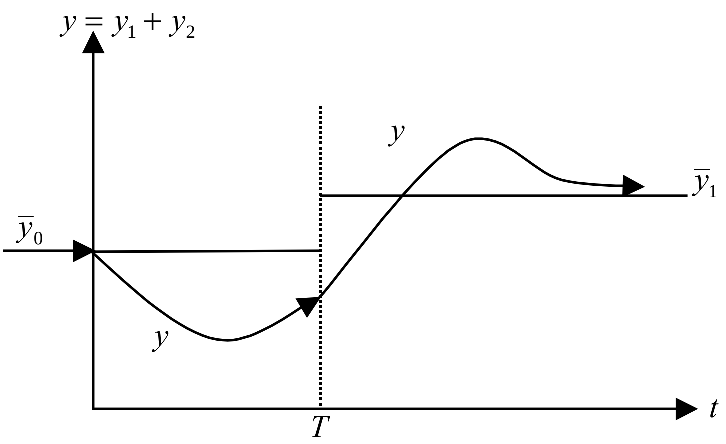

Figure 13: Response of aggregate output to $dg_1 = dg_2 > 0$ with lagged output adjustment

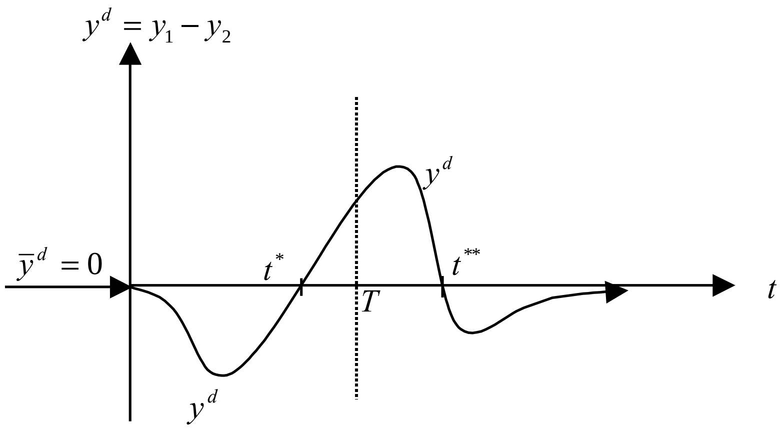

Figure 14: Response of differential output to $dg_1 = dg_2 > 0$ with lagged output adjustment

40 The reversal point t^* is identical with the one in the case of monetary policy.
41 In contrast with the case $\beta = \infty$, both reversal points may here lie in the interval $T < t < \infty$.

The analysis of the impact of monetary and fiscal policy constitutes a background for the now following discussion of the coordination between monetary and fiscal policy in Europe.

6. Coordination between Monetary and Fiscal Policy

This section analyzes the ability of monetary policy to neutralize the impact of symmetric unanticipated or anticipated demand shocks. These shocks may originate for example from a symmetric fiscal expansion in the union.

In the derivation of the monetary policy reaction function we assume that the primary goal of monetary policy is price stability. This is interpreted in our model as the effort to minimize consumer price inflation.[42] Furthermore, according to the treaty of Maastricht, the ECB is required to support overall macroeconomic stability in the euro area. We assume therefore that the ECB also aims to dampen business cycles at the aggregate level, and, to the extent possible, also divergences within the monetary union.[43]

6.1 Monetary policy reaction in the base model

The analytical solution to the aggregate and the difference system in the appendix shows that monetary policy can completely avoid cyclical divergences when it sets in the solution time path of the vector $(\tau, (m-p))'$ the constants A_1, A_2 and $\tilde{A}_1$ equal to zero. Under these conditions, anticipated demand shocks lead in T only to an immediate jump into the new steady state. Setting only $A_1 = A_2 = 0$ avoids announcement effects, i.e., the system stays in the initial equilibrium up to the implementation date T.

With an unanticipated demand shock ($T = 0$), complete stabilization at the aggregate and difference level only requires $\tilde{A}_1 = -\mathrm{d}\left(\overline{m-p}\right) = 0$, which means that the steady state level of the real money stock remains unchanged. Due to

$$(30)\quad \mathrm{d}\left(\overline{m-p}\right) = -l_2 \mathrm{d}\dot{m} - \frac{\alpha_3 - l_1 f_1}{b_5 + \lambda f_1}\mathrm{d}g$$

this requires a reduction in monetary growth as follows[44]

[42] The ECB targets consumer price inflation over the medium term. This is interpreted in terms of the model that it does not respond to one-time changes in the consumer price level, but monitors instead its rate of change. Another potential candidate in the evaluation of price stability is producer price inflation. It is shown below that the inflation rates based on the two price indices are proportional to each other in this model and require therefore qualitatively the same response of ECB policy.

[43] This can also be rationalized by numerous empirical analyses of the *Taylor* rule, which show that central banks typically also respond to cyclical conditions. A theoretical analysis of the optimal monetary policy design in the presence of macroeconomic asymmetries and central bank preferences of the *Barro-Gordon* type is provided by *De Grauwe* (2000).

[44] Here we assume $\alpha_3 > l_1 f_1$, which holds empirically. It then follows $\mathrm{d}\dot{m} < 0$.

$$\text{(31)}\ \mathrm{d}\dot{m} = -\frac{1}{l_2}\frac{\alpha_3 - l_1 f_1}{b_5 + \lambda f_1}\mathrm{d}g$$

This contractionary monetary policy means in conjunction with the expansionary fiscal impulse that both member countries immediately attain the new steady state values of their output variables y_1 and y_2. Furthermore, divergences in output developments across the union do not occur because real interest rates remain unchanged.[45] Note that differential output developments within the union can only be avoided by monetary policy as long as the initial demand disturbance is symmetric. With *symmetric* demand disturbances, the steady state values of the difference variables y^d and p^d remain unchanged such that the condition $\left(\mathrm{d}\left(\overline{m-p}\right)=0\right)$ stabilizes not only the aggregate, but also the difference system. With an *asymmetric* demand disturbance of, say $\mathrm{d}g_1 > 0 = \mathrm{d}g_2$, monetary policy (31) continues to perfectly stabilize the aggregate system. However, the time paths of the difference variables will only be dampened.[46]

The stabilizing impact of monetary policy (31) is illustrated in *Figure 15*. An *unanticipated* symmetric or asymmetric fiscal expansion leads to the adjustment process Q_0DQ_1, while the neutralizing monetary policy (31) has an exact opposite impact (Q_0DQ_1'). If both policies occur simultaneously, the system jumps immediately into the new steady state Q_2. The only difference compared with the initial equilibrium Q_0 is an increase in τ. The monetary policy (31) is consistent with the aim of price stability because this policy leads to a permanent fall in the level of inflation.[47]

Figure 15 also illustrates the monetary stabilization of an *anticipated* symmetric fiscal expansion. This fiscal policy has initially a contractionary and only after T an expansionary impact (cf. *Figure 6*). Thus, monetary policy has to react twice in order to stabilize the system in the anticipation period ($0 < t < T$) and in the period after implementation ($t > T$). The announcement effect of a future fiscal expansion on output is negative. In order to stabilize the system in the anticipation period, monetary policy has to announce in $t = 0$ an *expansionary* monetary policy for T. In the phase diagram, the contractionary implications of fiscal policy (Q_0BC) can be neutralized when monetary policy generates a mirror image time path up to T, i.e., $Q_0B'C'$. With these policies, the aggregate as well as the difference system remain in the initial equilibrium Q_0.

However, when the central bank actually raises monetary growth in T, it *reinforces* the already expansionary impulse, which results from the increase in government

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[45] With $\tilde{A}_1 = 0$, the aggregate and difference variables react as follows: $y \equiv \bar{y}_1 (> \bar{y}_0)$, $y^d \equiv \bar{y}^d = 0$ for *all* $t > 0$.

[46] In the case $\mathrm{d}g_1 > 0 = \mathrm{d}g_2$, we yield $\mathrm{d}\bar{p}^d > 0$ and $\mathrm{d}\bar{y}^d = (f_1 + 2f_2)\mathrm{d}\bar{p}^d > 0$. Then, $\tilde{A}_1 = 0$ no longer implies that the differential variables y^d and p^d remain constant. These variables adjust monotonically to their new steady state levels, which do not depend on the aggregate variable $\dot{\tau}$ (see appendix).

[47] In the long-run, inflation is determined by the rate of monetary growth such that the policy goal of price stability generally requires to keep monetary growth as low as possible. Policy conflicts arise when monetary policy has to turn more expansionary in order to neutralize the impact of contractionary shocks.

spending in T.[48] In addition, the rate of inflation $\dot{p}^c$ increases permanently. As long as price stability is the primary goal, monetary policy can only stabilize output in $t > T$ when the central bank deviates from the announcement and pursues instead an unanticipated *reduction* in the rate of monetary growth in T. The appropriate size of the monetary contraction $\dot{m}$ is identical to the one derived for an unanticipated fiscal expansion and therefore determined by the reaction function (31).

In the phase diagram, the failure to implement the previously announced expansionary expansion implies that the system moves in T vertically onto the saddle path S_1 (Point D). The implementation of the unanticipated monetary contraction leads to a further vertical move into Q_2. When the appropriate reduction in the rate of monetary growth $\dot{m}$ perfectly neutralizes the expansionary impact of fiscal policy[49], no further adjustment dynamics take place, so that Q_2 – as in the case $T = 0$ – represents the final equilibrium. This monetary policy reaction to a symmetric fiscal expansion also stabilizes the difference system such that output and inflation behave symmetrically across the monetary union. Furthermore, the monetary contraction leads to a permanent reduction in the rate of inflation within the monetary union.

