Parental Time Allocation: Implications of the Generalized Alchian-Allen Theorem

Junichi Minagawa[*†] **Thorsten Upmann**[‡]

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Abstract

In this paper we generalize the Alchian-Allen substitution theorem so as to account for income and endowment effects. We then apply the generalized version to the labor-leisure-child-care decision of a mother with young children. In this framework we find: (i) For mothers who work only a few hours a week, a rise in wages increases (under mild qualifications) the demand for leisure relative to child care, an effect in line with the Alchian-Allen result; (ii) given empirical evidence that an increase in wages induces mothers to spend less time on leisure and more with children, the generalized Alchian-Allen theorem shows that this requires the income elasticity of child care to be substantially higher than that of leisure. This finding, though, imposes substantial restrictions on reasonable specifications of a mother's utility function.

Keywords: The Alchian-Allen theorem; Income and endowment effects; Labor-leisure-child-care choice; Asymmetrical income elasticities; Specification of a utility function

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[*] 1-873-12 Ogawa, Kodaira, Tokyo 187-0032, Japan. Email: jminagawa@gmail.com

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[‡] University Duisburg-Essen, Mercator School of Management, Lotharstraße 65, D–47057 Duisburg, Germany. Email: Thorsten.Upmann@uni-duisburg-essen.de

1. Introduction

Time allocation problems are among the most important problems of individuals, affecting economic, social and educational aspects of life. In particular, the time allocation decision of a (single) parent[1] is not only important for herself, but also exerts overwhelming influence on her children. Since the consequences of various activities such as working time, time spent together with the children and pure leisure time are highly diverse, it is indispensable to distinguish between these activities in modeling the time allocation decision of a mother with young children. Notably, we should distinguish between, at least, two non-labor activities (or leisure time activities in a broad sense): pure leisure time (time without children) and parental child care (leisure time spent together with children). These modes of time use should not only be treated separately as they affect utility differently, but also from the cost point of view: While the price of pure leisure time includes the price of non-maternal (i. e., external) child care, parental child care merely covers the foregone wage income. As a consequence, the former is higher than the latter by the cost of external child care.

However, once we treat pure leisure time and parental child care as two different modes of time use, it is immediate that a rise in wage rates not only interferes with a mother's labor-supply-leisure decision but also with the division of non-working time between different modes of leisure time activities (in a broad sense). Since pure leisure time becomes relatively less expensive when compared with parental child care, a rise in the wage rate arguably induces a mother to reallocate time from maternal child care to pure leisure time in relative terms. This effect may be thought of as a direct consequence of the well-known and in the literature intensively discussed[2] Alchian-Allen substitution theorem (see Alchian and Allen, 1964, p. 74–75): a per unit charge (here: the wage rate) onto two similar, that means substitutable goods (here: pure leisure time and parental child care), lowers the relative price of the more expensive good and therefore raises the relative demand for that good.

[1]For illustrative purpose we subsequently speak of a *young mother*, but this should be understood more broadly as a (single) parent with young children.

[2]See, for example, Gould and Segall (1969), Borcherding and Silberberg (1978), Umbeck (1980), Bertonazzi and Maloney (1993), Cowen and Tabarrok (1995), Sobel and Garrett (1997), Bauman (2004), Hummels and Skiba (2004), Lawson and Raymer (2006) and Saito (2008).

While the Alchian-Allen result has been confirmed empirically for consumption goods, see, for example, Sobel and Garrett (1997) and Hummels and Skiba (2004),[3] there are empirical studies which report seemingly opposite results for parental time allocation decisions: for example, Kimmel and Connelly (2007), using the 2003-2004 American Time Use Survey to estimate a time allocation model, find that higher maternal wages decrease leisure and increase child care; and Guryan, Hurst, and Kearney (2008), using the 2003-2006 American Time Use Survey, find the patterns in the data that higher-educated mothers spend less time in leisure and more time with children.[4] To see why empirical findings for consumption goods and time allocation problems are seemingly contradictory,[5] it is important to realize that in due compliance with the Alchian-Allen theorem, the confirmations of Sobel and Garrett (1997) and Hummels and Skiba (2004) are based on the assumption that income effects are insignificant. While this is a reasonable assumption if we consider (the traditional example of) good and bad apples, income effects are clearly present in time allocation decisions. Therefore, the data base of the empirical analyses investigating parental time allocation decisions represents the image of a mixture of substitution and income effects. It is thus not surprising that the empirical analyses of consumption decisions and those of time allocation decisions arrive at varying conclusions.

