

Currency unions, fiscal policy, and reversibility risk

Alexander Kriwoluzky, Gernot J. Müller, and Martin Wolf*

November 2013

– Preliminary –

Abstract

The adoption of a common currency is not irreversible. In this paper, we develop a model of a small open economy which is initially part of a currency union. We show that, first, expectations of regime change arise necessarily in equilibrium, if fiscal policy fails to stabilize public debt. A regime change implies an alternative fiscal policy or, through exit from the union, monetary autonomy. Second, if monetary policy is expected to revalue debt after exit, yield spreads rise prior to exit, reflecting reversibility risk. We explore the macroeconomic implications of reversibility risk by calibrating the model to Greek data.

Keywords: Currency union, fiscal policy, regime change, exit, reversibility risk, euro, Greek crisis

JEL-Codes: F41, E62

*University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany. Email: a.kriwoluzky@uni-bonn.de, gernot.mueller@uni-bonn.de, martin.wolf@uni-bonn.de. Müller is also affiliated with the CEPR, London.

1 Introduction

Currency unions provide its inflation-prone member states with a nominal anchor (Alesina and Barro 2002). Delegating monetary policy to a hawkish central bank reduces inflation bias and thus differences in nominal interest rates across member states. The euro area is a case in point. Figure 1 displays monthly yield spreads on government bonds for Italy, Spain, Ireland, and Greece relative to Germany: they fell strongly in the run up to the creation of the euro in 1999 and stayed close to zero for almost a decade. Against this background, their significant rise after 2008 is often interpreted as a compensation for credit risk (e.g. Lane 2012). Yet, according to the European Central Bank, these yield spreads may also be driven by “unfounded fears of a reversibility of the euro” (ECB 2013).¹

Indeed, expectations of a member state’s exit from the union can give rise to yield spreads, or reversibility risk, provided its new currency is expected to depreciate. To see why, consider securities issued under the jurisdiction of the exiting member state, converted at par into new currency upon exit. In order to eliminate arbitrage possibilities, yields on these securities will need to rise prior to exit, if the new currency is expected to depreciate vis-à-vis the former common currency. In this paper, we ask how expectations of both, exit and depreciation, emerge within a member state of a currency union. We identify a common cause, namely a failure of fiscal policy to stabilize public debt at given prices.

Building on the New Keynesian small open economy framework developed by Galí and Monacelli (2005) and others, we develop a model which allows policy regimes to change over time and assume that agents are fully aware of this possibility. Policy regimes are captured by simple feedback rules for monetary and fiscal policy. Initially, there is no independent monetary policy, as the economy is assumed to be part of a currency union. At the same time, it lacks fiscal discipline. In the terminology of Leeper (1991), fiscal policy is “active” as it does not adjust (sufficiently) in a “passive” manner to stabilize debt. In principle, an active fiscal policy is sustainable as long as the price level is free to adjust in order to bring about a change in the value of government debt (Sims 1994, Woodford 1995, and Cochrane 2001). Yet, in a small open economy which is a member of a currency union, purchasing power parity ties down the domestic price level in the long-run.

Against this background and similar in spirit to Davig and Leeper (2007a), we establish our first result: under the fiscal rule in place, an equilibrium obtains only if market participants

¹In line with this view, Shambaugh (2012) presents evidence from the online betting market Intrade according to which prices in March 2012 were consistent with a 40 percent probability of an exit of at least one country within 2013. In February 2012 Buiter and Rahbari (2012) use the term “Grexit” and suggest a “likelihood of a Greek exit to 50% over the next 18 months”. In May 2012 the German Ifo-think tank published a report on “Greece’s exit from European Monetary Union: historical experience, macroeconomic implications and organisational implementation”, see Born et al. (2012).

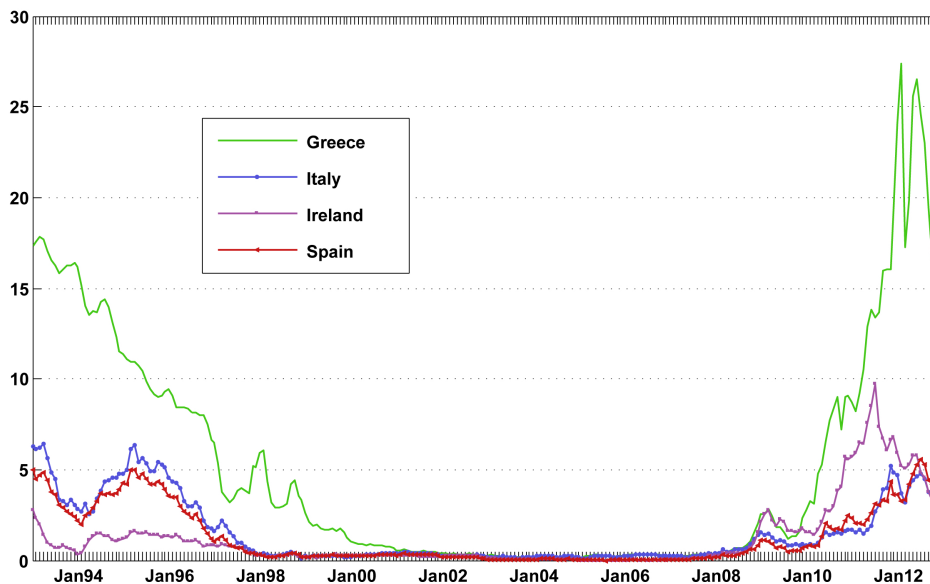


Figure 1: Interest rate spread vis-à-vis Germany. Notes: 10 year bond yields relative to German bond yields, monthly observations 1993–2012; source: ECB, long-term interest rate for convergence purposes.

expect a regime change to take place at some point.² Expectations about regime change arise necessarily in equilibrium, because active fiscal policy is inconsistent with permanent union membership. Regarding regime change, we consider two scenarios, allowing for expectations of either a change of the monetary or the fiscal regime.

Under the first scenario the country exits the currency union and starts operating an independent monetary policy which accommodates active fiscal policy. More precisely, it adjusts interest rates less than one-for-one to inflation thereby revaluing the debt stock which accumulated during union membership. This policy regime is inflationary, which we show, as a second result, to be necessary and sufficient for expectations of a depreciation to arise under the initial regime. Under the second scenario regarding regime change, the country remains part of the currency union, but alters its fiscal rule. The new fiscal rule is passive and ensures sufficient budget surpluses to service the outstanding debt. In addition, we assume that at the time of regime change there is a credit event, as a haircut is applied to the outstanding government debt. This gives rise to credit risk while the government accumulates debt under the initial regime, providing a natural benchmark against which we assess the implications of

²Formally, we allow for policy regimes to change within a Markov-switching linear rational expectations model. Davig and Leeper (2007a) use this framework to generalize the Taylor principle by showing that equilibrium determinacy obtains under a policy rule which would give rise to equilibrium indeterminacy in a fixed-regime model, provided there are expectations of a switch to a policy rule which is sufficiently aggressive towards inflation. In contrast, in our setup, the expected regime change ensures a mean square stable equilibrium as defined by Farmer et al. (2009) rather than determinacy.

reversibility risk.

We do so by considering the economy's adjustment under the initial regime to a deficit shock as a result of a lump-sum tax cut. In the presence of reversibility risk, the shock is recessionary. Intuitively, as domestic effective interest rates rise, private consumption and output fall. Furthermore, inflation takes off already before the actual exit takes place due to forward-looking price-setting decision. Higher prices, in turn, crowd out net exports, which leads to a further decline in domestic output. Hence, reversibility risk induces stagflationary effects of budget deficits. It is reminiscent of the classic inflation bias: a fundamental inconsistency in the policy framework induces a lack of credibility thereby worsening the trade-off faced by policy makers (Barro and Gordon 1983). In case there is only credit risk, instead, deficits have no allocative consequences. Ricardian equivalence obtains, because the accumulated debt stock is known to be serviced once the new fiscal regime is put in place. Up to first order, while the notional interest rate rises in line with credit risk, the effective interest rate remains unchanged. As a result, deficits are neutral for the allocation even under the initial policy regime.

We also interpret the European sovereign debt crisis through the lens of our model, notably the macroeconomic developments in Greece between late 2009 and early 2012. The upward revision of the fiscal deficit at the beginning of this period presumably supports the notion of an active fiscal policy. In due course, the macroeconomic outlook deteriorated further, fueling speculation of a Grexit. Eventually, debt was restructured in early 2012, as fiscal reform was supposedly under way. We calibrate the model to account for these developments, exposing it to the time series of actual primary deficits. In addition, we rely on time-series data for yield spreads and consumer prices to identify the beliefs of market participants regarding regime change. We find that reversibility risk explains half of the output decline and accounts for a quarter of the yield spread during the period under consideration.

Our analysis relates to earlier work on the stability of currency regimes. In fact, the notion that profligate fiscal policy fuels speculation regarding the duration of a fixed exchange rate regime dates back to at least Krugman (1979). In his seminal analysis, the depletion of foreign currency reserves precipitates the fall of an exchange rate peg, as otherwise arbitrage possibilities remain unexploited in equilibrium. In our analysis, instead, yields adjust to equalize expected returns on different interest-bearing securities—reflecting reversibility risk due to the steady accumulation of public debt. Whether the economy is part of a currency union or pegging its exchange permanently is immaterial for our results if capital is perfectly mobile. Absent this assumption, a central bank may be able to defend an exchange rate peg while only engineering a limited rise of domestic interest rates (Lahiri and Végh 2003).