Of course, the announcement of an *expansionary* monetary policy and the actual implementation of a *contractionary* policy can only be a successful strategy to stabilize the system as long as the announcement is credible[50], i.e., when the ECB has a reputation.[51] If the announcement is not credible then the private sector anticipates already in $t = 0$ the policy reaction (31) in $T > 0$. This anticipated monetary contraction in T *strengthens* the already contractionary impact of the announced fiscal expansion in the anticipation period $0 < t < T$. In addition, the period after implementation is not characterized by full stabilization. Instead, the output fluctuations after T are merely dampened.[52]

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[48] Graphically this means a movement along the saddle path belonging to the initial equilibrium S_0 from Q_0 to the new steady state Q_0', which, in comparison with the new steady state Q_1 under a passive monetary policy ($\mathrm{d}\dot{m} = 0$), is characterized by a larger decline in $\left(\overline{m - p}\right)$.

[49] With a passive monetary policy, the expansionary fiscal policy causes an adjustment process along the saddle path S_1 from D to Q_1. The restrictive monetary policy $\mathrm{d}\dot{m}(T) < 0$ leads after the initial discontinuous movement DQ_2 to the mirror adjustment process Q_2E along the saddle path S_2. With both policies taking place simultaneously, the aggregate system remains in the steady state Q_2.

[50] The debate about the time inconsistency of optimal policy rules initiated by *Kydland* and *Prescott* (1977) showed that credibility, rational expectations and time inconsistency of *systematic* monetary policy rules are logically inconsistent. Given rational expectations, a time-inconsistent monetary policy rule cannot be credible. However, deviations from a systematic policy rule are still consistent with rational expectations as long as the policies are unanticipated. The private sector still regards the policy rule as credible as long as the central bank enjoys a high level of reputation. Obviously, if the central bank deviates from the policy rule too frequently, it loses its reputation and the policy rule loses its credibility (see more in *Barro* and *Gordon* (1983)).

[51] The ECB appears to have already established substantial credibility in its anti-inflationary stance. This can be inferred from the low level of inflationary expectations revealed in surveys and from the relatively low level of long-term nominal interest rates in Europe.

[52] The condition for stabilization in $t > T$ ($\tilde{A}_1 = 0$) implies the following reaction function for monetary policy: $\mathrm{d}\dot{m}(T) = -\frac{1}{l_2}\left(\frac{\alpha_3 - l_1 f_1}{b_5 + \lambda f_1}\mathrm{d}g + \frac{1}{h_{11}}\frac{1}{b_5 + \lambda f_1}\left(1 - e^{(r_2 - r_1)T}\right)\right)$. Due to $h_{11} < 0$ and $r_2 > 0 > r_1$, this implies a stronger reduction in $\dot{m}$ than the reaction function (31). The anticipation of (31) means that the system does not jump in T but follows in *Figure* 15 a movement along the saddle path S_2 from

The unanticipated monetary policy reversal creates a further problem that the central bank loses reputation such that monetary policy announcements cease to be credible. On the other hand, this monetary policy may be appropriate when the European economy is initially in a *recession*, when fiscal policy is too inflexible to be immediately implemented and when the ECB supports the expansionary fiscal stance in order to overcome the recession. The ECB policy then prevents a further worsening of the recession caused by the contractionary announcement effects of fiscal policy and *stabilizes expectations*. The deviation from the previously announced monetary policy may be rationalized on the grounds that this prevents a political business cycle. Real output is stabilized at a permanently higher level and the contractionary stance of monetary policy avoids a permanently higher level of inflation.

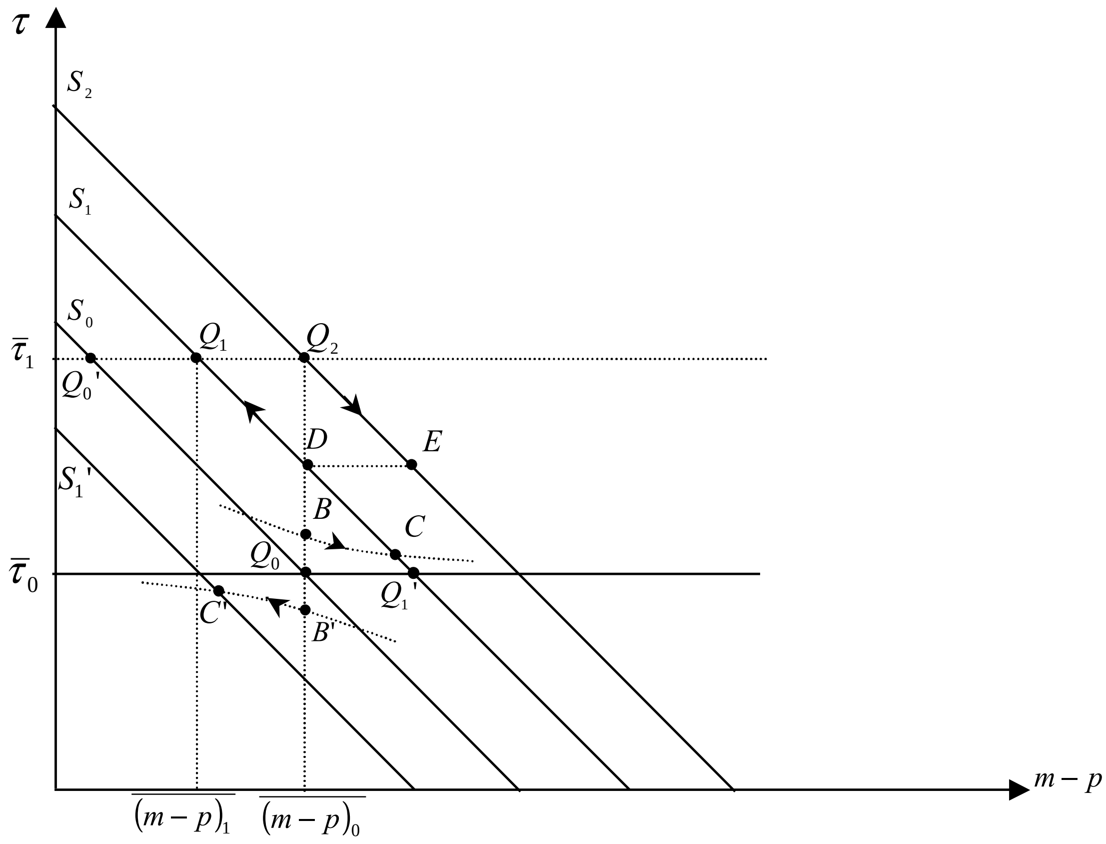

Figure 15: Reaction of monetary policy to $dg_1 = dg_2 > 0$

The implications of an announced fiscal expansion for inflation provide another rationale for a time-inconsistent reaction of monetary policy (*Figure 16*). The announcement of the expansionary fiscal package leads to an immediate fall in consumer and producer price inflation ($\dot{p}^c$ and $\dot{p}$), which continues up to the date of

below to the new steady state Q_2. Then, the aggregate output level y falls for $t > T$. The difference compared with the new steady state $\bar{y}_1$ is smaller than in the case of a passive monetary policy $\mathrm{d}\dot{m}(T) = 0$.

implementation T, where $\dot{p}^c > \dot{p}$.[53]

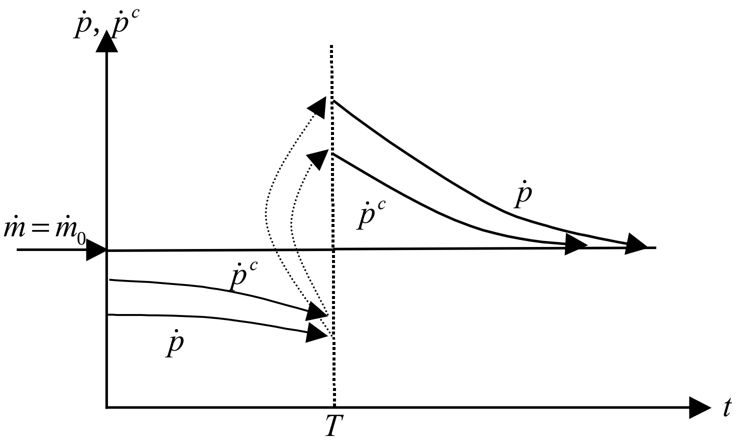

Figure 16: Response of inflation to $dg_1 = dg_2 > 0$

This deflationary episode can be avoided by the credible announcement in $t = 0$ about a suitable monetary expansion in T. The actual implementation of the expansionary fiscal policy induces an *inflationary* process, whereby $\dot{p} > \dot{p}^c$ for $t > T$. The implementation of the previously announced expansionary monetary policy gives a further boost to the inflationary process. On the other hand, an unanticipated reduction in $\dot{m}$ leads to an immediate and permanent fall in consumer and producer price inflation.

6.2 Modifications and robustness

In the case of a preference for the home good and *instantaneous* output adjustment $(\alpha_1 > \alpha_2; \beta = \infty)$, monetary policy continues to be able to completely neutralize *unanticipated* shocks, which leave the steady state values of the difference system unchanged[54]. The appropriate monetary policy is again determined by (31). A perfect stabilization of *anticipated* symmetric fiscal policies continues to be possible by the announcement of an expansionary monetary policy in $t = 0$ for $T > 0$ to be followed by an unanticipated reduction in the rate of monetary growth at the date of implementation T.[55] Ruling out such a time-inconsistent policy reaction, monetary

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[53] The price and wage adjustment equations imply $\dot{p} - \dot{p}^c = \delta(y - \bar{y}) = \alpha_3 \dot{\tau}$. For $t < T$, we have $y < \bar{y}_0$ and therefore $\dot{p} < \dot{p}^c$. For $t > T$ we find $y > \bar{y}_1$ and thus $\dot{p} > \dot{p}^c$. The behavior of $\dot{p}$ and $\dot{p}^c$ runs inversely to the development of the growth of the real money stocks $(\dot{m} - \dot{p})$ and $(\dot{m} - \dot{p}^c)$: Due to $(\dot{m} - \dot{p}) - (\dot{m} - \dot{p}^c) = -(\dot{p} - \dot{p}^c) = -\alpha_3 \dot{\tau}$ it follows $\dot{m} - \dot{p} > \dot{m} - \dot{p}^c > 0$ for $t < T$ and $\dot{m} - \dot{p} < \dot{m} - \dot{p}^c < 0$ for $t > T$. The size of the jump in T does not depend on T.