Since the Alchian-Allen theorem is concerned with pure substitution effects it is not readily applicable to time allocation decisions, as the neglect of income effects would deprive the time allocation decision one of its most important features, and thus of its singularity. This argument becomes even stronger in view of the fact that individuals possess an initial endowment of time which they allocate between different modes of time use — and the induced endowment effect works in opposite to the income effect: While a higher price of a consumption good induces a negative income effect (provided the good is normal), a higher price of a good in which the consumer has some initial holding induces a positive income effect. Thus, whenever

[3]Bertonazzi et al. (1993) provide another confirmation, though in a somewhat different context, namely, in a household production framework, which results from an interpretation of the charge in the Alchian-Allen theorem. See Umbeck (1980) and Cowen and Tabarrok (1995) for various interpretations.

[4]These are consistent with the earlier findings of Hill and Stafford (1974) and Leibowitz (1975) that higher status or educated mothers spend more time in child care.

[5]Moreover, conflicting findings are reported in gasoline markets: Nesbit (2007) confirms the Alchian-Allen theorem, while Lawson and Raymer (2006) do not.

time allocation problems are concerned, substitution effects are not only accompanied by and mixed with income effects, but are also superimposed by endowments effects. For this reason a generalization of the Alchian-Allen theorem is required which simultaneously accounts for income *and* endowments effects in supplement to the pure substitution effects. While such a generalization of the Alchian-Allen theorem is an interesting and challenging theoretical task on its own, it may also serve as a theoretical vehicle for an empirical analysis of time allocation decisions. In particular, it may help to explain empirical findings which are seemingly at variance with the predictions of the original Alchian-Allen theorem.

The aim of this work is therefore twofold. First, we derive a generalized version of the Alchian-Allen theorem so as to account for all of the crucial determinants of observed economic behavior: income effects, endowment effects and substitution effects as well as their mutual interactions. To achieve at this, we build on the work of Gould and Segall (1969), Borcherding and Silberberg (1978), and Bauman (2004) who have previously extended the Alchian-Allen theorem to the case of three or more goods, in terms of compensated demand functions.[6] In our approach, though, we use ordinary instead of compensated demand functions to capture both income and endowment effects, and show that the Alchian-Allen result may be extended to the presence of income and endowment effects (under rather mild qualifications). In this sense, the Alchian-Allen theorem survives a generalization. In particular, our generalized Alchian-Allen formula concurs with the pure substitution version provided by Borcherding and Silberberg (1978), if either the income elasticities for the goods under consideration coincide or if "on average" a consumer consumes his/her endowments.

The second aim of this paper is to demonstrate how the generalized Alchian-Allen theorem may provide valuable conditions on the elasticities of demand — and thus on utility functions — which must be fulfilled for the predictions of a theoretical model to be consistent with empirical findings. To this end, we consider a time allocation problem, more precisely the labor-leisure-child-care decision of a parent,[7] and show how our generalized version of the Alchian-Allen theorem can be applied to this framework. Building upon our aforementioned results of consumer

[6]Saito (2008) considers a version of the Alchian-Allen theorem with (zero) income effects under some specific assumptions.

[7]It should be noted that the above empirical studies on parental time allocation decisions, Kimmel and Connelly (2007) and Guryan, Hurst, and Kearney (2008), consider basically the four categories of time: labor, leisure, child care, and home production, but they find that leisure and

theory, we show that, under mild qualifications, a higher wage rate induces a parent who works only a few hours a week to spend relatively more time on pure leisure when compared with time spent on parental child care. Moreover, we use empirical estimates to deduce income elasticities from our version of the Alchian-Allen formula: Since empirical estimates strongly suggest that a rise in wages decreases the demand for leisure relative to child care, we are able to infer that the income elasticity of child care must substantially exceed the income elasticity of leisure. In this way, the cited empirical literature, when applied to our multi-time-use-allocation model, provides a warning against the use of either quasi-linear or homothetic utility functions in parental time-allocation models, because both imply that the income effects of different types of leisure activity coincide.