A number of papers have analyzed the conduct of fiscal policy in currency unions from the perspective of the fiscal theory of the price level, which is also operative in parts of our analysis. The focus of these contributions are the implications of the fiscal rule in one or several large member countries for the entire union (Woodford 1996, Sims 1997, Bergin 2000). One noteworthy result is that pursuing an active fiscal policy may be in a member state's interest, as it allows to attain a permanent increase in wealth at the expense of the rest of the union. In contrast, we analyze the case of a small open economy and abstract from developments in the rest of the union. In this regard, we find that an active fiscal policy is a reason for reversibility risk to arise in equilibrium, inducing stagflationary effects to public debt and to deficit shocks. This being said, we acknowledge the possibility that other factors may drive speculation of exit and depreciation and, hence, reversibility risk premiums. The remainder of the paper is organized as follows. Section 2 presents the regime-switching model and characterizes the properties of a solution. Section 3 establishes our main results and illustrates the macroeconomic implications of reversibility risk. In Section 4, we apply the model to Greek data and decompose the Greek yield spread into credit and reversibility risk. Section 5 concludes.

2 The model

Our model builds on the New Keynesian small open economy framework (Galí and Monacelli 2005). We focus on a single country which is sufficiently small so as to have a negligible impact on the rest of the world. Within the country a representative household consumes, saves and works, while monopolistically competitive firms produce a variety of goods while being constrained in their pricing-decisions à la Calvo. The country relates to the rest of the world insofar as consumption is a composite of goods produced at home and abroad and firms export part of their production. Furthermore, saving takes place via a complete set of internationally traded state-contingent securities. The government issues one period debt in order to finance lump-sum transfers. Government debt is nominally riskless in the baseline version of the model, an assumption which we relax in our analysis below. We capture monetary and fiscal policy through simple feedback rules, distinguishing two possibilities in each case. Regarding monetary policy, the options are either to maintain a currency union with the rest of the world or to operate an independent monetary policy. The fiscal rule either stabilizes public debt at given prices or fails to do so.

The new feature of our analysis is that our model permits these policy rules to change as part of the equilibrium process, in a way consistent with agents' expectations.³ Indeed, as

³The framework underlying our model has been used extensively to contrast the properties of alternative

stressed by Davig and Leeper (2007a), once it is recognized that policy regimes may differ across time, it is desirable to endow agents in the model economy with this very insight. In order to keep the analysis tractable, we assume exogenously given probabilities of regime change within a Markov-Switching Linear Rational Expectations (MS-LRE) model. In what follows, we directly present the model in MS-LRE form and delegate its underlying non-linear framework to Appendix A.

2.1 The model

Our analysis is based on a first-order approximation to the optimality conditions of households and firms, the market clearing conditions as well as to the policy rules. The approximation is valid around a deterministic steady state, which is the same for every policy regime, with balanced trade, zero inflation and purchasing power parity. In what follows, small letters denote relative deviations from this steady state. Note also that we only consider shocks which arise in the domestic economy, leaving the rest of the world unaffected.

A first set of equilibrium conditions is *invariant across policy regimes*. The dynamic IS equation and the open-economy New Keynesian Phillips curve are, in turn, given by

$$y_t = E_t y_{t+1} - \frac{\varpi}{\gamma} (r_t - E_t \pi_{H,t+1}), \quad (2.1)$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa \left(\varphi + \frac{\gamma}{\varpi} \right) y_t. \quad (2.2)$$

Here $\pi_{H,t}$ denotes domestic (producer price) inflation ($\pi_{H,t} = p_{H,t} - p_{H,t-1}$), r_t is the nominal interest rate⁴ and y_t denotes output. As for deep parameters, the discount factor of the household is given by β , the coefficient of constant relative risk aversion by γ and the inverse of the Frisch elasticity by φ . We further define $\varpi := 1 + \omega(2 - \omega)(\sigma\gamma - 1)$ and $\kappa := (1 - \beta\xi)(1 - \xi)/\xi$, where σ denotes the trade price elasticity and ω the weight of imports in the production of final goods. $1 - \xi$ is the fraction of firms which are randomly selected to adjust prices within a given period.

Under complete international financial markets output is tied to the terms of trade s_t , the price of exports relative to imports,

$$y_t = -\frac{\varpi}{\gamma} s_t, \quad (2.3)$$

$$s_t = p_{H,t} + e_t, \quad (2.4)$$

policy rules within fixed-regime models (see, for instance, Galí and Monacelli 2005 and Corsetti et al. 2013b).

⁴Strictly speaking, within the currency union r_t is the nominal interest rate on securities which are issued under domestic jurisdiction. See the discussion further below.

where the second equation relates the terms of trade to domestic producer prices and the variable e_t . It represents the nominal exchange rate, measured as the price of domestic currency in terms of foreign currency in case of independent monetary policy. If, instead, the country is member of a currency union it has a natural interpretation as a shadow exchange rate: it corresponds to the exchange rate that would prevail were the country to exit the union (Flood and Garber 1984). The shadow exchange rate helps distinguish two types of securities, namely securities “issued under domestic jurisdiction”, which we assume will be converted at par into new currency upon exit and “foreign jurisdiction” securities, which we assume will not.⁵

As regards fiscal policy, we posit that the government levies lump sum taxes and issues one-period debt. Real public debt (\hat{d}_t) and tax receipts (\hat{t}_t^r) are both stated in terms of steady-state output, and their evolution is measured in percentage point deviation from steady state (indicated by a hat). ζ denotes the debt-to-GDP ratio in steady state. In turn, bond yields (i_t) are equal to the nominal interest rate as we assume the government issues debt in its own currency, or equivalently, under domestic jurisdiction

$$\beta \hat{d}_t^r = \hat{d}_{t-1}^r + \zeta(\beta i_t - \pi_{H,t}) - \hat{t}_t^r, \quad (2.5)$$

$$i_t = r_t. \quad (2.6)$$

This last assumption is important as it guarantees that, in principle, inflation is a mean to service the debt stock. We provide a more detailed discussion at the end of section 3.2.

A second set of equilibrium relationships *varies across policy regimes*. Specifically, regarding tax collections we posit the following fiscal rule:

$$\hat{t}_t^r = \psi_{\varsigma_t} \hat{d}_{t-1}^r - \varepsilon_t^d, \quad (2.7)$$

where the ς_t indicates that the parameter ψ (which measures the responsiveness of taxes to the level of debt) follows a discrete-time Markov chain $\{\varsigma_t\}$ which determines the evolution of policy regimes over time. Monetary policy also possibly differs across regimes. In case of membership in the currency union, we impose $e_t = 0$. Alternatively, if monetary policy is independent, we assume it to follow an interest rate feedback rule, while the exchange rate adjusts to clear the foreign exchange market.

⁵In fact, while intuitively appealing, this labeling is somewhat arbitrary as it is of no importance in itself precisely which type of security is converted into new currency. Yet, the discussion of a possible Grexit suggests that securities issued under Greek jurisdiction are indeed likely to be converted one-for-one into new currency upon exit (see, for example, Buiter and Rahbari 2012). Similarly, historical examples of “forcible conversions” of debt issued in foreign currency, but under home law, indicate a role for jurisdictions for the issue at hand (Reinhart and Rogoff 2011).

Altogether we consider three different regimes:

$$\text{Union AF:} \quad e_t = 0, \quad \psi < 1 - \beta \quad (2.8 - 1)$$

$$\text{Union PF:} \quad e_t = 0, \quad \psi > 1 - \beta \quad (2.8 - 2)$$

$$\text{Float AF:} \quad r_t = \phi_\pi \pi_{H,t}, \quad \psi < 1 - \beta, \quad \phi_\pi < 1 \quad (2.8 - 3)$$

In the first and third regime, ψ is small such that taxes adjust not sufficiently to stabilize outstanding debt, that is, fiscal policy is active (AF). Instead, given the specific assumptions on the Markov chain that we impose below, tax collections suffice to stabilize the level of outstanding debt at given prices in regime two, a situation of passive fiscal policy (PF). The “AF/PF” suffix thus characterizes the fiscal rule. Regimes one and two are associated with membership in a currency union. In regime three the country operates an independent, but passive monetary policy, accommodating active fiscal policy: it adjusts nominal interest rates less than one-for-one to inflation ($\phi_\pi < 1$).⁶

2.2 Equilibrium and stability

We are now in a position to define an *equilibrium*, following Farmer et al. (2011). First, we restate equations (2.1) - (2.8) more compactly:

$$\Gamma_{\varsigma_t} x_t = E_t x_{t+1} + \Psi_{\varsigma_t} \varepsilon_t^d, \quad \varsigma_t \in \{\text{Union AF, Union PF, Float AF}\}, \quad (2.9)$$

where $x_t = (y_t, r_t, i_t, \pi_{H,t}, p_{H,t}, e_t, s_t, \hat{t}_t^r, \hat{d}_t^r)'$. The matrices Γ_{ς_t} and Ψ_{ς_t} contain the model's deep parameters and ς_t indicates that they are regime dependent. Regime transitions are governed by a matrix $P = [p_{ij}] = [Prob(\varsigma_t = j; \varsigma_{t-1} = i)]$ specified below.