[54] This result holds for symmetric shocks as long as the level of foreign interest rates remains unaffected.

[55] See the appendix.

policy is unable to achieve complete stabilization in *both* intervals $0 < t < T$ and $t > T$. As a modification of our results in the case $\alpha_1 = \alpha_2$, it is no longer possible for monetary policy to use policy announcements in order to neutralize in the interval $t > T$ the effects of a symmetric fiscal impulse.[56]

The ability of monetary policy as a stabilization tool is significantly reduced by the presence of lagged output adjustment ($\beta < \infty$). This introduces rich dynamics (cf. (27)), which pose a challenge for a stability oriented monetary policy. In contrast with the case $\beta = \infty$, monetary policy no longer achieves complete stabilization of unanticipated shocks.[57] Furthermore, even allowing for time-inconsistent monetary policy responses it is no longer possible to completely stabilize anticipated symmetric fiscal policies in both periods. The room for monetary stabilization is confined to the announcement effects in the anticipation period where the contractionary announcement effects of a symmetric fiscal expansion can be neutralized by the announcement of a future monetary expansion.[58]

7. Conclusions

The results of this paper can be summarized as follows: The country with the relatively larger interest sensitivity of aggregate demand is *initially* more strongly affected by the common monetary policy by the ECB. This holds irrespective whether monetary policy is implemented by an increase in the level or in the growth rate of the European money stock and also irrespective of whether the monetary policy is unanticipated or anticipated. The presence of asymmetric interest rate transmission causes one intertemporal reversal in the relative effectiveness of ECB policy on output across the EMU member countries. The paper shows that the timing of the policy effectiveness reversal is fairly robust across various types of monetary policy. Unanticipated fiscal policy does also lead to a reversal in policy effectiveness, which happens to be identical to the reversal point in the case of monetary policy. Anticipated fiscal policy leads to two reversals, the first of which being either identical to the one found previously or being at the date of implementation while the second one necessarily occurs after the implementation of fiscal policy.

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[56] In the case $\alpha_1 = \alpha_2$, complete stabilization in the interval $t > T$ requires setting $\tilde{A}_1 = 0$. With $\alpha_1 > \alpha_2$, complete stabilization requires to set two independent constants ($\tilde{A}_1$ *and* $\tilde{A}_0$) equal to zero, which is impossible to achieve with only one monetary instrument ($\dot{m}$). For details see the analytical appendix.

[57] Dynamic adjustment processes in response to exogenous shocks can only be avoided by a countercyclical monetary policy, when in the general solution to the dynamic system all the constants $\tilde{A}_1,...,\tilde{A}_4$ are equal to zero. In contrast with the case $\beta = \infty$, where (31) achieved complete stabilization with unanticipated shocks ($T = 0$) by setting $\tilde{A}_0 = \tilde{A}_1 = 0$, there is no monetary policy strategy available, which can set $\tilde{A}_1,...,\tilde{A}_4$ simultaneously equal to zero.

[58] With $\beta < \infty$, it is possible to neutralize the announcement effects of symmetric shocks in the interval $0 < t < T$ via a suitable adjustment of $\dot{m}$. It can be derived from the general solution for $t < T$ by setting the constants $A_1,...,A_5$ equal to zero. $A_1,...,A_4$ stay in proportion with A_5. Thus, stabilization in $0 < t < T$ is achieved when monetary policy reacts in a way that sets $A_5 = 0$. In principle, this is possible because a change in $\dot{m}$ leads to a permanent change in the steady state level of the real money stock $m - p$, which in turn is included in A_5. For details see the analytical appendix.

Finally, the paper analyzes interactions between monetary and fiscal policy. It is assumed that the common monetary policy tries to stabilize the European economy after a symmetric fiscal shock occurring in all EMU member countries simultaneously. This fiscal shock may be unanticipated or anticipated. It is shown that the ECB may be able to stabilize output around the equilibrium level of output in the case of unanticipated fiscal policy. In contrast, anticipated fiscal policies pose a problem for the design of ECB policy. Complete stabilization requires both, a time-inconsistent reaction of monetary policy as well as a high credibility of ECB policy. While a time-inconsistent reaction of monetary policy has considerable merits in the case of anticipated fiscal policy, it is likely to undermine ECB credibility over time. Imposing the requirement that monetary policy has to be time-consistent implies that the ECB faces an intertemporal stabilization policy trade-off. Stabilization in one part of the adjustment process is necessarily associated with more pronounced fluctuations in other parts of the adjustment process.

This positive analysis of the dynamic effects of monetary and fiscal policies in a monetary union represents a useful starting point for an interesting research agenda about the conduct of stabilization policy in asymmetric three-country models of monetary unions. The current paper looks only at one particular asymmetry, which attracted widespread attention. One further route would be to look at other types of macroeconomic asymmetries and to investigate their implications for the dynamic effects of monetary and fiscal policy. Another next step would be to introduce dynamic optimizing behavior of the authorities in an effort to stabilize the economy in response to shocks. This extension allows to determine the optimal response of the ECB to different kinds of shocks and to evaluate the country-specific costs and benefits of asymmetric macroeconomic structures. This extension would also allow for an analysis of the interactions between monetary and fiscal policy in a dynamic game-theoretic context. Finally, the current setup takes the structure and decision rules of the private sector as given. A logical next step would be to introduce optimizing behavior of the private sector, which is likely to affect in the case of fiscal policy the choice between present and future consumption and, consequently, alters the impact of fiscal policy. These modifications have been applied to simpler small country and two-country models of international policy transmission. It will be interesting to learn how these approaches and results translate to the conduct of stabilization policy within a monetary union.

8. Appendix

Aggregate and difference system

The model (1)-(12) is first conveniently transformed into aggregate and difference variables and then decomposed as in *Aoki* (1981):

(A1) $\lambda \cdot y = (1-\alpha_3)a_2\dot{\tau} - 2a_2 i^* - b_5\tau + g + k$

(A2) $\alpha_3 \cdot \dot{\tau} = \delta(y - \bar{y})$

(A3) $p^c = p - \alpha_3\tau$

(A4) $\bar{y} = 2f_0 + f_1\bar{\tau}$

(A5) $m - p^c = 2l_0 + l_1 y - 2l_2(i^* + \dot{e})$

(A6) $\mu \cdot y^d = \tilde{a}_2(1-\alpha_3)\dot{\tau} - 2\tilde{a}_2 i^* + a_{01} - a_{02} + g_1 - g_2 - (2b_4 + b_5)p^d$

(A7) $(1-\alpha_1+\alpha_2)\dot{p}^d = \delta(y^d - \bar{y}^d)$

(A8) $\bar{y}^d = (f_1 + 2f_2)\bar{p}^d$

with

$$\lambda = (1-a_1+b_1-b_2),\ \mu = (1-a_1+b_1+b_2),\ a_2 = \frac{1}{2}(a_{21}+a_{22}),$$

$$\tilde{a}_2 = \frac{1}{2}(a_{21}-a_{22}) > 0,\ k = a_{01}+a_{02}+2b_0+2b_3 y^*,\ \alpha_1 = \alpha_2,$$

$$y = y_1 + y_2,\ \tau = \tau_1+\tau_2,\ g = g_1+g_2,\ p^c = p_1^c + p_2^c,\ p_1^c = p_2^c,$$

$$p = p_1 + p_2,\ y^d = y_1 - y_2,\ p^d = p_1 - p_2.$$

(A1) to (A5) reduce into the state equations (13) and (14) for the aggregate system; (A6) to (A8) collapse into the single state equation (17) for the difference system.

The unique convergent *solution to the aggregate system* is given by

(A9) $\begin{pmatrix} \tau \\ m-p \end{pmatrix} = \begin{pmatrix} \bar{\tau}_0 \\ (\overline{m-p})_0 \end{pmatrix} + A_1 h_1 e^{r_1 t} + A_2 h_2 e^{r_2 t}$ for $t < T$ and

(A10) $\begin{pmatrix} \tau \\ m-p \end{pmatrix} = \begin{pmatrix} \bar{\tau}_1 \\ (\overline{m-p})_1 \end{pmatrix} + \tilde{A}_1 h_1 e^{r_1 t}$ for $t > T$

where $e^{r_j t} := \exp(r_j t)$. The characteristic roots satisfy $r_1 < 0 < r_2$. The corresponding characteristic vectors are defined as h_1 and h_2, where $c > 0$ is given in the main text:

$$\text{(A11)}\ h_1 = \begin{pmatrix} h_{11} \\ 1 \end{pmatrix},\ h_{11} = \frac{r_1 - r_2}{c} < 0,\ h_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The constants A_1, A_2 and $\tilde{A}_1$ are determined by the well-known conditions that the jump variable τ stays continuous at the date of implementation T, whereas the predetermined variable $(m - p)$ remains continuous at both, at $t = 0$ and $t = T$ (see *Turnovsky*, 2000):[59]

$$\text{(A12)}\ A_1 = -A_2 = -\left(\mathrm{d}\left(\overline{m - p}\right) - \frac{1}{h_{11}} \mathrm{d}\overline{\tau} \right) e^{-r_2 T},$$

$$\text{(A13)}\ \tilde{A}_1 = A_1 - \frac{1}{h_{11}} \mathrm{d}\overline{\tau} e^{-r_1 T}$$

$$= -\mathrm{d}\left(\overline{m - p}\right) e^{-r_2 T} + \frac{1}{h_{11}} \mathrm{d}\overline{\tau} \left(e^{-r_2 T} - e^{-r_1 T} \right).$$

In the special case $T = 0$, i.e., with an unanticipated disturbance, only (A10) characterizes the solution with $\tilde{A}_1 = -\mathrm{d}\left(\overline{m - p}\right)$.