2. The Model

Consider the standard model of consumer behavior with three goods. Suppose that the consumer's preferences may be represented by a differentiable, monotonic, strictly quasi-concave utility function $u : \mathbb{R}^3_+ \to \mathbb{R} : (x_1, x_2, x_3) \mapsto u(x_1, x_2, x_3)$. We allow for the consumer to have non-negative initial endowments, denoted by $\bar{x}_1$, $\bar{x}_2$, and $\bar{x}_3$ for the three goods respectively and a positive money income m. Moreover, let p_1, p_2, and p_3 denote the respective (before-tax) prices, where by assumption $p_1 > p_2$. For the purpose of a more compact notation, we define the following vectors $\mathbf{x} := (x_1, x_2, x_3)^{\mathsf{T}}$ and $\mathbf{p} := (p_1, p_2, p_3)$. (We write commodity vectors as column and price vectors as row vectors.)

By assumption, a fixed charge t is added to the prices of the first two goods, such that gross consumer prices may be written as $q_1 \equiv p_1 + t$, $q_2 \equiv p_2 + t$, and $q_3 \equiv p_3$; or more compactly $\mathbf{q} := \mathbf{p} + \mathbf{t}$ with $\mathbf{q} := (q_1, q_2, q_3)$, $\mathbf{t} := (t, t, 0)$, where consumer prices are assumed to be positive. The consumer's total income (or wealth) level given by $\mathbf{q} \cdot \bar{\mathbf{x}} + m$ depends on the price vector and the money income and thus we may define a function $I(\mathbf{q}, m) := \mathbf{q} \cdot \bar{\mathbf{x}} + m$. Then, as usual, the consumer's utility maximization problem is given by

$$\max_{x_1, x_2, x_3} \; u(\mathbf{x}) \quad \text{s.t.} \quad \mathbf{q} \cdot \mathbf{x} \leqq \mathbf{q} \cdot \bar{\mathbf{x}} + m, \tag{1}$$

yielding ordinary demand functions $\mathbf{x}^o(\mathbf{q}, I(\mathbf{q}, m))$. Taking into account that consumer prices depend on the charge t, we define demand as a function of t (and

home production react in similar ways to wages or education; consequently, we may safely focus on the first three categories.

exogenous income m) rather than of prices (and full income I): $\hat{\mathbf{x}}^o(t,m) := \mathbf{x}^o(p_1 + t, p_2 + t, p_3, I(p_1 + t, p_2 + t, p_3, m)) \equiv \mathbf{x}^o(\mathbf{q}(t), I(\mathbf{q}(t), m))$.

Correspondingly, the expenditure minimization problem may be expressed as

$$\min_{x_1,x_2,x_3} \mathbf{q} \cdot [\mathbf{x} - \bar{\mathbf{x}}] \quad \text{s. t.} \quad u(\mathbf{x}) \geqq v, \tag{2}$$

the solution of which is given by the compensated demand function $\mathbf{x}^*(\mathbf{q}, v)$. Similarly, it is helpful to write compensated demand as a function of the charge t (and v): $\hat{\mathbf{x}}^*(t,v) := \mathbf{x}^*(p_1 + t, p_2 + t, p_3, v) \equiv \mathbf{x}^*(\mathbf{q}(t), v)$.

In the remainder of our analysis, we use the following notation. The compensated price elasticity of good i with respect to the consumer price of good j is defined as $\varepsilon^*_{ij}(\mathbf{q}, v) := (q_j/x^*_i(\mathbf{q}, v))(\partial x^*_i(\mathbf{q}, v)/\partial q_j)$. The income elasticity of good i is defined by $\varepsilon_{iI}(\mathbf{q}, I) := (I/x^o_i(\mathbf{q}, I))(\partial x^o_i(\mathbf{q}, I)/\partial I)$. Whenever it is clear at which point of the domain these elasticities are evaluated, we suppress their arguments; a similar hint applies to consumer prices where we frequently suppress the argument t and simply write q_i or $\mathbf{q}$. Likewise, in order to save notational effort we shall henceforth simply write x_i and $\mathbf{x}$ to denote the demand level (or the image) of the demand function under consideration, rather than the generic consumption level. Since in the subsequent analysis we exclusively deal with the solution of the consumer choice problem only, no ambiguity should arise, though.