Definition 1. A rational expectations equilibrium is a mean square stable (MSS) stochastic process that, given the Markov chain $\{\varsigma_t\}$, satisfies equation (2.9).

Definition 2. An n -dimensional process $\{x_t\}$ is MSS if there exists an n -vector μ and an $n \times n$ matrix Σ such that

- $\lim_{n \rightarrow \infty} E_t[x_{t+n}] = \mu$
- $\lim_{n \rightarrow \infty} E_t[x_{t+n} x_{t+n}'] = \Sigma$.

Note that the concept of *stability* as defined above thus differs from stability as it is commonly applied in fixed-regime models. Intuitively, explosive trajectories in some regimes are not an

⁶We do not allow for “Float PF” as a possible fourth regime, as it would add no additional insights to our results. We provide intuition in section 3.1 below, where we analyze the consequences for reversibility risk if we effectively replaced Float AF by a regime Float PF.

issue, if the economy does not stay in these regimes for too long. What matters is that trajectories be not globally explosive, which is captured by MSS. The duration of a regime is thus key for stability. It is governed by the transition matrix on which we impose a specific structure, reflecting the particular interest of our analysis:

$$P = \begin{pmatrix} \mu & (1-\mu)\lambda & (1-\mu)(1-\lambda) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad 0 \leq \mu \leq 1, \quad 0 < \lambda < 1. \quad (2.10)$$

It implies that regime one is transitory (unless $\mu = 1$), while regimes two and three are absorbing states. λ determines the likelihood of being absorbed into regime two, and we restrict it to the open interval $(0, 1)$. Graphically, our Markov chain prescribes the following sequence of regime transitions:

$$\text{Union AF}_{\odot\mu} \quad \xrightarrow{1-\mu} \quad \begin{cases} \lambda & \text{Union PF}_{\odot 1} \\ 1-\lambda & \text{Float AF}_{\odot 1} \end{cases}$$

Initially, there is thus membership in a monetary union paired with an active fiscal policy. In any period, the economy stays in Union AF with probability μ , and leaves this regime with probability $1 - \mu$. λ , in turn, is the probability weight of a change in the fiscal rule. By contrast, a change in the conduct of monetary policy, that is, exit from the monetary union, is expected with a probability of $1 - \lambda$. In this case, the fiscal rule is assumed to remain unchanged, which leads to “default by inflation”, associated with a nominal depreciation. Importantly, both Union PF and Float AF are absorbing states, in the sense that the regimes will remain in place indefinitely.

Generally, the solution of MS-LRE models is obtained through specific algorithms (Farmer et al. 2011). Under our assumptions on the transitions probabilities, the problem simplifies considerably. Since the two target regimes are absorbing, we are able to solve the model backwards using the method of undetermined coefficients.⁷ This is particularly welcome, because we can thereby ensure the *uniqueness* of our solution, as the method of undetermined coefficients always delivers all candidate solutions. For the parameter specifications which we consider, we find that at most one of the candidate solutions satisfies MSS.⁸

Figure 2 illustrates that solutions to problem (2.9)-(2.10) do exist for given transition probabilities. It shows results based on a numerically evaluation, adjusting μ and λ such that the

⁷Appendix B solves the MS-LRE in its most general form, including default as introduced in section 3.2.

⁸Note that in general MS-LRE models may have multiple fundamental (‘non-sunspot’) equilibria, see Farmer et al. (2011) for an example. In our analysis, we consider mean square stable solutions of the form $x_t = F_{\zeta_t} x_{t-1} + G_{\zeta_t} \varepsilon_t \quad \forall \zeta_t$, see Appendix B.

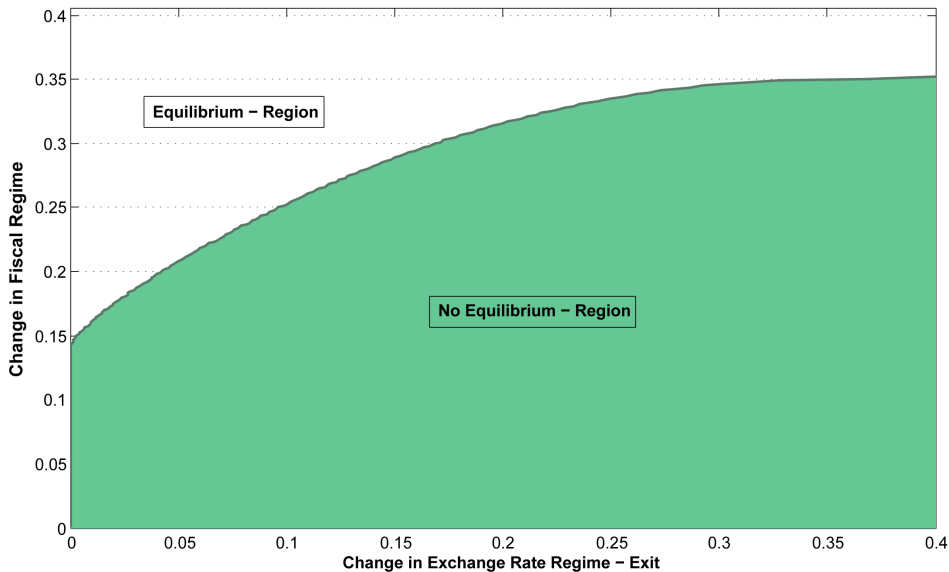


Figure 2: Stability and instability regions. Notes: The x-axis displays the transition-probability of exiting the union, $(1 - \mu)(1 - \lambda)$, whereas the y-axis displays the transition-probability of default and changing the fiscal regime, $(1 - \mu)\lambda$.

transition probabilities range between 0 and 0.4. The origin corresponds to the case where no regime change is expected to take place ($\mu = 1$). All other parameter values are set according to the values established in Section 4.2, where we calibrate the model to Greece (see Table 1 below). The area below the curve (including the curve itself) represents the combinations of transition-parameters for which no equilibrium exists. Figure 2 also shows that an increase in the probability of changing the fiscal regime stabilizes the economy, while an increase in the exit probability tends to destabilize the economy. In the following section we shed light on this observation.

3 Reversibility risk

We now investigate why reversibility risk may arise in a currency union and explore its consequences. In a nutshell, we show that reversibility risk reflects an inconsistency in the policy framework which is bound to be resolved at some point. In terms of consequences, reversibility risk turns out to have far reaching macroeconomic implications, as it induces budget deficits to be stagflationary.

3.1 Why reversibility risk arises

We start from the basic observation that interest rates reflect expectations of future policies via a version of the uncovered interest parity (UIP) condition. Combine equations (2.1),(2.3)

and (2.4) to obtain

$$r_t = -E_t(\Delta e_{t+1}). \quad (3.11)$$

This condition holds under all policy regimes, but the case of a currency union is of particular interest. In this case e_t represents the shadow exchange rate. Note, moreover, that r_t is the spread in the yield of securities issued under domestic jurisdiction relative to those issued under foreign jurisdiction, because variables are expressed in terms of deviation from steady state and we only consider shocks originating in the domestic economy.

Condition (3.11) holds in equilibrium and rules out arbitrage possibilities as market participants are able to trade both types of state-contingent securities. Intuitively, if exit from the currency union cannot be ruled out and if, upon exit, the newly created currency is expected to depreciate, the domestic-currency return on foreign-jurisdiction securities increases in those states of the world where depreciation takes place. Thus, in equilibrium, the return on securities issued under domestic jurisdiction (to be converted at par into the new currency upon exit) must also be higher on average, reflecting reversibility risk. In our model, high levels of outstanding debt give rise to reversibility risk in regime Union AF, as we establish in the following proposition.

Proposition 1. *Given the transition matrix (2.10), any rational expectations equilibrium satisfying the conditions summarized in system (2.9) features expectations of a policy regime change (that is, it requires $\mu < 1$). Moreover, expectations about currency depreciation upon exit increase in the level of outstanding public debt.*

Proof. We proof the first part by assuming to the contrary that there are no expectations of a regime change ($\mu = 1$). We show that in this case there is no rational expectations equilibrium, exploiting the fact that absent regime change the existence of a MSS process requires variables to be on non-explosive trajectories in each regime (Farmer et al. 2009). We proceed by showing that public debt is on an explosive trajectory in regime Union AF. First, absent expectations about regime change, $r_t = 0$ by (3.11). Second, combine (2.2),(2.3) and (2.4) to obtain

$$\beta E_t(p_{H,t+1}) = (1 + \beta + \frac{\kappa\varphi\varpi}{\gamma} + \kappa)p_{H,t} - p_{H,t-1}, \quad (3.12)$$

with a unique non-explosive solution given by $p_{H,t} = \phi p_{H,t-1}$, $\phi = \phi_{\text{aux}}/2\beta - \sqrt{\phi_{\text{aux}}^2/4\beta^2 - 1/\beta}$ with $\phi_{\text{aux}} = 1 + \beta + \kappa\varphi\varpi/\gamma + \kappa$. It is easily established that ϕ lies between zero and one. This expression illustrates that purchasing power parity pins down the domestic price level in the long run. Third, combine the equations for debt (2.5) and taxes (2.7) to obtain

$$\beta \hat{d}_t^r = (1 - \psi) \hat{d}_{t-1}^r + \zeta(\beta r_t - \pi_{H,t}) + \varepsilon_t^d, \quad (3.13)$$

where we use $i_t = r_t$. The last equation shows that debt is on an explosive trajectory, as $1 - \psi > \beta$ and both the evolution of r_t and $\pi_{H,t}$ are isolated from the level of debt and of deficit shocks under Union AF. Thus, there is no equilibrium for $\mu = 1$.