The *steady-state impact* of exogenous changes in the growth rate of the money stock $\dot{m}$ and government expenditure g_1 and g_2 on the state variables τ and $m - p$ is

$$\text{(A14)}\ \mathrm{d}\overline{\tau} = 0 \cdot \mathrm{d}\dot{m} + \frac{1}{b_5 + \lambda f_1} (\mathrm{d}g_1 + \mathrm{d}g_2)$$

$$\text{(A15)}\ \mathrm{d}\left(\overline{m - p}\right) = -l_2 \mathrm{d}\dot{m} - \frac{\alpha_3 - l_1 f_1}{b_5 + \lambda f_1} (\mathrm{d}g_1 + \mathrm{d}g_2)$$

The stable (convergent) saddle path S for the aggregate system follows from (A10) as

$$\text{(A16)}\ \tau - \overline{\tau} = h_{11} \left((m - p) - \left(\overline{m - p} \right) \right)$$

The solution to the state equation for the *difference system* (17) is continuous everywhere and given by

[59] These conditions for continuity imply that (A9) and (A10) also hold at $t = T$.

(A17) $p^d = \bar{p}_0^d + \gamma r_1 A_1 h_{11} \frac{1}{r_1 - r_0}\left(e^{r_1 t} - e^{r_0 t}\right)$ for $t < T$,

(A18) $p^d = \bar{p}_1^d - (\bar{p}_1^d - \bar{p}_0^d)e^{r_0(t-T)} + \gamma r_1 (A_1 - \tilde{A}_1) h_{11} \frac{1}{r_1 - r_0} e^{(r_1 - r_0)T} e^{r_0 t}$

$+ \gamma r_1 h_{11} \frac{1}{r_1 - r_0}\left(- A_1 e^{r_0 t} + \tilde{A}_1 e^{r_1 t}\right)$ for $t > T$.

The characteristic root $r_0 < 0$ is defined in (17) and $\gamma = \frac{\delta \tilde{a}_2 (1-\alpha_3)}{(1-\alpha_1+\alpha_2)\mu} > 0$.

The logarithmic-linear specification of the model (1)-(12) assumes the initial values of corresponding variables to be identical. This means that, for example, the initial value of the price differential p^d is $\bar{p}_0^d = 0$. Since $a_{21} > a_{22}$, or equivalently $\tilde{a}_2 > 0$, we assume without loss of generality in the IS-equation (A6) $a_{01} > a_{02}$. The steady-state values of the difference variables only respond to asymmetric disturbances such as $\mathrm{d}g_1 \neq \mathrm{d}g_2$ or to changes in the level of world interest rates $i*$. Using the notation $\mathrm{d}\bar{p}^d = \bar{p}_1^d - \bar{p}_0^d$ the model yields

(A19) $\mathrm{d}\bar{p}^d = \frac{1}{\mu(f_1 + 2f_2) + 2b_4 + b_5}(\mathrm{d}g_1 - \mathrm{d}g_2 - 2\tilde{a}_2 \mathrm{d}i^*)$.

Effects of Monetary Policy

An increase in the growth rate of the money stock by one unit ($d\dot{m} = 1$) implies for the constants A_1 and $\tilde{A}_1$ in (A12) and (A13) in connection with (A14) and (A15) $A_1 = \tilde{A}_1 = l_2 \cdot e^{-r_2 T} > 0$. The *impact effect* at the date of the announcement (0+) of the subsequent monetary expansion in $T > 0$ is:

$$\tau(0+) = \bar{\tau}_0 + A_1 h_{11} < \bar{\tau}_0 \quad (h_{11} < 0)$$

(A20) $$\dot{\tau}(0+) = A_1 h_{11} r_1 > 0 \quad (r_1 < 0)$$

$$y(0+) = \bar{y}_0 + \frac{\alpha_3}{\delta}\dot{\tau}(0+) > \bar{y}_0$$

On impact, the common currency depreciates ($\tau(0+) < \bar{\tau}_0$) but immediately starts to re-appreciate ($\dot{\tau}(0+) > 0$). In conjunction with (18) we know that real interest rates fall in both member countries. Furthermore, aggregate output increases beyond the full employment level (cf. (15)). In the course of the adjustment process, the union experiences a real appreciation and a decline in the real money stock (*Figure 1*). The link with the consumption based real money stock $m-p^c$ is given by $m-p^c=m-p+\alpha_3\tau$. With $\tau(0+)<\bar{\tau}_0$, $m-p^c$ falls on impact. Using (A9) and (A10), it can be shown that $m-p^c$ declines further throughout the subsequent adjustment process as long as the weak condition $1+\alpha_3 h_{11}>0$ holds. In the long-run and as a result of $\mathrm{d}\bar{\tau}/\mathrm{d}\dot{m}=0$, both price indices fall by the same amount.

The impact behavior of the output differential $y^d=y_1-y_2$ follows from (A6) and $\dot{\tau}(0+)>0$, which imply that y^d increases

(A21) $$y^d(0+)=\frac{\tilde{a}_2(1-\alpha_3)}{\mu}\dot{\tau}(0+)>0\,.$$

In the subsequent adjustment process, gradual increases in the price differential $p^d=p_1-p_2$ resulting from excess demand in the goods market (cf. (A7)) lower y^d again (cf. (A6)) and lead to a cyclical reversal. Since $\bar{y}^d=0$ in the case $d\dot{m}>0$, (A7) implies $y^d=\bar{y}^d=0$ if and only if $\dot{p}^d=0$. Setting the derivative of (A17) with respect to time equal to zero allows for the determination of the exact timing of this reversal. In the interval $0 < t < T$ we yield:

(A22) $$\begin{aligned} y^d=0 &\Leftrightarrow \dot{p}^d=\gamma r_1 A_1 h_{11}\frac{1}{r_1-r_0}(r_1e^{r_1t}-r_0e^{r_0t})=0\\ &\Leftrightarrow r_1e^{r_1t}-r_0e^{r_0t}=0\end{aligned}$$

The solution to this equation is labeled *t**. It is given in (20) and independent from *T*. Note that the same reversal point *t** can be derived for *t* > *T* from (A18) by again setting $\dot{p}^d=0$. In the case $d\dot{m}>0$, we previously derived $A_1=\tilde{A}_1$ and $d\bar{p}^d=0$ so that (A18) is equivalent to (A17) and (20) follows immediately.[60]

[60] This equivalence does not hold for other disturbances such as fiscal policy (see in the following (A26)).

Effects of Fiscal Policy

A symmetric increase in government expenditure ($dg_1 = dg_2 > 0$) leads in the *steady state* to an increase in the terms of trade τ (cf. A14). The impact on the long-run real money stock $\left(\overline{m-p}\right)$ is indeterminate. Under the realistic assumption $\alpha_3 > l_1 f_1$, (A15) implies $\mathrm{d}\left(\overline{m-p}\right)/\mathrm{d}g < 0$. In contrast, the long-run consumption based real money stock unambiguously increases as (A4) and (A5) jointly imply

(A23) $$\frac{\mathrm{d}\left(\overline{m-p^c}\right)}{\mathrm{d}g} = l_1\,\mathrm{d}\overline{y}/\mathrm{d}g = l_1 f_1 \frac{\mathrm{d}\overline{\tau}}{\mathrm{d}g} > 0.$$

Concerning the signs of the constants (A12) and (A13), it can be shown in the case of fiscal policy using the results for the characteristic roots r_1 and r_2 that, irrespective of $sign(\alpha_3 - l_1 f_1)$, $\mathrm{d}\left(\overline{m-p}\right) - \frac{1}{h_{11}}\mathrm{d}\overline{\tau} > 0$. It then follows from (A12) that $A_1 < 0$. Assuming $\alpha_3 > l_1 f_1$ implies $\tilde{A}_1 > 0$ for all $T \geq 0$.

Using (A20) for the *impact effect* of an announced fiscal expansion, we find $\tau(0+) > \overline{\tau}_0, \dot{\tau}(0+) < 0, y(0+) < \overline{y}_0$, i.e., a real appreciation, a rise in real interest rates and a fall in aggregate output. At the date of implementation *T*, aggregate output rises discontinuously. The size of the jump in *T* does *not* depend on the time span between the announcement and the implementation *T* and it is unambiguously larger than the steady state impact $d\overline{y} = \overline{y}_1 - \overline{y}_0$:

(A24) $$y(T+) - y(T-) = \mathrm{d}\overline{y} + \frac{\alpha_3}{\delta}(\dot{\tau}(T+) - \dot{\tau}(T-))$$

with

(A24a) $$\dot{\tau}(T+) - \dot{\tau}(T-) = (\tilde{A}_1 - A_1) h_{11} r_1 e^{r_1 T} = -r_1 \mathrm{d}\overline{\tau} > 0$$

The output differential falls $(y^d(0+) < 0)$, which follows from (A21) and $\dot{\tau}(0+) < 0$. Using (A7), $\dot{p}^d(0+) < 0$, such that in the adjustment process the price differential $p^d = p_1 - p_2$ starts to fall allowing the output differential y^d to gradually rise again. If

T is sufficiently large, we find a reversal point t^* for y^d in the interval $0 < t < T$. t^* is *identical* with (20).[61]

At the date of implementation T, the output differential y^d increases discontinuously as (A6) and (A24a) imply

$$\text{(A25)}\; y^d(T+) - y^d(T-) = \frac{\tilde{a}_2(1-\alpha_3)}{\mu}(\dot{\tau}(T+) - \dot{\tau}(T-))$$

$$= -r_1 \frac{\tilde{a}_2(1-\alpha_3)}{\mu} \mathrm{d}\overline{\tau} > 0$$

Using (A7), we know that the jump in the output differential is accompanied by a jump in the inflation differential. The price differential $p^d = p_1 - p_2$ starts to rise after T allowing the output differential y^d to gradually fall again (cf. *Figure 7*). In contrast with the case of monetary policy, symmetric fiscal policy gives rise to an additional reversal point t^{**} where the output differential y^d again changes sign. t^{**} is a reversal point of y^d in the period $t > T$. The same reversal point holds for $\dot{p}^d$. Setting the derivative of (A18) with respect to time equal to zero yields the following expression for t^{**}:

$$\text{(A26)}\; t^{**} = t^* + \frac{1}{r_0 - r_1} \ln\left(\frac{A_1 - \dfrac{1}{h_{11}}\mathrm{d}\overline{\tau}e^{-r_1 T}}{A_1 - \dfrac{1}{h_{11}}\mathrm{d}\overline{\tau}e^{-r_0 T}} \right).$$

In the special case $T = 0$, i.e., an unanticipated symmetric increase in government expenditure, t^{**} and t^* coincide such that only a single date exists at which a sign reversal of y^d occurs. Note that this result also holds in the case of a change in the growth rate of the money stock $\dot{m}$ since $\partial\overline{\tau}/\partial\dot{m} = 0$ implies $t^{**} = t^*$.