3. Results

Before we derive our central result, Proposition 2, it is advantageous to reformulate a result of Borcherding and Silberberg (1978), which can also be found in e. g., Silberberg and Suen (2000, p. 339), in our notation. Following our notational convention, we here write $x_i = \hat{x}^*_i(t,v)$, ε^*_{ij} instead of $\varepsilon^*_{ij}(\mathbf{q}(t), v)$ and q_i instead of $q_i(t)$. With this note of caution we arrive at

Proposition 1 (Borcherding/Silberberg, 1978)**.**

$$\frac{\partial}{\partial t} \frac{\hat{x}^*_1}{\hat{x}^*_2}(t,v) = \frac{x_1}{x_2} \frac{1}{q_2} \left[(\varepsilon^*_{11} - \varepsilon^*_{21}) \left(\frac{q_2}{q_1} - 1 \right) + (\varepsilon^*_{23} - \varepsilon^*_{13}) \right].$$

Proof. As long as compensated demand functions are concerned, the demonstration of Borcherding and Silberberg holds without modification except for the use of consumer prices rather than before-tax prices. □

Proposition 1 implies that if the two goods are not perfect complements (i. e., $\varepsilon_{11}^* < \varepsilon_{21}^*$) and good 1 is not a much stronger substitute for the third good than is good 2 (i. e., $\varepsilon_{23}^* - \varepsilon_{13}^* > -\alpha$ for some small positive α), then the Alchian-Allen result, that the partial derivative $(\partial/\partial t)(\hat{x}_1^*/\hat{x}_2^*)$ is positive, generalizes to this more general framework. (Recall that we assumed $p_1 > p_2$ and thus $q_1 > q_2$.) In addition, negligible price differences between good 1 and good 2 (i. e., $p_1 \approx p_2$ and hence $q_1 \approx q_2$) will weaken the substitution effect. Borcherding and Silberberg (1978) argue that if the two goods under consideration are close substitutes for each other, then they should be similarly related to the third good, thus, the term $\varepsilon_{23}^* - \varepsilon_{13}^*$ should be small.

In order to formulate Proposition 2 it is expedient to write demand as a function of $\mathbf{q}$ and m: $\tilde{\mathbf{x}}^o(\mathbf{q}, m) := \mathbf{x}^o(\mathbf{q}, I(\mathbf{q}, m))$. Similarly, we define the expenditure function $E(\mathbf{q}, v) := \mathbf{q} \cdot [\mathbf{x}^*(\mathbf{q}, v) - \bar{\mathbf{x}}]$, with $m = E(\mathbf{q}, v)$. It then follows as a familiar duality result that $x_i = x_i^*(\mathbf{q}, v) \equiv \tilde{x}_i^o(\mathbf{q}, E(\mathbf{q}, v))$, or more compactly $\mathbf{x} = \mathbf{x}^*(\mathbf{q}, v) \equiv \tilde{\mathbf{x}}^o(\mathbf{q}, E(\mathbf{q}, v))$. Alternatively, this identity may also be expressed as $\hat{\mathbf{x}}^*(t, v) \equiv \hat{\mathbf{x}}^o(t, E(\mathbf{q}(t), v))$. We are now prepared to derive a version of the Alchian-Allen theorem which accounts for both income and endowment effects.[8]

Proposition 2 (The Generalized Alchian-Allen Formula)**.**

$$\frac{\partial}{\partial t}\frac{\hat{x}_1^o}{\hat{x}_2^o}(t, m) = \frac{\partial}{\partial t}\frac{\hat{x}_1^*}{\hat{x}_2^*}(t, v) + \frac{x_1}{x_2}\frac{1}{I}\left(\varepsilon_{1I} - \varepsilon_{2I}\right)\sum_{j=1}^{2}(\bar{x}_j - x_j).$$

The proof of Proposition 2 makes use of the following lemma, which is a generalization of the well-known Hicks-Slutsky equation when endowment effects are present, that is when the consumer's income is given by the value of its initial endowment (in some or all of the commodities) in addition to money income.