Now turn to the second part of the proposition. We focus on Float AF. As we establish in Appendix B, output and inflation evolve as

$$\begin{aligned}\pi_{H,t} &= \phi_{\pi,d}\hat{d}_{t-1}^r + \phi_{\pi,\varepsilon}\varepsilon_t^d, \\ y_t &= \phi_{y,d}\hat{d}_{t-1}^r + \phi_{y,\varepsilon}\varepsilon_t^d,\end{aligned}$$

where $\{\phi_{\pi,d}, \phi_{\pi,\varepsilon}, \phi_{y,d}, \phi_{y,\varepsilon}\}$ are strictly positive coefficients. Combining (2.3) and (2.4), we solve for the nominal exchange rate as a function of the endogenous state variables $p_{H,t-1}$ and \hat{d}_{t-1}^r and the shock ε_t^d :

$$\begin{aligned}e_t &= -\frac{\gamma}{\varpi}y_t - p_{H,t} \\ &= -p_{H,t-1} - \left(\frac{\gamma}{\varpi}\phi_{y,d} + \phi_{\pi,d}\right)\hat{d}_{t-1}^r - \left(\frac{\gamma}{\varpi}\phi_{y,\varepsilon} + \phi_{\pi,\varepsilon}\right)\varepsilon_t^d.\end{aligned}$$

Assuming that the economy operates under regime Union AF at time $t - 1$, the expected change in value of securities issued under domestic jurisdiction is given by

$$E_{t-1}(\Delta e_t) = -(1 - \mu)(1 - \lambda) \left(p_{H,t-1} + \left(\frac{\gamma}{\varpi}\phi_{y,d} + \phi_{\pi,d}\right)\hat{d}_{t-1}^r \right).$$

Here we use $e_{t-1} = 0$, $E_{t-1}(\varepsilon_t^d) = 0$ and the fact that the (shadow) exchange rate changes only in case of an exit from the currency union. Given that $\varpi > 0$ and $\varphi > 0$, expected depreciation increases in the level of outstanding debt. \square

The above result rests on the fact that public debt is on an explosive trajectory in case permanent union membership is coupled with active fiscal policy.⁹ Recalling the classic analysis of Leeper (1991), union membership for a small country thus appears as an instant of “active” monetary policy: it is not allowing the price level to adjust in order to stabilize public debt, because its conduct is decided at the union level and by assumption unresponsive to developments in a small member state. In this regard, Proposition 1 makes a positive statement: in case of union membership an active fiscal policy may still be consistent with an equilibrium, provided that market participants expect a regime change to take place at some point.¹⁰ Recall that we assume whenever expectations of regime change arise, expectations about an exit cannot be ruled out ($\lambda < 1$). Hence, it follows immediately from Proposition 1 that exit

⁹Note that while private sector optimality conditions do not constrain public debt to be on a non-explosive path in the present setup, it is unappealing to allow governments to run Ponzi-schemes (Sims 1997). In any case, we restrict our analysis to (mean square) to stable equilibria as defined above.

¹⁰Davig and Leeper (2011) also allow a policy regime which features active monetary and fiscal policy to be maintained for a limited period within a regime-switching model.

must be possible for an equilibrium to obtain. At the same time, there will be expectations of a depreciation upon exit, whenever monetary policy is expected to revalue the debt stock upon exit. Under Union AF, a build-up of public debt will therefore be accompanied by a rise in reversibility risk.

Our result hinges critically on the assumption that the domestic economy is small. Sims (1997, 1999) and Bergin (2000) analyze the implications of an active fiscal policy in large member states of a currency union. They are quite different. In fact, a large member state may sustain an active fiscal policy indefinitely, provided monetary policy is passive at the union level, thereby allowing the inflationary consequences of a member state's active fiscal policy to be felt across the entire union. The resulting incentive of a member state to pursue an active fiscal policy provides a rationale for constraining the conduct of fiscal policy within a currency union. Our analysis, instead, shows that pursuing an active fiscal policy is not necessarily in the interest of a small member state to the extent that it may fuel speculation of an exit from the union.¹¹

The above discussion suggests that it is active fiscal policy, in combination with a passive monetary stance upon exit, which drives our results. In the following we establish this formally. For this purpose, we consider an alternative scenario where fiscal policy is passive in all regimes. In this case, reversibility risk in the initial policy regime is absent in all equilibria. In addition, there is an equilibrium where exit is not expected.

Proposition 2. *Consider the equilibrium conditions summarized in system (2.9), but assume that $\psi > 1 - \beta$ in all regimes. Given the transition matrix (2.10), an equilibrium does not necessarily feature expectations of a regime change ($\mu = 1$). Moreover, there are no expectations of a depreciation upon exit in any equilibrium.*

Proof. We prove the first part of the proposition by recognizing that absent regime change ($\mu = 1$), the existence of an MSS process is equivalent to all variables being on non-explosive trajectories in all regimes in isolation. Start with union membership. Along the lines of the proof of Proposition 1, $r_t = 0$ by (3.11) and $p_{H,t} = \phi p_{H,t-1}$ with $\phi < 1$. Given $1 - \psi < \beta$, the autoregressive root in equation (3.13) is strictly smaller than one. Hence, public debt is on a non-explosive trajectory. Next, we establish non-explosiveness under the float. Combining

¹¹As a technical matter, the small open economy which we consider is of measure zero (Galí and Monacelli 2005) such that variables, even those on explosive trajectories, have no impact on the rest of the world. Nevertheless, in the present context, one may question the small-open-economy assumption on conceptual grounds. Still, if we were to relax the assumption, the results in Bergin (2000) suggest that Proposition 1 still holds provided that monetary policy is active at the union level and permanent transfers across member states are ruled out.

(2.1), (2.2) and the feedback rule for monetary policy implies

$$\begin{pmatrix} 1 & \frac{\varpi}{\gamma} \\ 0 & \beta \end{pmatrix} E_t \begin{pmatrix} y_{t+1} \\ \pi_{H,t+1} \end{pmatrix} = \begin{pmatrix} 1 & \frac{\varpi}{\gamma} \phi \pi \\ -\kappa(\varphi + \frac{\gamma}{\varpi}) & 1 \end{pmatrix} \begin{pmatrix} y_t \\ \pi_{H,t} \end{pmatrix}. \quad (3.14)$$

The minimum state variable solution to (3.14) is given by $y_t = 0$ and $\pi_{H,t} = 0$. As a consequence, debt evolves as follows: $\beta \hat{d}_t^r = (1 - \psi) \hat{d}_{t-1}^r + \varepsilon_t^d$. Again, it is non-explosive as fiscal policy is passive: $1 - \psi < \beta$.

Now turn to the second part of the proposition. Under the float we have $y_t = 0$ and $\pi_{H,t} = 0$ in all equilibria (that is: also for $\mu < 1$), and, by (2.3) and (2.4), $\Delta e_t = 0$. Hence, there is no expected depreciation prior to exit from the union. \square

Taken together Propositions 1 and 2 show to what extent an active fiscal policy causes expectations of an exit and depreciation upon exit to arise within a small member state of a currency union. Our argument hinges on the assumption that if a country exits the union for lack in fiscal discipline, it will likely accommodate active fiscal policy upon exit by means of its new monetary autonomy (passive monetary policy)—an assumption which strikes us plausible. That said, we stress that even though reversibility risk is fundamentally justified under Union AF, it also provokes a further deterioration of fundamentals through its impact on the government's financing cost (see Section 3.2). Thus, there is the possibility that an autonomous shift in expectations regarding regime change causes fiscal policy to become active, even if it is passive in the absence of such a shift. We do not analyze this possibility in the present paper.¹²

3.2 Reversibility risk matters

Throughout the paper, we focus on deficit shocks as an exogenous source triggering adjustment dynamics. We now provide details on the transmission mechanism while the economy operates under the Union AF regime. In order to highlight the role of reversibility risk in the transmission mechanism we contrast it to that of credit risk. For this purpose we modify the model such that credit risk arises, because of a non-zero probability that the government applies a haircut to its outstanding liabilities (see, e.g., Uribe 2006 or Bi 2012). As a practical matter, we assume this credit event to take place at the time of the switch to the new fiscal regime, thereby capturing a scenario of fiscal reform coupled with a one-time default.¹³

¹²Specifically, in case of a membership in the currency union, condition $\psi > 1 - \beta$ is generally not sufficient for debt to be non-explosive, if expectations of an exit and depreciation arise. Because of the resulting risk premiums, the initial regime may become unsustainable, confirming expectations of the exit—the classic scenario of a self-fulfilling currency crisis (see, e.g., Obstfeld 1996). Above, however, we assume that the initial regime is unsustainable independent of expectations regarding regime change.

¹³Technically, the scenario of a one-time debt default at the time of the switch to Union PF introduces a new regime, see the solution to the full model in Appendix B.