Coordination of Monetary and Fiscal Policy

An *unanticipated* symmetric increase in government expenditure leads on impact to an overproportionate reaction of aggregate output ($y(0+) > \overline{y}_1$, if $T = 0$) and to

[61] As t^* is *independent* from T, a sufficiently small T (i.e., $T < t^*$) implies that in this case *no* reversal point exists in the interval $0 < t < T$. The same proposition holds in the case of a change in the rate of monetary growth.

divergent output developments within the union. An immediate stabilization of y at the *new* steady level $\bar{y}_1$ (i.e., $y \equiv \bar{y}_1$ for all $t > 0$) can be achieved as long as $\dot{\tau} \equiv 0$ (cf. A2). Since $\dot{\tau} = r_1 \tilde{A}_1 h_{11} e^{r_1 t}$, this requires a reaction of monetary policy, which generates $\tilde{A}_1 = 0$. In the case $T = 0$, the constant is $\tilde{A}_1 = -\mathrm{d}\left(\overline{m-p}\right)$. On this basis, we derive the reaction function of monetary policy (31) for $\mathrm{d}\dot{m}(0+)$. If the expansionary symmetric fiscal policy $\mathrm{d}g_1 = \mathrm{d}g_2 > 0$ is accompanied by a restrictive monetary policy (31), aggregate output will instantaneously jump into the new steady state $\bar{y}_1$. Correspondingly, also the state variables τ and $m-p$ immediately adopt their steady state values. As $\dot{\tau} = 0$ and $\mathrm{d}\bar{p}^d = 0$, (17) and (A7) imply $\dot{p}^d = y^d = 0$ for all $t > 0$. The policy rule (31) also prevents divergent output and inflation developments *within* the union. Overall, (31) achieves a complete stabilization, i.e., a stabilization of both, the aggregate as well as the difference system.[62]

An *anticipated* symmetric increase in government expenditure has implications before and after the implementation at date *T*. A complete stabilization of both, the aggregate as well as the difference system, requires a monetary policy reaction, which implies $A_1 = 0$ *and* $\tilde{A}_1 = 0$. $A_1 = 0$ can be achieved by the announcement at $t = 0$ that monetary policy turns expansionary at $T > 0$ using (A12), (A14) and (A15). The announcement is assumed to be credible such that the following rate of monetary growth from *T* onward is anticipated (*ant*) by the private sector:

$$\text{(A27)}\quad \mathrm{d}\dot{m}^{ant.}(T) = -\frac{1}{l_2}\frac{1}{b_5 + \lambda f_1}\left(\alpha_3 - l_1 f_1 + \frac{1}{h_{11}}\right)\mathrm{d}g(T) > 0 .$$

If $\dot{m}$ actually increases in *T*, it *intensifies* the already expansionary impact of fiscal policy in the period $t > T$. This is reflected in the constant $\tilde{A}_1$ in (A13), which turns out to be larger than under a passive monetary policy. The complete stabilization of the aggregate and the difference system before and after the implementation date *T*

[62] In the case of asymmetric fiscal policy ($\mathrm{d}g_1 \neq \mathrm{d}g_2$), full stabilization within the difference system is no longer possible since $\mathrm{d}\bar{p}^d \neq 0 \neq \mathrm{d}\bar{y}^d$. (A18) shows that even with $\tilde{A}_1 = 0$, differential adjustment processes still occur. These adjustment processes are independent from the aggregate variable $\dot{\tau}$, such that the output differentials within the union are “smoothed” compared with the case of passive monetary policy.

requires a *time inconsistent* monetary policy in T, i.e., not as announced an expansion but a *reduction* in the rate of growth of the money stock according to (31).

A time consistent monetary policy is not able to achieve complete output stabilization throughout the overall adjustment process. The ECB faces an *intertemporal* policy tradeoff: Either she stabilizes the overall system in the anticipation period before T, i.e., sets $A_1 = 0$ by announcing in $t = 0$ an appropriate *increase* in $\dot{m}$ for $T > 0$, which implies in the period after the implementation in comparison with a passive monetary policy a larger $\tilde{A}_1$ and larger cyclical imbalances. Or she stabilizes the system after T, i.e., sets $\tilde{A}_1 = 0$ by announcing in $t = 0$ for $T > 0$ an appropriate reduction in $\dot{m}$. However, this raises the absolute value of A_1 and amplifies the recession in the anticipation period already caused by the contractionary announcement effects of fiscal policy.

Robustness: The case $\alpha_1 > \alpha_2$

The state equations of the aggregate system now depend on the variable p^d in the difference system, and the dynamic equations for the aggregate variables τ and $(m-p)$ can no longer be solved independently. In the discussion of the modifications to system (22), we introduce for the sake of brevity the following abbreviations:

(A28) $\beta_1 := \dfrac{\alpha_3\lambda - \delta(1-\alpha_3)a_2}{\delta} > 0$ [63]

$$\beta_2 := -(\alpha_1 - \alpha_2)\tilde{a}_2 < 0$$

$$\beta_3 := -\delta(1-\alpha_3)\tilde{a}_2 < 0$$

$$\beta_4 := (1-\alpha_1+\alpha_2)\mu - \delta(\alpha_1-\alpha_2)a_2 > 0$$

$$\beta_5 := \frac{l_2\delta + l_1\alpha_3}{\delta} > 0$$

$\Delta := \beta_1\beta_4 - \beta_2\beta_3 > 0$ [64]

[63] $\beta_1 > 0$, if $\alpha_3\lambda - \delta(1-\alpha_3)a_2 > 0$. This assumption was already made in the case $\alpha_1 = \alpha_2$. It is a sufficient condition for the saddle path property of the aggregate system.

Then, the system matrix on the left side of (22) has a positive determinant $l_2\Delta$ such that (22) can be expressed in explicit form as:

(A29) $$\begin{pmatrix} \dot{\tau} \\ \dot{p}^d \\ \dot{m}-\dot{p} \end{pmatrix} = \begin{pmatrix} \gamma_1 & \gamma_2 & 0 \\ \gamma_3 & \gamma_4 & 0 \\ \gamma_5 & \gamma_6 & 1/l_2 \end{pmatrix} \begin{pmatrix} \tau-\overline{\tau} \\ p^d-\overline{p}^d \\ (m-p)-\left(\overline{m-p}\right) \end{pmatrix}$$

with

$$\gamma_1 = -\beta_4 b_5/\Delta < 0, \quad \gamma_2 = \beta_2\delta(2b_4+b_5)/\Delta < 0,$$

$$\gamma_3 = \beta_3 b_5/\Delta < 0, \quad \gamma_4 = -\beta_1\delta(2b_4+b_5)/\Delta < 0,$$

$$\gamma_5 = \frac{\beta_4\beta_5 b_5}{l_2\Delta} + \frac{\alpha_3}{l_2} > 0, \; \gamma_6 = -\frac{\beta_2\beta_5\delta(2b_4+b_5)}{l_2\Delta} > 0.$$

The determinant of (A29) is $(\gamma_1\gamma_4-\gamma_2\gamma_3)/l_2 = b_5\delta(2b_4+b_5)/(l_2\Delta) >$ 0. The characteristic roots r_0, r_1, r_2 are given by

(A30) $$r_{0,1} = \frac{\gamma_1+\gamma_4}{2} \pm \sqrt{\left(\frac{\gamma_1+\gamma_4}{2}\right)^2 - \frac{b_5\delta(2b_4+b_5)}{\Delta}}$$

$$r_2 = \frac{1}{l_2} > 0$$

The discriminant is positive, which implies that the characteristic roots r_0 and r_1 are real. Furthermore, because of $\gamma_1+\gamma_4<0$, both roots are negative with $r_1 < r_0 < 0$. It follows that the dynamic system (A29) has the saddle path property. The corresponding characteristic vectors h_0, h_1, h_2 are derived as

(A31) $$h_0 = \begin{pmatrix} h_{10} \\ h_{20} \\ 1 \end{pmatrix}, \; h_1 = \begin{pmatrix} h_{11} \\ h_{21} \\ 1 \end{pmatrix}, \; h_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

with

$$h_{1j} = \frac{(\gamma_4-r_j)(r_j-r_2)}{(\gamma_4-r_j)\gamma_5-\gamma_3\gamma_6} \quad (j=0,1)$$

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[64] $\Delta > 0$, as long as $\tilde{a}_2$ is not very large, i.e., that the differences in the semi-interest elasticities a_{21} and a_{22} are limited in size.

$$h_{2j} = \frac{-\gamma_3(r_j - r_2)}{(\gamma_4 - r_j)\gamma_5 - \gamma_3\gamma_6} \quad (j = 0,1).$$

The term $\gamma_3\gamma_6$ can be considered sufficiently small such that $h_{1j} < 0$. The sign of h_{2j} is indeterminate. Typically, $\gamma_4 - r_1 > 0 > \gamma_4 - r_0$. This implies that $h_{20} > 0 > h_{21}$.