Lemma 1 (The Hicks-Slutsky equation with endowment effects)**.**

$$\frac{\partial x_i^*(\mathbf{q}, v)}{\partial q_j} = \frac{\partial \tilde{x}_i^o(\mathbf{q}, m)}{\partial q_j} - (\bar{x}_j - x_j)\frac{\partial x_i^o(\mathbf{q}, I(\mathbf{q}, m))}{\partial I}.$$

Proof of Lemma 1. Differentiating both sides of $x_i^*(\mathbf{q}, v) \equiv \tilde{x}_i^o(\mathbf{q}, E(\mathbf{q}, v))$ with respect to q_j yields

$$\frac{\partial x_i^*(\mathbf{q}, v)}{\partial q_j} = \frac{\partial \tilde{x}_i^o(\mathbf{q}, m)}{\partial q_j} + \frac{\partial \tilde{x}_i^o(\mathbf{q}, m)}{\partial m}\frac{\partial E(\mathbf{q}, v)}{\partial q_j}. \tag{3}$$

[8] We here remark that as usual, by the duality approach, we can equally work with the indirect utility function $u^o(\mathbf{q}, m) := u(\tilde{\mathbf{x}}^o(\mathbf{q}, m))$, instead of the expenditure function.

Applying Shephard's lemma: $\partial E(\mathbf{q}, v)/\partial q_j = x_j^*(\mathbf{q}, v) - \bar{x}_j$ (see Cornwall (1984, p. 747)) and noting $\partial \tilde{x}_i^o/\partial m = \partial x_i^o/\partial I$, we obtain the required result. $\square$

Proof of Proposition 2. By definition we have $\hat{x}_i^*(t, v) = x_i^*(\mathbf{q}(t), v)$ and thus

$$\frac{\partial \hat{x}_i^*(t, v)}{\partial t} = \sum_{j=1}^{2} \frac{\partial x_i^*(\mathbf{q}, v)}{\partial q_j} \frac{dq_j(t)}{dt}. \tag{4}$$

Using $dq_j(t)/dt = 1$ and applying Lemma 1, we obtain

$$\frac{\partial \hat{x}_i^*(t, v)}{\partial t} = \sum_{j=1}^{2} \left[\frac{\partial \tilde{x}_i^o(\mathbf{q}, m)}{\partial q_j} - (\bar{x}_j - x_j) \frac{\partial x_i^o(\mathbf{q}, I(\mathbf{q}, m))}{\partial I} \right]. \tag{5}$$

Now it is straightforward to calculate the partial derivative of the demand ratio

$$\frac{\partial}{\partial t} \frac{\hat{x}_1^*}{\hat{x}_2^*}(t, v) = \frac{1}{(x_2)^2} \left[\frac{\partial \hat{x}_1^*(t, v)}{\partial t} x_2 - x_1 \frac{\partial \hat{x}_2^*(t, v)}{\partial t} \right]. \tag{6}$$

Substituting for the partial derivatives $\partial \hat{x}_i^*(t, v)/\partial t$, $i = 1, 2$ and expressing in terms of income elasticities, we arrive at

$$\begin{aligned} \frac{\partial}{\partial t} \frac{\hat{x}_1^*}{\tilde{x}_2^*}(t, v) = \frac{1}{(x_2)^2} & \left\{ \sum_{j=1}^{2} \left[\frac{\partial \tilde{x}_1^o(\mathbf{q}, m)}{\partial q_j} \right] x_2 - x_1 \sum_{j=1}^{2} \left[\frac{\partial \tilde{x}_2^o(\mathbf{q}, m)}{\partial q_j} \right] \right\} \\ & - \frac{x_1}{x_2} \frac{1}{I} \left[\varepsilon_{1I}(\mathbf{q}, I(\mathbf{q}, m)) - \varepsilon_{2I}(\mathbf{q}, I(\mathbf{q}, m)) \right] \sum_{j=1}^{2} (\bar{x}_j - x_j). \end{aligned} \tag{7}$$

Since $\partial \hat{x}_i^o(t, m)/\partial t = \sum_{j=1}^{2} [\partial \tilde{x}_i^o(\mathbf{q}, m)/\partial q_j]$, we know that the first part on the right hand side equals $(\partial/\partial t)(\hat{x}_1^o(t, m)/\hat{x}_2^o(t, m))$. Finally, transposing the last term to the left hand side completes the proof. $\square$

Remark 1. From our formulation of Proposition 2 and Lemma 1 it is apparent that our model covers the baseline scenario where the consumer has no initial endowments and income is thus equal to exogenous monetary income (i. e., $I \equiv m$). In this case, endowment effects do not appear and only ordinary income effects are present. If, however, income is price dependent, endowment effects arise reflecting the fact that higher prices of these goods directly increase the consumer's income.