Specifically, in case of a credit event, the government repudiates the amount $\delta_t > 0$ of its debt obligations. It is assumed to be proportional to the outstanding debt in excess of the steady-state level:

$$\delta_t = \zeta^{-1} \delta \hat{d}_{t-1}^r, \quad (3.15)$$

where $\delta \in [0, 1]$ is the haircut applied to excess debt. Otherwise, we assume $\delta_t = 0$. As a result, the flow budget constraint (2.5) of the government and bond yields (2.6) are now given by

$$\beta \hat{d}_t^r = \hat{d}_{t-1}^r + \zeta(\beta i_t - \delta_t - \pi_{H,t}) - \hat{t}_t^r \quad (3.16)$$

$$i_t = r_t + E_t(\delta_{t+1}), \quad (3.17)$$

such that the government pays the nominal interest rate plus a premium which depends on the amount of debt which is repudiated. Nominal interest rates in turn are determined through the UIP condition, as established in the previous section. Insert (3.11) into (3.17) and apply the law of iterated expectations to obtain

$$i_t = -(1 - \mu)(1 - \lambda)E_t(e_{t+1}|\text{Float AF}) + (1 - \mu)\lambda\delta_{t+1}, \quad (3.18)$$

where we use that $e_t = 0$ today and that the exchange rate fluctuates only in case of an exit from the currency union. Note that in case market participants expect the newly created currency to depreciate in the event of exit, we have $e_{t+1} < 0$, such that today's bond yields rise through an increase in the nominal interest rate. The above expression thus decomposes sovereign yield spreads (l.h.s.) into reversibility risk (r.h.s.: first term) and credit risk (r.h.s.: second term).

Reversibility risk driving nominal interest rates reflects a no-arbitrage condition which is implicit in the UIP condition, as discussed in the previous section. It therefore also affects the economy's private sector through its impact on effective borrowing conditions. Credit risk, by contrast, does not change the economy-wide nominal interest rate r_t , but only raises the refinancing costs of the government. Investors hold government bonds only if they are compensated by a higher notional return. Up to first order, however, the *effective* interest rate remains unaffected by credit risk.¹⁴

These considerations are key to understand the macroeconomic effects of a one-time deficit shock while the economy operates in regime Union AF. Figure 3 displays impulse response functions computed for the calibration obtained in Section 4.2 below (except for the parameters μ, λ and δ). It contrasts the two polar cases: reversibility risk only versus credit risk

¹⁴Through a sovereign risk channel (Corsetti et al. 2013a) sovereign credit risk may affect the effective borrowing conditions in the private sector, too. We also note that in our complete-markets setup there are no distributional effects associated with government default.

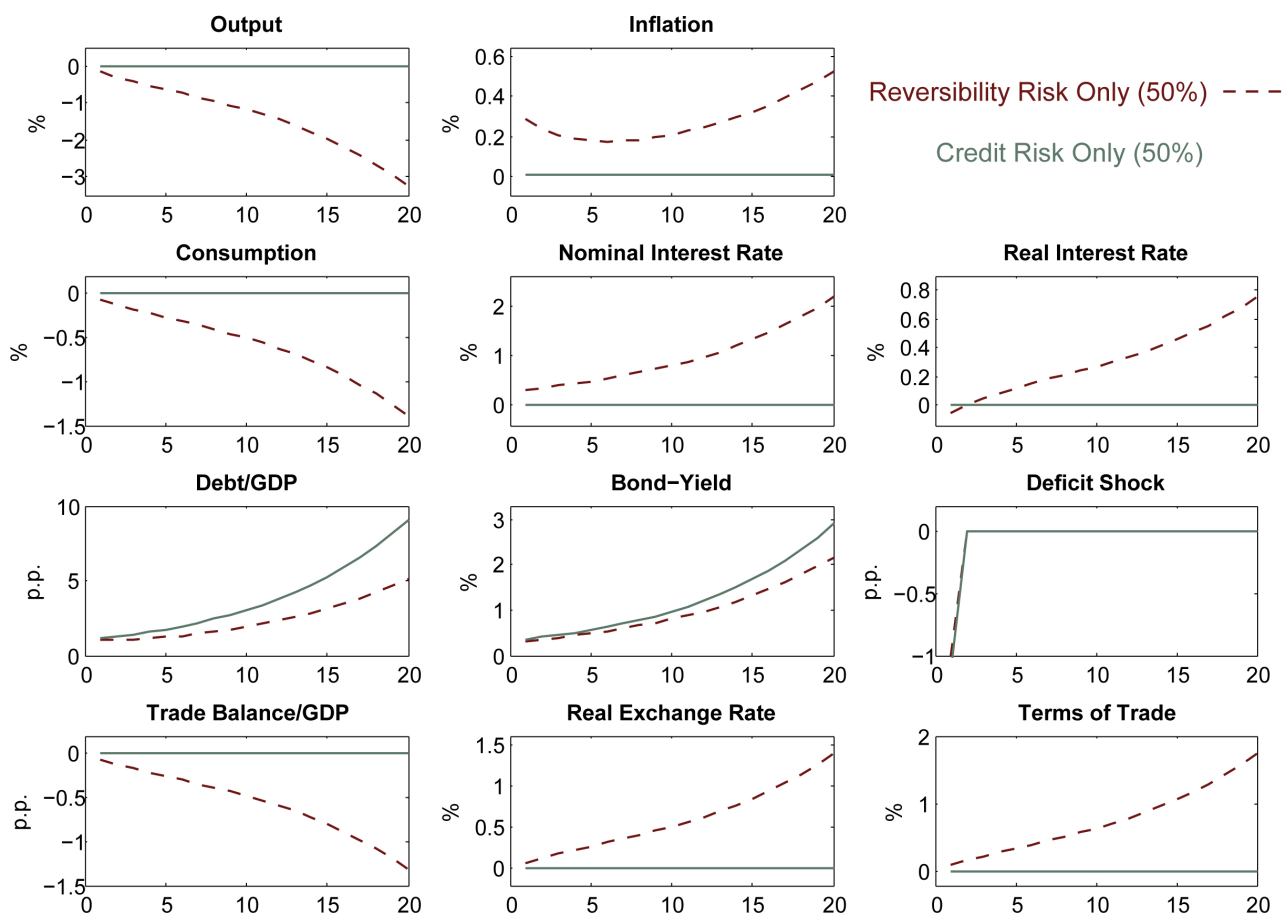


Figure 3: Impulse responses to deficit shock conditional on staying in regime Union AF. Notes: deficit shock equal to one percent of (annual) steady-state GDP. Horizontal axes measure quarters. Vertical axes measure deviations from steady state in percent, and percentage points in case of debt/GDP, trade balance/GDP and the deficit shock (annual steady-state GDP in all cases). Interest rates and inflation are annualized.

only. Solid lines represent the case where there is only credit risk ($\lambda \rightarrow 1$). Dashed lines, in turn, represent the case where there is only reversibility risk. In this case, we assume $\delta = 0$ at all times. In each instant, we fix the belief parameter μ at 0.8.¹⁵

For the default scenario (solid lines) we assume a haircut of $\delta = 0.5$. We find that in this case a deficit shock does not affect any variables, except for government debt, the deficit and government bond yields (shown in the third row of Figure 3). Importantly, as discussed above, private sector borrowing rates are unaffected by the deficit shock, as only the government's financing costs reflect credit risk. Furthermore, debt is known to be serviced eventually, once the switch to Union PF has taken place. Ricardian equivalence thus obtains even under the Union AF regime: deficits are neutral in the sense that they have no allocative consequences.¹⁶ Note also that bond yields and thus credit risk rise endogenously with the level of outstanding debt.

For the exit scenario we set $\lambda = 0.5$. Agents thus attach a probability of 50% to the event of an exit at some point in time, and of $(1 - 0.8)0.5 = 10\%$ within the next period.¹⁷ In this case, a deficit shock is non-neutral. In fact, we find that a one-time deficit shock induces long-lasting effects—the model generates substantial internal propagation. Moreover, the deficit shock is inflationary. The inflationary consequences of a deficit shock are due to forward-looking price-setting behavior. All else equal, firms tend to raise prices given that they expect inflation and depreciation upon exit which, in turn, will raise future marginal costs.¹⁸ It is reminiscent of the classic inflation bias: agents understand an inflationary shift in policy is not ruled out, which generates inflation already today (Barro and Gordon 1983). Finally, we find the deficit shock to be recessionary. To understand this result, note first that higher inflation induces a loss of competitiveness and thus a crowding out of net exports. Under our calibration this accounts for about half the output decline. Importantly, however, output also declines, because domestic consumption declines reflecting the rise in real interest rates due to reversibility risk.

Importantly, Figure 3 also illustrates that credit and reversibility risk are a function of the state of the economy. As debt levels increase and economic fundamentals deteriorate, yield spreads also increase. Intuitively, the longer the government procrastinates in terms of adjust-

¹⁵Note that although market participants attach some probability on leaving the regime Union AF in any period, in this experiment the regime is held fixed, which induces a series of forecast errors. A similar experiment is conducted in for example Davig and Leeper (2011).

¹⁶As the government's financing costs rise, this neutrality result would break down if taxes were distortionary (Bi 2012).