The unique convergent solution of the state vector $(\tau, p^d, m-p)'$ follows as

(A32) $$\begin{pmatrix} \tau \\ p^d \\ m-p \end{pmatrix} = \begin{pmatrix} \bar{\tau}_0 \\ \bar{p}_0^d \\ (\overline{m-p})_0 \end{pmatrix} + A_0 h_0 e^{r_0 t} + A_1 h_1 e^{r_1 t} + A_2 h_2 e^{r_2 t} \quad \text{for } t < T$$

(A33) $$\begin{pmatrix} \tau \\ p^d \\ m-p \end{pmatrix} = \begin{pmatrix} \bar{\tau}_1 \\ \bar{p}_1^d \\ (\overline{m-p})_1 \end{pmatrix} + \tilde{A}_0 h_0 e^{r_0 t} + \tilde{A}_1 h_1 e^{r_1 t} \qquad \text{for } t > T.$$

The five constants are determined using the conditions that the predetermined variables p^d and $m-p$ are continuous in t = 0 and t = T while the jump variable τ is continuous in T:

$$A_0 h_{20} + A_1 h_{21} = 0$$

$$A_0 + A_1 + A_2 = 0$$

(A34) $$\bar{p}_0^d + A_0 h_{20} e^{r_0 T} + A_1 h_{21} e^{r_1 T} = \bar{p}_1^d + \tilde{A}_0 h_{20} e^{r_0 T} + \tilde{A}_1 h_{21} e^{r_1 T}$$

$$(\overline{m-p})_0 + A_0 e^{r_0 T} + A_1 e^{r_1 T} + A_2 e^{r_2 T} = (\overline{m-p})_1 + \tilde{A}_0 e^{r_0 T} + \tilde{A}_1 e^{r_1 T}$$

$$\bar{\tau}_0 + A_0 h_{10} e^{r_0 T} + A_1 h_{11} e^{r_1 T} = \bar{\tau}_1 + \tilde{A}_0 h_{10} e^{r_0 T} + \tilde{A}_1 h_{11} e^{r_1 T}$$

The first two equations in (A34) imply

(A35) $$A_0 = -\frac{h_{21}}{h_{21} - h_{20}} A_2, \quad A_1 = \frac{h_{20}}{h_{21} - h_{20}} A_2.$$

The third and the fifth equation in (A34) define a subsystem, which can be solved for $\tilde{A}_0 - A_0$ and $\tilde{A}_1 - A_1$:

(A36) $$\begin{pmatrix} \tilde{A}_0 - A_0 \\ \tilde{A}_1 - A_1 \end{pmatrix} = \frac{1}{\chi e^{(r_0 + r_1)T}} \begin{pmatrix} h_{21} e^{r_1 T} & -h_{11} e^{r_1 T} \\ -h_{20} e^{r_0 T} & h_{10} e^{r_0 T} \end{pmatrix} \begin{pmatrix} \bar{\tau}_0 - \bar{\tau}_1 \\ \bar{p}_0^d - \bar{p}_1^d \end{pmatrix}$$

with $\chi = h_{10}h_{21} - h_{11}h_{20} > 0$.

It follows:

(A37) $\tilde{A}_0 = A_0 + \frac{1}{\chi} e^{-r_0 T} (-h_{21} \mathrm{d}\overline{\tau} + h_{11} \mathrm{d}\overline{p}^d)$

(A38) $\tilde{A}_1 = A_1 + \frac{1}{\chi} e^{-r_1 T} (h_{20} \mathrm{d}\overline{\tau} - h_{10} \mathrm{d}\overline{p}^d)$

Inserting (A36) into the fourth equation of (A34), we yield for A_2 the following expression, which replaces (A12):

(A39) $A_2 = e^{-r_2 T} \left(\mathrm{d}\left(\overline{m-p}\right) + \frac{1}{\chi} \left((h_{20} - h_{21}) \mathrm{d}\overline{\tau} + (h_{11} - h_{10}) \mathrm{d}\overline{p}^d \right) \right)$

In the case of *monetary policy* $d\dot{m} > 0$, $A_2 < 0$ and owing to (A32) and (A35)

(A40) $\tau(0+) = \overline{\tau}_0 + A_0 h_{10} + A_1 h_{11} = \overline{\tau}_0 - \frac{\chi}{h_{21} - h_{20}} A_2 < \overline{\tau}_0$

Furthermore,

(A41) $\dot{\tau}(0+) = A_0 h_{10} r_0 + A_1 h_{11} r_1 = \frac{1}{h_{21} - h_{20}} \left(h_{20} h_{11} r_1 - h_{21} h_{10} r_0 \right) A_2$

$$= \frac{1}{h_{21} - h_{20}} \left(-\gamma_1 \chi \right) A_2 = \frac{(r_0 - r_2)(r_1 - r_2)}{\gamma_3 \gamma_6 - \gamma_5 (\gamma_4 - r_2)} \gamma_1 A_2 > 0$$ [65]

and

(A42) $\dot{p}^d(0+) = A_0 h_{20} r_0 + A_1 h_{21} r_1 = -\frac{h_{20} h_{21}}{h_{21} - h_{20}} (r_0 - r_1) A_2 > 0$

since $\frac{h_{20} h_{21}}{h_{21} - h_{20}} = \frac{\gamma_3 (r_0 - r_2)(r_1 - r_2)}{(r_0 - r_1)(\gamma_5 (\gamma_4 - r_2) - \gamma_3 \gamma_6)} > 0$

Due to *sign* $y(0+) =$ *sign* $\dot{\tau}(0+)$ and *sign* $y^d(0+) =$ *sign* $\dot{p}^d(0+)$ we find that on impact - as in the case $\alpha_1 = \alpha_2$ - an increase in y as well as y^d and a fall in τ. Because of

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[65] The following relationship holds: $h_{20} h_{11} r_1 - h_{21} h_{10} r_0 = -\gamma_1 \chi$. This follows from the characteristic equations $\gamma_1 h_{11} + \gamma_2 h_{21} = r_1 h_{11}$, $\gamma_1 h_{10} + \gamma_2 h_{20} = r_0 h_{10}$, by multiplying them with h_{20} and h_{21} respectively and subtracting them from each other.

(A43) $i_1 - \dot{p}_1^c = i^* - \frac{1}{2}(1-\alpha_3)\dot{\tau} - \frac{1}{2}(\alpha_1 - \alpha_2)\dot{p}^d$

and

(A44) $i_2 - \dot{p}_2^c = i^* - \frac{1}{2}(1-\alpha_3)\dot{\tau} + \frac{1}{2}(\alpha_1 - \alpha_2)\dot{p}^d$

we yield in the case $\alpha_1 > \alpha_2$ different developments in real interest rates. On impact, $i_1 - \dot{p}_1^c$ falls more strongly than $i_2 - \dot{p}_2^c$. In the subsequent adjustment process, we find qualitatively the same adjustment patterns for y and y^d as in the case $\alpha_1 = \alpha_2$. Interestingly, we find the same reversal point t^* in y^d. Following (A35) we have

(A45) $\dot{y}^d = 0 \Leftrightarrow \dot{p}^d = 0 \Leftrightarrow$

$$A_0 h_{20} r_0 e^{r_0 t} + A_1 h_{21} r_1 e^{r_1 t} = 0 \Leftrightarrow \frac{h_{20} h_{21}}{h_{21} - h_{20}}\left(-r_0 e^{r_0 t} + r_1 e^{r_1 t}\right) A_2 = 0 \Leftrightarrow$$

$$-r_0 e^{r_0 t} + r_1 e^{r_1 t} = 0$$

The last equation is identical with (A22) such that again equation (20) determines t^*.

In the case of a *symmetric fiscal expansion* and analogous to the case $\alpha_1 = \alpha_2$ it is reasonable to assume $A_2 > 0$. With $h_{20} > 0 > h_{21}$ it follows that $A_0 < 0, A_1 < 0$. Under the previous assumption $\mathrm{d}\left(\overline{m-p}\right)/\mathrm{d}g < 0$ (i.e., $\alpha_3 > l_1 f_1$), we find the constants $\tilde{A}_0$ and $\tilde{A}_1$ for all $T \geq 0$ to be positive. Using (A40) to (A43) we find as a mirror image to the case of monetary policy:

(A46) $\tau(0+) > \bar{\tau}_0$, $\dot{\tau}(0+) < 0$, $\dot{p}^d(0+) < 0$, $\left(i_1 - \dot{p}_1^c\right)(0+) > i^*$.

This implies for the output variables $y(0+) < \bar{y}_0$ and $y^d(0+) < 0$. The subsequent development of y and y^d is qualitatively identical with the case $\alpha_1 = \alpha_2$. At the date of implementation *T*, the output variables jump as follows:

(A47) $y(T+) - y(T-) = \mathrm{d}\bar{y} + \frac{\alpha_3}{\delta}\left(\dot{\tau}(T+) - \dot{\tau}(T-)\right)$

with

(A48) $\dot{\tau}(T+) - \dot{\tau}(T-) = \left(\tilde{A}_0 - A_0\right) h_{10} r_0 e^{r_0 T} + \left(\tilde{A}_1 - A_1\right) h_{11} r_1 e^{r_1 T}$

$$= \frac{1}{\chi}\left(-h_{21}h_{10}r_0 + h_{20}h_{11}r_1\right)d\overline{\tau}$$

$$= \frac{1}{\chi}\left(-\gamma_1\chi\right)d\overline{\tau} = -\gamma_1 d\overline{\tau} > 0$$

and

(A49) $y^d(T+) - y^d(T-) = \mathrm{d}\overline{y}^d + \frac{1-\alpha_1+\alpha_2}{\delta}\left(\dot{p}^d(T+) - \dot{p}^d(T-)\right)$

with $\mathrm{d}\overline{y}^d = 0$ in the case $\mathrm{d}g_1 = \mathrm{d}g_2 > 0$ and

(A50) $\dot{p}^d(T+) - \dot{p}^d(T-) = \left(\tilde{A}_0 - A_0\right)h_{20}r_0e^{r_0T} + \left(\tilde{A}_1 - A_1\right)h_{21}r_1e^{r_1T}$

$$= \frac{1}{\chi}h_{20}h_{21}(r_1 - r_0)\mathrm{d}\overline{\tau} > 0$$

For y^d we find - as in the case $\alpha_1 = \alpha_2$ - two sign reversals. If a reversal point *t** exists in the interval $0 < t < T$, it is determined by - following (A45) - equation (20). For $t > T$, we find the following reversal point *t*** in the behavior of y^d:

(A51) $y^d = 0 \Leftrightarrow \dot{p}^d = 0 \Leftrightarrow$

$$\tilde{A}_0h_{20}r_0e^{r_0t} + \tilde{A}_1h_{21}r_1e^{r_1t} = 0 \Leftrightarrow e^{(r_0-r_1)t} = -\frac{\tilde{A}_1h_{21}}{\tilde{A}_0h_{20}}\frac{r_1}{r_0} \Leftrightarrow$$

$$t^{**} = \frac{1}{r_0 - r_1}\ln\left(\frac{r_1}{r_0}\right) + \frac{1}{r_0 - r_1}\ln\left(-\frac{\tilde{A}_1h_{21}}{\tilde{A}_0h_{20}}\right) = t^* + \frac{1}{r_0 - r_1}\ln\left(-\frac{\tilde{A}_1h_{21}}{\tilde{A}_0h_{20}}\right)$$

In the special case *T*=0, the positive constant $-\frac{\tilde{A}_1h_{21}}{\tilde{A}_0h_{20}}$ is equal to unity such that *t*** = *t**.