Suppose that the summation term $\sum_{j=1}^{2}(\bar{x}_j - x_j)$, representing the household's aggregate excess supply, is positive.[9] Then, Proposition 2 implies that if good 2 does not feature much stronger income effects than good 1 (i. e., $\varepsilon_{1I} - \varepsilon_{2I} > -\beta$ for some small positive β), the Alchian-Allen result, that $(\partial/\partial t)(\hat{x}_1^*/\hat{x}_2^*)$ is positive, continues to hold — and is even strengthened — if we account for both income and endowment effects; that is, we have $(\partial/\partial t)(\hat{x}_1^o/\hat{x}_2^o) > (\partial/\partial t)(\hat{x}_1^*/\hat{x}_2^*) > 0$. In particular, when both goods feature identical income elasticities, the generalized Alchian-Allen formula, given in Proposition 2, coincides with the substitution result of Borcherding and Silberberg (1978), stated in Proposition 1, irrespective of whether consumption falls short of or exceeds initial endowment. Similarly, if the sum of the household's excess supplies is sufficiently small, in the sense that "on average" the household consumes its initial endowment, both formulae also coincide — and the Alchian-Allen result continues to hold under the same qualifications given above (see our discussion of Proposition 1 on page 5). Consequently, we find that the Alchian-Allen result, that an increase of a per unit charge levied on two goods raises the relative demand for the more expensive good, continues to hold (under mild qualifications, though) if we acknowledge for income and endowment effects.

4. Application to the Labor-Leisure-Child-Care Choice

Consider a straightforward extension of the labor-leisure choice in standard form.[10] Let $\bar{T}$ be the disposal time of a mother with young children, which is assumed to be allocated to the following three categories of time: labor, leisure (time without children), and parental child care (time with children), the (generic) quantities of which we denote by h, l, and c, respectively. Suppose that she derives utility from leisure, child care, and the consumption level z of some composite good; and her preferences can be represented by the utility function $u : \mathbb{R}_+^3 \to \mathbb{R}$.

Let w be the wage rate and p_{ec} be the price of external child care per time unit. Her income is composed of the amount of time devoted to working, $h = \bar{T} - l - c$, multiplied by the net wage $w_n := w - p_{ec}$ and exogenous non-labor income m. (Observe that the child, by assumption, must be under supervision while the mother is either at work or consumes leisure time.) Denoting the price of the composite

[9] While, for reasons of later applications in Section 4, we refer here to the case where the sum is positive, we may also discuss the case where the sum is negative. However, since this discussion and its implications are obvious, they are omitted.

[10] For a comprehensive survey in this area, see, e. g., Blundell and MaCurdy (1999).

good by p_z, the budget constraint is written by $p_{ec}l + p_z z = (w - p_{ec})(\bar{T} - l - c) + m$. The mother's problem is then

$$\max_{l,c,z}\ u(l,c,z) \quad \text{s. t.} \quad (w_n + p_{ec})l + w_n c + p_z z = w_n \bar{T} + m. \tag{8}$$

Carefully observe that with suitable substitutions — set $x_1 = l$, $x_2 = c$, $x_3 = z$, $p_1 = p_{ec}$, $p_2 = 0$, $p_3 = p_z$ and $t = w_n$ —, the labor-leisure-child-care choice problem then has the form described in Section 2. Hence, assuming an interior solution, Propositions 1 and 2 apply.

It follows from Proposition 1 that if leisure (good 1) and maternal child care (good 2) are not perfect complements (i. e., $\varepsilon^*_{ll} < \varepsilon^*_{lc}$) and leisure is not a much stronger substitute for the composite good than is maternal child care (i. e., $\varepsilon^*_{cz} - \varepsilon^*_{lz} > -\alpha$ for some small positive α), then the Alchian-Allen result for this problem, $(\partial/\partial w_n)(\hat{l}^*/\hat{c}^*) > 0$, will hold. Moreover, the substitution effect (of the demand ratio) will decrease when the cost of (pure) leisure time, $q_1 = p_{ec} + w_n$, converges to the the opportunity cost of maternal child care, $q_2 = w_n$. This happens if either child care is available at virtually no cost, which, for example, happens if child care is provided by relatives or neighbors, or the wage rate substantially exceeds the cost of child care, which, for example, occurs for top earners.