¹⁷The unconditional probability of exiting the union is given by $\text{Prob}(\text{Exit}) = \sum_{i=0}^{\infty} \mu^i (1 - \mu)(1 - \lambda) = 1 - \lambda$.

¹⁸A similar channel operates in a closed economy model: Davig and Leeper (2007b) find that deficit shocks are inflationary in a regime of passive fiscal policy, if agents anticipate a switch to a regime of active fiscal policy, where the latter regime is associated with high levels of inflation.

ing its policy regime, the more severe the consequences will be, as stressed by Uribe (2006). In the case of outright default, the amount of obligations it is expected to default on rises as time elapses. In the case of an exit, the level of inflation and depreciation which are necessary to stabilize debt upon exit increases over time.

The differential impact of credit and reversibility risk on the dynamics triggered by the deficit shock also sheds light on the results displayed in Figure 2 above. As reversibility risk affects the entire economy's response to the deficit shock, the possibility of unstable dynamics increases in λ , for given values of the structural model parameters. We also note that once both types of risk are present simultaneously, their effects will reinforce each other, as both impact public finances adversely. For instance, in the presence of reversibility risk, a rise in credit risk will, all else equal, deepen the recession which results from a deficit shock.

Finally, the above discussion illustrates the role of our assumption, in line with actual practice in much of the euro area, that the government issues debt under domestic jurisdiction.¹⁹ In fact, if the government were to issue debt under foreign jurisdiction, the debt stock would not be revalued as a result of inflation upon exit and, as a result, expectations of a depreciation upon exit would not arise.²⁰

4 The case of Greece 2009–2012

In this section we use the model to analyze key aspects of the macroeconomic developments in Greece during the period 2009–2012. The model, in particular, allows us to gauge to which extent bond yield spreads have been driven by expectations of default and to which extent by expectations of an exit. As discussed in the introduction, the Greek government faced spiralling financing costs starting in 2010, as did several other governments in the euro area (see Figure 1 above). Yet the experience of Greece is most dramatic in terms of spreads. Moreover, the scenario of an exit from the euro area was arguably most plausible in the case of Greece—reflected in widely used neologism “Grexit”.²¹ Finally, the size and persistence of fiscal deficits arguably support the notion of an active fiscal policy at the time, both prior to and during the crisis period under consideration. This makes the case of Greece

¹⁹In fact, most euro area governments issued debt under their own jurisdiction, including Greece up to the restructuring in 2012, see the discussion in Buiters and Rahbari (2012) and Buchheit et al. (2013).

²⁰More formally, equation (3.16) would feature an additional term containing $-\Delta e_t$, such that inflation associated with a depreciation of the currency would not affect the debt stock. However, we stress that there is no such assumption involved for the private sector. The UIP condition (3.11) ensures that agents are indifferent between foreign and home-jurisdiction securities. The composition of securities within the exiting country is thus of no relevance for our results; in particular, we do not require that there be only securities issued under domestic jurisdiction in the private sector of the exiting country.

²¹See footnote 1. Occasionally, commentators also contemplate a “Spexit”, although at the time of writing the term is certainly less present in the policy discourse.

particularly suitable to be studied through the lens of our model. In what follows we provide a short summary about the developments in Greece which are most relevant for our analysis, calibrate the model to capture key features of the data and perform a decomposition of yield spreads.

4.1 The Greek crisis

By late 2009, the Greek crisis escalated as the newly elected government of George Papandreou announced a substantial overshooting in the previous government's projection for Greece's 2009 deficit, from 6 to 12.7 percent of GDP (Gibson et al. 2012). Following the fast rise of yields and the debt-to-GDP ratio, rating agencies downgraded Greek debt obligations to junk-bond status in April 2010. At this point, Greece had lost access to international financial markets.

In May 2010, official lending by the EU and the IMF provided a substitute, as a support package amounting to 110 billion euros (or about 50 percent of Greek GDP) was agreed upon. At the same time, austerity measures and various structural reforms were initiated in order to stabilize fiscal imbalances. Yet the success of these measures has been limited—at least to the extent that sovereign yield spreads relative to Germany continued to widen over the course of 2011.

In July 2011 a second support package for Greece was discussed, and eventually ratified in the beginning of 2012. It involved a substantial restructuring of privately held debt. In March 2012, 200 billion euros, about 56% of the end-2011 debt total, were renegotiated, reaching an initial net decline of the debt-to-GDP ratio from 170% to about 120% (Zettelmeyer et al. 2012). However, in order to recapitalize Greek banks which had experienced large losses—no least because of the restructuring—new borrowing was required. As a result, the actual debt reduction was considerably lower. Indeed, IMF (2012) predicted end-2012 debt to exceed end-2011 debt.

These developments were also reflected in the yield spreads, which fell strongly in March 2012, but started to rise again to reach a new record high by June 2012. Instead, a longer lasting reduction of yield spreads ensued at about the same time as ECB president Mario Draghi's announcement to contain yield spreads through purchases of government bonds in July. In fact, this policy (“outright monetary transactions”) arguably meant to confront “unfounded fears of the reversibility of the euro”.

In what follows we focus on the period 2009Q4–2012Q1. The first quarter of this period coincides with the take-off of sovereign yield spreads following the correction of the 2009 budget deficit. We limit our analysis to the period up to 2012Q1, because we are interested

in studying the repercussions of an expected regime change, rather than the effect of the regime change itself, notably for the evolution of yield spreads. Moreover, in line with our modelling assumption, prior to the 2012 restructuring almost all ($> 90\%$) of Greek public debt (privately held) was issued under domestic jurisdiction, see Buiter and Rahbari (2012). We still include 2012Q1, as the restructuring of debt has taken place only at the end of that quarter.

4.2 Calibration

We use observations for the Greek economy, if available, to pin down the parameter values of the model. They are displayed in Table 1. A period in the model corresponds to one quarter. The discount factor β is set to 0.99. We assume that the coefficient of relative risk aversion, γ , takes a value of one, consistent with balanced growth. We set $\varphi = 3$, implying a Frisch elasticity of labor supply of $1/3$ in line with evidence provided by Domeij and Flodén (2006). The trade price elasticity σ is set to 1.5, in line with estimates for Greece by Bennett et al. (2008), and ω to 0.2, corresponding to the 2009 export-to-GDP ratio in Greece.

Price rigidities play an important role for our quantitative results, and we perform robustness checks at the end of the next section. We assume a fairly flat Phillips curve, by setting $\xi = 0.925$. Note that such a parametrization apparently conflicts with evidence from microeconomic studies such as Nakamura and Steinsson (2008). Nonetheless, the choice of a relatively high degree of price rigidities seems appropriate in the context of our framework, as we abstract from several model features that would imply a flatter Philips curve for any given value of ξ , e.g., non-constant returns to scale in the variable factor of production or non-constant elasticities of demand (see, e.g., Eichenbaum and Fisher (2007)). Recent evidence by IMF (2013) suggests that Phillips curves indeed have been flat in the time span under consideration.

We set $\epsilon = 11$, such that the steady-state markup is equal to 10 percent. Regarding the conduct of monetary policy in case of an exit, we assume $\phi_\pi = 0.9$ such that monetary policy is passive. At the same time, we assume $\psi = 0.009$ in case fiscal policy is active, whereas we assume $\psi = 0.02$ for the regimes where fiscal policy is passive. Below, we perform a sensitivity analysis with respect to these parameter values. **XXX Nur fuer zwei der drei XXX**

We pin down a last set of parameter values by calibrating the model to match key features of the Greek economy during the period 2009Q4–2012Q1. Specifically, as spreads have been close to zero prior to 2009Q4, we assume that the Greek economy has been in steady state in 2009Q3 and set $\zeta = 5.13$ in order to match the debt-to-GDP ratio of 128.3 percent at that

Table 1: Model calibration

Parameter	Parameter description	Value	Target / Source
β	Discount factor (steady state)	0.99	Annual interest rate 4.1%
γ	risk aversion	1	Balanced growth
φ	Inverse Frisch elasticity	3	Domeij and Flodén (2006)
σ	Trade-price elasticity	1.5	Bennett et al. (2008)
ω	Home Bias	0.2	Export-to-GDP ratio 2009
ξ	Fraction of unchanged prices	0.925	Flat Phillips curve
ϵ	Elasticity of substitution	11	Mark-up 10%
ϕ_π	Taylor-rule coefficient	0.9	PM
ψ	Tax-rule coefficient	0.009	AF
ζ	Steady-state debt-to-GDP ratio	5.13	128.3% Debt 2009Q3
δ	Haircut	0.519	51.9% Haircut 2012Q1
μ	Probability of staying in initial regime	0.78	Spread 2009Q4–2012Q1
λ	Default vs exit	0.885	CPI 2009Q4–2012Q1

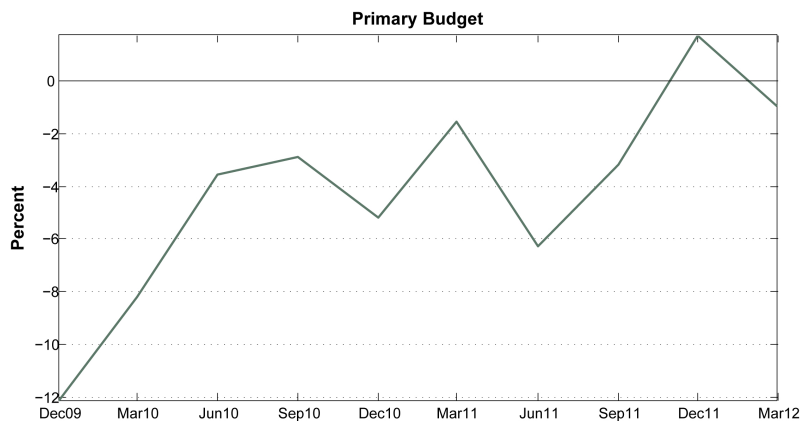


Figure 4: Primary budget balance in Greece 2009Q4–2012Q1. Source: Eurostat. Notes: quarterly observations in percent of GDP.

time.²² For each of the 10 quarters of the period under consideration, we specify a value for the deficit shock ε_t^d so as to generate a primary budget deficit in the model which is of the same size as the one observed for Greece. Figure 4 displays the actual time series. Deficits have been large and persistent throughout the period under consideration, except for the last quarter of 2011 for which a primary surplus has been recorded.