Monetary policy reaction to $\mathrm{d}g_1 = \mathrm{d}g_2 > 0$ in the case $\alpha_1 > \alpha_2$

In the case of an *unanticipated* symmetric fiscal expansion (*T* = 0), complete stabilization requires $\tilde{A}_0 = \tilde{A}_1 = 0$ (cf. (A33)). The constants are

(A52) $\tilde{A}_0 = -\frac{h_{21}}{h_{21}-h_{20}} \mathrm{d}(\overline{m-p})$, $\tilde{A}_1 = \frac{h_{20}}{h_{21}-h_{20}} \mathrm{d}(\overline{m-p})$,

such that an unanticipated monetary policy, which generates $\mathrm{d}(\overline{m-p}) = 0$, stabilizes the overall system.[66] As in the case $\alpha_1 = \alpha_2$, the appropriate response is determined by the reaction function (31). In the case of an *anticipated* symmetric fiscal expansion (T > 0), complete stabilization in the interval $0 < t < T$ requires $A_0 = A_1 = A_2 = 0$. Using (A35), this condition is fulfilled as long as monetary policy sets $A_2 = 0$. From (A39), the previous reaction function for monetary policy (A27) now takes the more general form

(A53) $\mathrm{d}\dot{m}^{\text{ant}}(T) = -\frac{1}{l_2}\frac{1}{b_5+\lambda f_1}\left(\alpha_3 - l_1 f_1 + \frac{h_{11}-h_{20}}{\chi}\right)\mathrm{d}g(T) > 0$.

The actual implementation of the expansionary monetary policy (A53) in T reinforces the expansionary fiscal policy in the period $t > T$; a complete stabilization requires in T an unanticipated, time-inconsistent restrictive monetary policy. Following (A33) and (A52), stabilization in the interval $t > T$ requires the two conditions $\tilde{A}_0 = 0$ and $\tilde{A}_1 = 0$ to hold, which can be attained via the reaction function (31).

Ruling out this time-inconsistent reaction as a feasible policy option, monetary policy can stabilize the overall system only in the anticipation period $0 < t < T$, with a policy reaction of the form (A53). In contrast with the case $\alpha_1 = \alpha_2$, it is impossible to achieve via a monetary policy announcement in $t = 0$ complete stabilization in the period after the fiscal policy implementation, $t > T$, because the constants $\tilde{A}_0$ and $\tilde{A}_1$ in (A37) and (A38) can not be set simultaneously equal to zero.[67]

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[66] In the case of an *asymmetric* fiscal expansion as, for example, $\mathrm{d}g_1 > 0 = \mathrm{d}g_2$, it follows $\mathrm{d}\overline{p}^d \neq 0$. In this case, the equations for $\tilde{A}_j$ ($j = 0,1$) in (A52) are augmented by the term $(-1)^j \frac{1}{h_{21}-h_{20}} \mathrm{d}\overline{p}^d$. As a result, a change in $\mathrm{d}\dot{m}$ no longer achieves complete stabilization of the aggregate system since it is impossible to set $\tilde{A}_0$ and $\tilde{A}_1$ simultaneously to zero.

[67] To see this, insert (A35) and (A39) into (A37) and (A38). In the case of monetary policy or a symmetric fiscal policy, $\mathrm{d}\overline{p}^d = 0$ holds. Then, setting $\tilde{A}_0$ and $\tilde{A}_1$ equal to zero implies different reaction functions for $\dot{m}$. With $\alpha_1 > \alpha_2$, only *unanticipated* fiscal policy can be completely stabilized via monetary policy. With *anticipated* fiscal policy, even an unanticipated monetary policy in T fails to achieve complete stabilization in the period after implementation $t > T$.

Sluggish output adjustment in goods markets

The determinant of the system matrix D in (27) is

$$\text{(A54)}\quad \det D = \beta^2 b_5 (2b_4 + b_5) \frac{\delta^2}{\alpha_3 (1 - \alpha_1 + \alpha_2)} \cdot \frac{1}{l_2} > 0\,.$$

The number of stable characteristic roots is inferred to be even. With four predetermined state variables ($y, y^d, p^d, m - p$) we have at maximum four stable roots (r_1, ..., r_4). Furthermore, one unstable root $r_5 = 1/l_2$ exists, such that in the case $r_i \in \Re$ with $r_i < 0$ (i = 1,...,4) a saddle path equilibrium prevails.[68] In this case exists a unique convergent solution $x = (y, y^d, \tau, p^d, m - p)'$ in the following form:

$$\text{(A55)}\quad x = \bar{x}_0 + \sum_{j=1}^{5} A_j h_j e^{r_j t} \quad \text{for } t < T$$

$$\text{(A56)}\quad x = \bar{x}_1 + \sum_{j=1}^{4} \tilde{A}_j h_j e^{r_j t} \quad \text{for } t > T$$

The characteristic vector $h_j = (h_{1j}, \ldots, h_{5j})'$ refers to the respective characteristic root r_j $(1 \le j \le 5)$, while the constants $A_1, \ldots, A_5, \tilde{A}_1, \ldots, \tilde{A}_4$ are determined by the following conditions:

1. The five state variables are continuous in T.
2. The four predetermined variables $y, y^d, p^d, m - p$ are continuous in t = 0.

The characteristic vector h_5 belonging to the unstable root $r_5 = 1/l_2$ can be defined as a unit vector $h_5 = (0,0,0,0,1)'$ and the fifth component h_{5j} of the characteristic vector h_j can be normalized to unity $(h_{51} = \ldots = h_{55} = 1)$. The conditions for continuity follow as[69]

$$\text{(A57)}\quad \sum_{j=1}^{5} A_j h_j e^{r_j T} = \mathrm{d}\bar{x} + \sum_{j=1}^{4} \tilde{A}_j h_j e^{r_j T}$$

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[68] Numerical simulations with realistic parameter values show that the system has typically four stable characteristic roots. Complex conjugate solutions are also conceivable. However, in some parameter settings we find merely two stable roots. This case leads to explosive solutions to the dynamic system (27) (*Turnovsky* (2000), p. 147; *Buiter* (1984)).

[69] (A57) and (A58) represent a direct generalization of (A34).

(A58) $0 = \sum_{j=1}^{5} A_j h_{ij} \quad ; \; i = 1\,,\, 2,\, 4,\, 5$

As $h_{i5} = 0$ for $1 \le i \le 4$ and $h_{5j} = 1$ for $1 \le j \le 5$, (A58) can be written alternatively as:

(A59) $$\begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{41} & h_{42} & h_{43} & h_{44} \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -A_5 \end{pmatrix}$$

Denoting the matrix preceding $(A_1, \ldots, A_4)'$ with H, we find a proportional relation between A_i $(1 \le i \le 4)$ and A_5 of the following form[70]

(A60) $A_i = \chi_i A_5$

$$\text{with } \chi_i = -\frac{1}{\det H}(-1)^{4+i} cof_{4i} = (-1)^{5+i} \frac{cof_{4i}}{\det H} \qquad (1 \le i \le 4)$$

where cof_{4i} $(1 \le i \le 4)$ denotes the (4, i) cofactor of H, i.e., the subdeterminant of H, which results from the deletion of the 4th row and the ith column of the matrix H.

The conditions for continuity (A57) can be rewritten as

(A61) $\mathrm{d}\overline{x} + \sum_{j=1}^{4} (\tilde{A}_j - A_j) h_j e^{r_j T} = A_5 h_5 e^{r_5 T}$

The five-dimensional equation system decomposes into

(A62) $$\begin{pmatrix} h_{11}e^{r_1 T} & h_{12}e^{r_2 T} & h_{13}e^{r_3 T} & h_{14}e^{r_4 T} \\ h_{21}e^{r_1 T} & h_{22}e^{r_2 T} & h_{23}e^{r_3 T} & h_{24}e^{r_4 T} \\ h_{31}e^{r_1 T} & h_{32}e^{r_2 T} & h_{33}e^{r_3 T} & h_{34}e^{r_4 T} \\ h_{41}e^{r_1 T} & h_{42}e^{r_2 T} & h_{43}e^{r_3 T} & h_{44}e^{r_4 T} \end{pmatrix} \begin{pmatrix} A_1 - \tilde{A}_1 \\ A_2 - \tilde{A}_2 \\ A_3 - \tilde{A}_3 \\ A_4 - \tilde{A}_4 \end{pmatrix} = \begin{pmatrix} \mathrm{d}\overline{y} \\ \mathrm{d}\overline{y}^d \\ \mathrm{d}\,\overline{\tau} \\ \mathrm{d}\overline{p}^d \end{pmatrix}$$

and

(A63) $(A_1 - \tilde{A}_1)e^{r_1 T} + (A_2 - \tilde{A}_2)e^{r_2 T} + (A_3 - \tilde{A}_3)e^{r_3 T} + (A_4 - \tilde{A}_4)e^{r_4 T} + A_5 e^{r_5 T} = \mathrm{d}\left(\overline{m - p}\right)$

For A_5 it follows

[70] (A60) substitutes for (A35) in the case of sluggish output adjustment $(\beta < \infty)$.