Also, we conclude from Proposition 2 that, under the same qualifications as given in the course of discussion of Proposition 1 (see page 6), the Alchian-Allen result arguably holds for mothers who spend most of their disposal time on leisure and child care, for this implies that the sum of the excess supplies $\sum_{j=1}^{2}(\bar{x}_j - x_j) = T - l - c$ is close to zero.[11] Next, recall that empirical analyses have demonstrated that a rise in wages decreases leisure and increases child care, implying that $(\partial/\partial w_n)$ $(\hat{l}^o/\hat{c}^o) < 0$ (see Kimmel and Connelly, 2007, and also Guryan, Hurst, and Kearney, 2008). In view of Proposition 2, this finding requires that the income elasticity of child care must be substantially higher than that of leisure, $\varepsilon_{cI} \gg \varepsilon_{lI}$, provided that the Alchian-Allen result for compensated demand, $(\partial/\partial w_n)(\hat{l}^*/\hat{c}^*) > 0$, holds true. Consequently, Proposition 2 in combination with the empirical finding that a rise in wages decreases the demand for leisure relative to child care, provides evidence that income effects must be strongly asymmetric between different modes of time use. The presence of an asymmetry in income effects, however, indicates that quasilinear preferences (linear in the composite good) and homothetic preferences must be

[11]Note that in our time-allocation model the sum of the excess supplies is necessarily nonnegative, as $\sum_{j=1}^{2}(\bar{x}_j - x_j) = T - l - c \geqq 0$.

rejected as a proper specification of mothers' preferences, since the former imply no income effects in leisure and child care and the latter have unitary income elasticities for both goods. This observation imposes substantial restrictions on plausible specifications of young mothers' utility functions — and it seemed fundamental to us to point upon these consequences for conceivable preferences.

5. Concluding Remarks

In this paper we have contributed to the discussion and the advancements of the Alchian-Allen theorem (see Alchian and Allen, 1964). In particular, we accomplished to derive a version of this theorem which uses uncompensated rather than compensated demand, and which, in addition to usual income effects, also accounts for endowment effects. Our generalization of the Alchian-Allen theorem shows that (provided that consumption does not exceed initial endowment) the Alchian-Allen result for compensated demand continues to hold for uncompensated demand, unless the income elasticity of the lower priced good substantially exceeds the income elasticity of the higher priced good.

One might argue that two similar, substitutable goods should feature roughly identical income effects, and therefore little could be gained by taking into account income effects. While this line of reasoning seems to be plausible for consumption goods on which the expenditures represent only a small fraction of total income, (recall the traditional example of good and bad apples), it may well be deceptive when the expenditures or revenues are substantial in terms of total income or endowment, as in this case income and endowment effects (generically) do play a major role. This is shown in the second part of the paper where we apply our framework to a situation where income and endowment effects arguably are substantial: the time-allocation problem of a (single) parent. There we combine an application of the generalized Alchian-Allen theorem to the parental time allocation decision with results of the empirical literature. In particular, exploiting the well-established empirical fact that an increase in the wage rate reduces the demand for leisure relative to that of parental child care, our generalized Alchian Allen theorem shows that this empirical finding implies that the income effects of different modes of time use must be strongly asymmetric.[12] In particular, we infer that the income elasticity of

[12]Kimmel and Connelly (2007) and Guryan, Hurst, and Kearney (2008) suggest this asymmetry as a potential explanation of their findings mentioned in the Introduction of this paper.

parental child care must substantially exceed that of leisure so as to bring about the observed behavior.

The consequences of the asymmetry between the income elasticities of pure leisure time and parental child care are twofold. First, they constitute a warning against a rather routine application of utility functions which imply symmetric income elasticities, as do, for example, quasilinear and homothetic preferences. Also, asymmetric income elasticities are important for assessing policy projects. An income transfer to a (single) parent, or more specifically, a fixed child care subsidy, such as baby bonus or child benefits, increases the time the parent spends together with the child relative to pure leisure time. The parental time allocation, however, is of great importance, especially for the child and its development. Therefore the implications identified in this paper should be carefully acknowledged in both economic theory and public policies.

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