Given this sequence of shocks, the model predicts an increase in sovereign risk which, given the other parameter values, depends on δ , μ and λ . We set $\delta = 0.519$ implying an effective haircut of 51.9 percent, corresponding to the actual value in 2012Q1, according to calculations by

²²To be precise, Greek yield spreads in 2009Q3 have been about 1.3 percent, see Figure 1. This number is small relative to the levels observed shortly afterwards. In our model, spreads are zero in steady state only. Moreover the assumption that the economy is in steady state initially eliminates additional degrees of freedom.

Zettelmeyer et al. (2012).²³ We identify μ and λ by targeting the increase in the risk premium and the CPI over the period under consideration. Recall that deficit shocks raise the price level only in the presence of reversibility risk, so targeting the change in the CPI allows us to identify λ .²⁴

Our calibration yields values for $\mu = 0.78$ and $\lambda = 0.885$. These values imply a probability of exit of 2.53 percent from one quarter to the next, and of 19.47 percent of government default. The probability of leaving the euro area within one year implied by these values corresponds to 7.25 percent.²⁵

4.3 Accounting for credit and reversibility risk during the Greek crisis

We are now in the position to offer a structural account of the crisis dynamics which took off in Greece in late 2009. The panels of Figure 5 display the behavior of yield spreads, the CPI (detrended), output, and the debt-to-GDP ratio over the period 2009–2012, contrasting actual developments and those predicted by the calibrated model, driven only by deficit shocks.²⁶

The evolution of the yield spreads and the CPI are shown in the top panels of Figure 5. While the changes between 2009Q3 and 2012Q1 have been used as calibration targets, we note that the model prediction tracks the actual evolution of the spread rather closely. Admittedly, this is less so in case of the CPI, where the model fails to predict a first peak in late 2010.

The bottom panels show the evolution of output and the build-up of government debt. Again, we note that the model prediction for debt tracks actual developments rather closely.²⁷ Regarding output, however, we note that the model also accounts for half the output decline since 2009Q3. This is a noteworthy result and testifies to the importance of reversibility risk, because in the absence of exit expectations deficit shocks would have no bearing on real activity (see Figure 3 above). Nevertheless, credit risk, by raising financing costs and thus deficits, also contributes to a deterioration of public finances which, in presence of exit expectations, further amplifies reversibility risk and its effect on the economy at large.

Eventually, we aim to isolate the distinct contribution of credit and reversibility risk to the dynamics of sovereign yield spreads in Greece. Given our calibrated model this is straight-

²³In our model the haircut applies only to debt in excess of steady-state debt. Steady-state debt, instead, is riskless, as $\delta \leq 1$. **XXX Wieso ist wichtig wie gross δ ist? XXX**

²⁴The CPI increase is **XXX new data description XXX**, as there is zero trend inflation in our model.

²⁵This value appears small in light of the numbers discussed in the policy debate at the height of the crisis (see footnote 1). It is likely to be the result of our assumption that the exit probability is constant for the period under consideration. It may equally indicate that market beliefs about Grexit have actually been smaller than conveyed in popular debate. Yet, and despite the small transition probability, we find that reversibility did have a strong bearing on the Greek economy, see the discussion below.

²⁶Actual data are normalized in line with our assumption that the economy has been in steady state initially.

²⁷As discussed above, the restructuring of Greek debt has taken place at the end of 2012Q1, explaining the sudden decline in debt in Figure 5. By contrast, the average spread in 2012Q1 did not decline (spreads were 24.1 percent in January, 27.4 percent in February and 17.2 percent in March, see Figure 1).

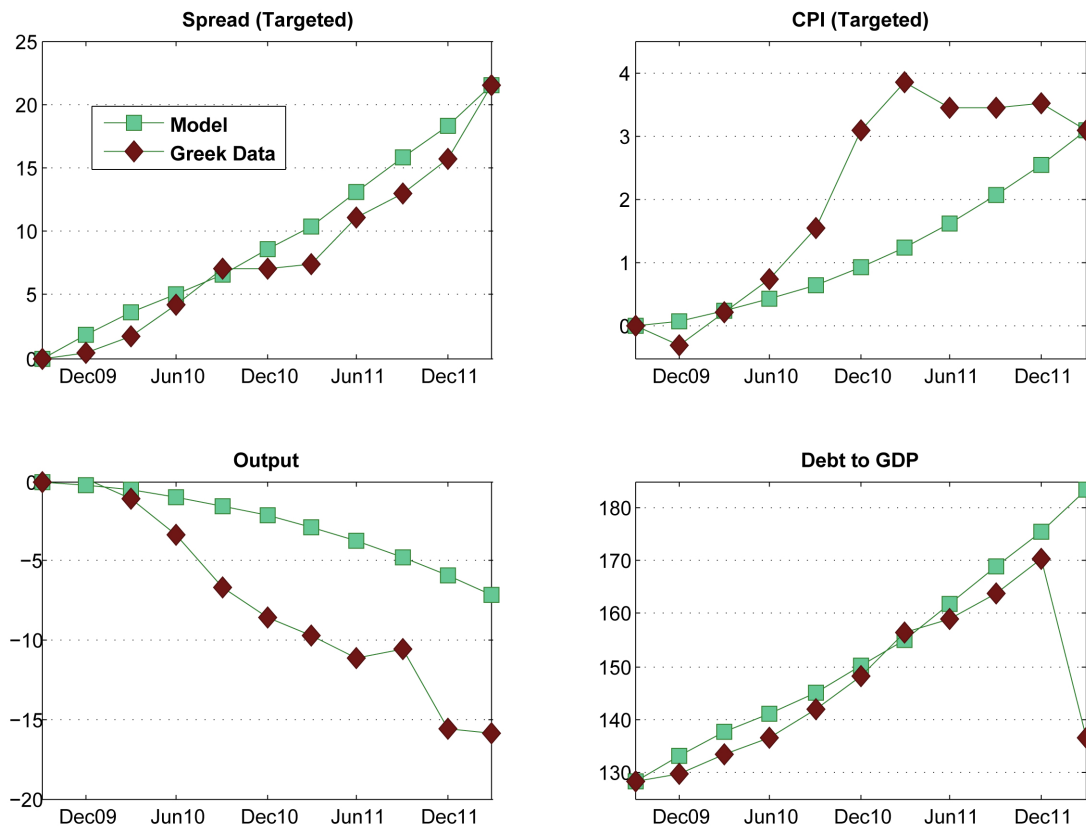


Figure 5: Evolution of key variables during crisis period. Notes: yield spread and the CPI in 2012Q1 (first row) serve as calibration target. Line with squares: model prediction; line with diamonds: data (normalized to zero in 2009Q3, except for debt); first observation: 2009Q4. Vertical axis measures percentage points for the spread, relative change in percent for the CPI and output, and levels for debt-to-GDP; data sources: Eurostat and IMF.

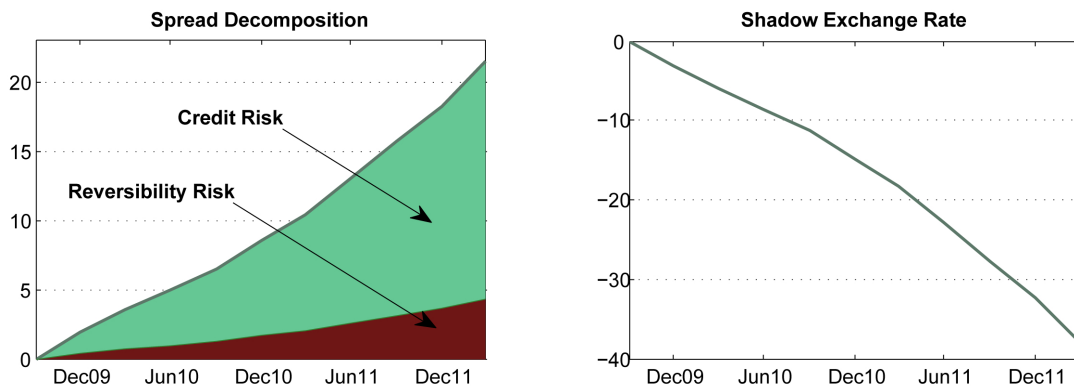


Figure 6: Decomposition of yield spread (left) and depreciation upon exit (right). Notes: reversibility risk measured by return on securities issued under domestic jurisdiction; credit risk: difference between yield spread and spread due to reversibility risk (remember equation (3.18); all measured in percentage points); depreciation upon exit (shadow exchange rate) measured in percent.

forward: while the government’s borrowing costs include both reversibility and credit risk, the private sector borrowing costs only include the former. The left panel of Figure 6 shows the result. We find that reversibility risk accounts for slightly less than a quarter of the total spread, with the rest made up by credit risk. The right panel of Figure 6 reports the shadow exchange rate, that is, the source of reversibility risk. At each point in time, it corresponds to the amount of depreciation were the economy to exit the currency union. It rises over time in close sync with the evolution of debt, as inflation upon exit will be higher, the higher the debt level—in line with the fundamental insight of the fiscal theory of the price level.