(A64) $A_5 = e^{-r_5 T}\left(\mathrm{d}\left(\overline{m-p}\right) + \sum_{j=1}^{4} (\tilde{A}_j - A_j) e^{r_j T} \right).$

Denoting in (A62) the matrix belonging to $(A_1 - \tilde{A}_1, \ldots, A_4 - \tilde{A}_4)'$ by $\tilde{H}_T$, it follows

(A65) $\begin{pmatrix} A_1 - \tilde{A}_1 \\ A_2 - \tilde{A}_2 \\ A_3 - \tilde{A}_3 \\ A_4 - \tilde{A}_4 \end{pmatrix} = \tilde{H}_T^{-1} \begin{pmatrix} \mathrm{d}\overline{y} \\ \mathrm{d}\overline{y}^d \\ \mathrm{d}\overline{\tau} \\ \mathrm{d}\overline{p}^d \end{pmatrix}$

In the case $\mathrm{d}\overline{y}^d = \mathrm{d}\overline{p}^d = 0$ we find, as a result of $\mathrm{d}\overline{y} = f_1 \cdot \mathrm{d}\overline{\tau}$, $A_i - \tilde{A}_i$ to be proportional with respect to $\mathrm{d}\overline{\tau}$:

(A66) $A_i - \tilde{A}_i = \Psi_i \cdot \mathrm{d}\overline{\tau} \quad (1 \le i \le 4)$

with

$$\Psi_i = \frac{1}{\det \tilde{H}_T} (-1)^{1+i} (c\tilde{o}f_{1i} \cdot f_1 + c\tilde{o}f_{3i})$$

where $c\tilde{o}f_{1i}$ and $c\tilde{o}f_{3i}$ represent the (1, *i*) and (3, *i*) cofactor of $\tilde{H}_T$.

In the case of *monetary policy* $\mathrm{d}\dot{m} > 0$, $\mathrm{d}\overline{\tau} = 0$ and in conjunction with (A66) $A_i = \tilde{A}_i$ $(1 \le i \le 4)$. Then, (A64) implies $A_5 = e^{-r_5 T}\mathrm{d}\left(\overline{m-p}\right) < 0$.

The characteristic roots r_1, ..., r_5 are assumed real and the corresponding characteristic vectors can be expressed in a general form by using the definition of h_j

(A67) $D \cdot h_j = r_j h_j \quad (j = 1, \ldots, 5)$

with $D = (d_{ij})$ and solved analogously to (27). h_5 represents the unit vector $(0,0,0,0,1)'$, while the components $h_{1j}, \ldots, h_{5j}$ of the characteristic vector h_j $(1 \le j \le 4)$ satisfy:

(A68) $h_{1j} = \dfrac{r_j (r_j - r_5)}{d_{51} r_j + d_{53} d_{31}}$

$$h_{2j} = -\frac{d_{21} r_j^2 (r_j - r_5)}{\left((d_{22} - r_j) r_j + d_{24} d_{42}\right)(d_{51} r_j + d_{53} d_{31})}$$

$$h_{3j} = \frac{d_{31}(r_j - r_5)}{d_{51}r_j + d_{53}d_{31}}$$

$$h_{4j} = -\frac{d_{42}d_{21}r_j(r_j - r_5)}{\left((d_{22} - r_j)r_j + d_{24}d_{42}\right)(d_{51}r_j + d_{53}d_{31})}$$

$$h_{5j} = 1$$

The following relationship holds:

(A69) $h_{2j} = \frac{r_j}{d_{42}} h_{4j}, \quad h_{1j} = \frac{r_j}{d_{31}} h_{3j} \qquad (1 \le j \le 4)$

As a result of $d_{51}r_j + d_{53}d_{31} > 0$, $h_{1j} > 0$ and $h_{3j} < 0$. Furthermore, $d_{42}d_{21}r_j(r_j - r_5) > 0$, $d_{24}d_{42} < 0$, $d_{22} < 0$, but $(d_{22} - r_j)r_j + d_{24}d_{42} \gtrless 0$. It follows that the sign of h_{4j} is indeterminate and, owing to (A69), the sign of h_{2j} is also indeterminate. This implies that the constants A_1,..., A_4 can no longer be signed (cf. (A60)).

Numerical simulations with realistic parameter values show that the jump variable τ behaves qualitatively in the same fashion as with instantaneous output adjustment ($\beta = \infty$). The output variables y and y^d are here continuous everywhere. On impact, y and y^d no longer jump, but respond gradually. The output differential y^d remains continuous at time T and has - as in the case $\beta = \infty$ - a reversal point t^*, which does not depend on T. Using (A60) and (A69), t^* is the solution to the following equation:[71]

(A70) $y^d = 0 \Leftrightarrow \dot{p}^d = 0 \Leftrightarrow$

$$\sum_{j=1}^{4} A_j h_{4j} r_j e^{r_j t} = 0 \Leftrightarrow$$

$$A_5 \left(\sum_{j=1}^{4} \chi_j h_{4j} r_j e^{r_j t}\right) = 0 \Leftrightarrow$$

$$\sum_{j=1}^{4} \chi_j h_{4j} r_j e^{r_j t} = 0 \Leftrightarrow$$

[71] (A70) replaces (A45). It can no longer be solved analytically for t because it now includes four exponentials.

$$\sum_{j=1}^{4}(-1)^{5+j}cof_{4j}h_{4j}r_je^{r_jt}=0 \Leftrightarrow$$

$$\sum_{j=1}^{4}(-1)^{j}cof_{4j}h_{4j}e^{r_jt}=0$$

With a s*ymmetric fiscal policy* ($\mathrm{d}g_1=\mathrm{d}g_2>0$), y^d experiences two sign reversals. As long as one sign reversal occurs before the date of implementation *T*, it is determined by (A70).

The dynamic effects of an *unanticipated* symmetric fiscal expansion can only be neutralized by monetary policy as long as the conditions $\tilde{A}_1=0, \tilde{A}_2=0, \tilde{A}_3=0$ and $\tilde{A}_4=0$ simultaneously hold. From (A57) and (A58) it follows in the case $T=0$

(A71) $$\sum_{j=1}^{5}A_jh_{ij}=0=d\bar{x}_i+\sum_{j=1}^{4}\tilde{A}_jh_{ij} \qquad i=1, 2, 4, 5$$

because the four predetermined variables remain continuous at $t=T=0$. In extensive form:

(A72) $$\begin{pmatrix}-\mathrm{d}\bar{y}\\-\mathrm{d}\bar{y}^d\\-\mathrm{d}\bar{p}^d\\-\mathrm{d}\overline{(m-p)}\end{pmatrix}=\underbrace{\begin{pmatrix}h_{11}&h_{12}&h_{13}&h_{14}\\h_{21}&h_{22}&h_{23}&h_{24}\\h_{41}&h_{42}&h_{43}&h_{44}\\1&1&1&1\end{pmatrix}}_{H}\begin{pmatrix}\tilde{A}_1\\\tilde{A}_2\\\tilde{A}_3\\\tilde{A}_4\end{pmatrix}$$

Due to $d\bar{y}=f_1\mathrm{d}\bar{\tau}$ and $\mathrm{d}\bar{y}^d=0=\mathrm{d}\bar{p}^d$, $\tilde{A}_i$ is

(A73) $$\tilde{A}_i=\frac{1}{\det H}\left\{(-1)^i cof_{1i}\cdot f_1\mathrm{d}\bar{\tau}+(-1)^{i+1}cof_{4i}\mathrm{d}\left(\overline{m-p}\right)\right\}$$

$$=\frac{(-1)^{i+1}cof_{4i}}{\det H}\left\{-\frac{cof_{1i}}{cof_{4i}}f_1\mathrm{d}\bar{\tau}+\mathrm{d}\left(\overline{m-p}\right)\right\} \qquad (1\le i\le 4)$$

In contrast with the previous case of instantaneous output adjustment ($\beta=\infty$), $\tilde{A}_i$ now depends on $\mathrm{d}\bar{\tau}$ (cf. (A52)), because the cofactor cof_{1i} is not equal to zero. With considerable algebra, it is possible to show that for $i\neq j$, $cof_{1i}/cof_{4i}\neq cof_{1j}/cof_{4j}$ holds. Therefore, in the case $T=0$ it is not possible for monetary policy to set $\tilde{A}_1=\tilde{A}_2=\tilde{A}_3=\tilde{A}_4=0$ by using a change of $\dot{m}$, i.e., to achieve complete stabilization.

This implies that also in the case of *anticipated* fiscal policy it is impossible to achieve complete stabilization even with a time-inconsistent monetary policy. However, it is possible to neutralize the impact of the announcement effect of fiscal policy. Using (A60), this requires setting $A_5 = 0$. (A60) and (A66) imply

$$\text{(A74)} \quad A_5 = e^{-r_5 T}\left(\mathrm{d}\left(\overline{m-p}\right) - \left(\sum_{\mathrm{j}=1}^{4} \Psi_{\mathrm{j}} e^{r_j T} \right) \mathrm{d}\overline{\tau} \right).$$

The requirement $A_5 = 0$ implies the following monetary policy reaction in $T > 0$ to be credibly announced in $t = 0$:[72]

$$\text{(A75)} \quad \mathrm{d}\dot{m}^{\mathrm{ant}}(T) = -\frac{1}{l_2}\frac{1}{b_5 + \lambda f_1}\left(\alpha_3 - l_1 f_1 + \sum_{j=1}^{4} \Psi_j e^{r_j T} \right) \mathrm{d}g(T)\,.$$

As long as the effect of fiscal policy ($dg(T) > 0$) in the interval (0, T) on output y is negative as in *Figure 13*, monetary policy has to be expansionary ($\mathrm{d}\dot{m}^{\mathrm{ant}}(T) > 0$). Since we assume $\alpha_3 > l_1 f_1$, this requires $\sum_{j=1}^{4} \Psi_j e^{r_j T} < l_1 f_1 - \alpha_3 < 0$.

[72] (A75) is a generalization of (A53).

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