Our results so far suggest a limited role for reversibility risk in accounting for Greek yield spreads. We will now explore to what extent these results hold up if we alter parameter values that are hard to pin down empirically and thus may appear controversial, such as the high degree of price stickiness. The degree of price stickiness governs inflation dynamics and thus the amount of reversibility risk necessary to generate the increase in prices observed in the data.²⁸ We consider a value of $\xi = 0.85$ instead of $\xi = 0.925$, effectively reducing the average price duration from 13 to about 6 quarters.

Results are shown in the upper left panel of Figure 7. Even though the model now overshoots somewhat the rise in yield spreads, we note that the relative weights of reversibility and credit risk that make up the spread remain largely unchanged. A similar finding obtains once we lower the (counterfactual) Taylor-coefficient upon exit, ϕ_π , from 0.9 to 0.7, which implies a

²⁸In this and the following robustness checks, we perform the same targeting exercise as before, using the parameter values reported in Table 1 (except for the parameter under consideration). **XXX Aber hier treffen wir die targets nicht XXX**

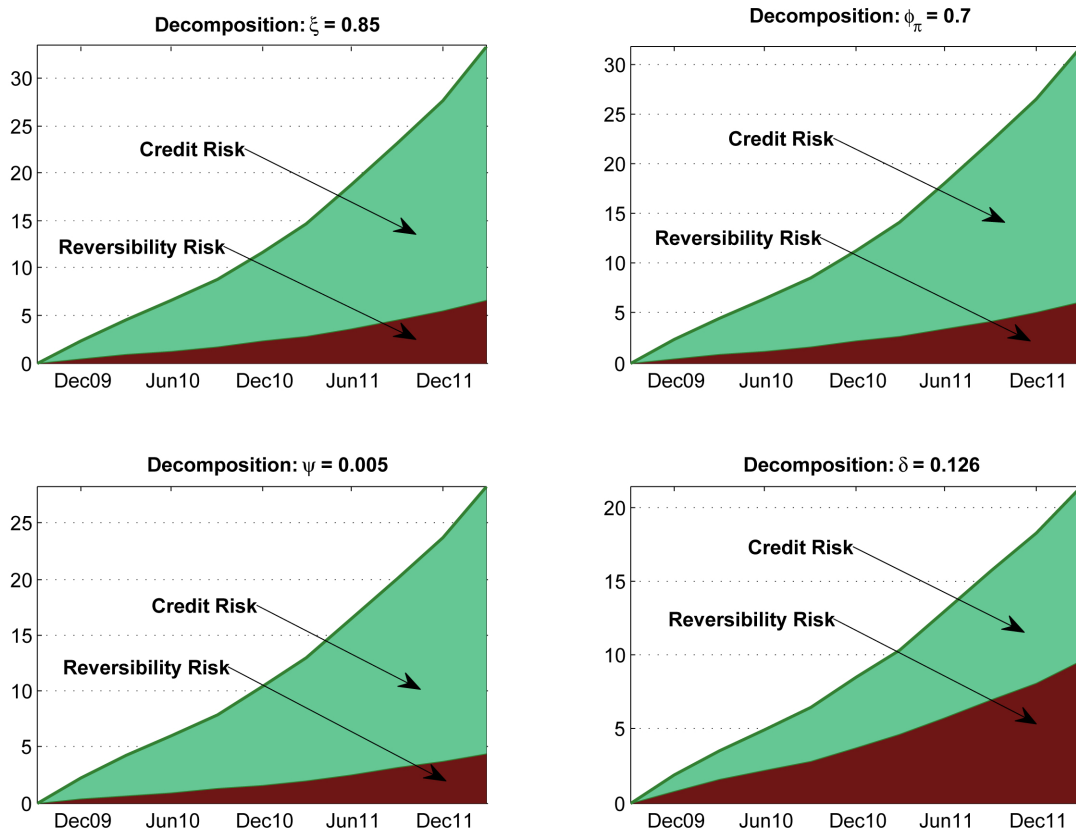


Figure 7: Robustness of the spread decomposition towards changes in $\xi, \phi_\pi, \psi, \delta$. Notes: Targets are the spread and CPI increase 2009–2012, see Figure 5.

more accommodative monetary policy (upper right panel). We further consider a decline in ψ , the tax collection parameter, from 0.009 to 0.005, so that fiscal policy becomes “more active”. Less tax collections imply that upon exit, higher levels of depreciation are necessary in order to restore equilibrium. Once again, we find that altering ψ does not significantly change the composition of credit versus reversibility risk in yield spreads (lower left panel). Finally, we illustrate that, in principle, the model is capable of producing high risk premiums driven by reversibility risk. In the lower right panel of Figure 7 we alter the expected fraction of default, δ , in such a way that only one quarter of the actual haircut in 2012Q1 was expected by investors. In this case, reversibility risk makes up about half of the rise in yields from 2009–2012.

5 Conclusion

In this paper, building on the standard New Keynesian small open economy framework, we have developed a Markov-Switching Linear Rational Expectations model of changing policy regimes. In particular, policy regimes differ in terms of government budget policies as well as in terms of the exchange rate regime. As a first result, we show that a budget policy which does not stabilize debt is not sustainable for a member of a currency union. However, such a policy regime may nevertheless be consistent with an equilibrium if market participants expect a regime change to take place at some point.

A lack in credibility in the fiscal rule in place gives rise to reversibility risk and credit risk. In our setup, credit risk emerges because of a possible haircut on outstanding debt. Reversibility risk, instead, emerges because of a large scale depreciation in case the country exits the currency union. We find that the macroeconomic implications of the two sources of risk differ fundamentally. If only credit risk is present, a deficit shock affects the borrowing conditions of the government, but has no further bearing on the equilibrium outcome. Instead, deficit shocks are stagflationary in the presence of reversibility risk.

We analyze key developments in Greece during the period 2009Q4–2012Q1 through the lens of the model. Specifically, we use the increase in yield spreads and the CPI during that period to pin down the beliefs of market participants of a credit event and an exit from the euro area. We find probabilities of 20 and 3 percent, respectively, for these events to take place from one quarter to the next. A decomposition of sovereign yield spreads suggests that about one quarter of the spread has been due to reversibility risk. Nevertheless, we stress that reversibility risk has a strong effect on the economy: it explains about half of the output decline during the period under consideration.

Still, our results suggest a limited role for reversibility risk in accounting for Greek yield

spreads. This result is particularly noteworthy in light of the rationale provided by the ECB for its promise of unlimited purchases in secondary sovereign bond market (“Outright monetary transactions” or OMT, for short), namely to restore the monetary transmission mechanism by confronting “unfounded fears of the reversibility of the euro”. That said, our findings are not inconsistent with the apparent success of the ECB’s OMT policy in reducing sovereign yield spreads as such. However, we leave a more detailed analysis of this policy, as well as of the developments in other European countries for future research.

References

- Alesina, A. and Barro, R. J. (2002). Currency unions. *Quarterly Journal of Economics*, 117(2):409–436.
- Barro, R. J. and Gordon, D. B. (1983). A positive theory of monetary policy in a natural rate model. *Journal of Political Economy*, 91(4):589–610.
- Bennett, H., Escolano, J., Fabrizio, S., Gutiérrez, E., Ivaschenko, I., Lissovlik, B., Moreno-Badia, M., Schule, W., Tokarick, S., Xiao, Y., and Zarnic, Z. (2008). Competitiveness in the southern euro area: France, greece, italy, portugal, and spain. IMF Working paper WP/08/112.
- Bergin, P. R. (2000). Fiscal solvency and price level determination in a monetary union. *Journal of Monetary Economics*, 45(1):37–53.
- Bi, H. (2012). Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy. *European Economic Review*, 56:389–410.
- Born, B., Buchen, T., Carstensen, K., Grimme, C., Kleemann, M., Wohlrabe, K., and Wollmershuser, T. (2012). Greece’s exit from european monetary union: historical experience, macroeconomic implications and organisational implementation. Technical report.
- Buchheit, L. C., Gulati, G. M., and Tirado, I. (2013). The Problem of Holdout Creditors in Eurozone Sovereign Debt Restructurings.
- Buiter, W. and Rahbari, E. (2012). Global Economics View, February 6.
- Cochrane, J. H. (2001). Long-term debt and optimal policy in the fiscal theory of the price level. *Econometrica*, 69(1):69–116.
- Corsetti, G., Kuester, K., Meier, A., and Müller, G. (2013a). Sovereign Risk, Fiscal Policy, and Macroeconomic Stability. *Economic Journal*, 123:F99–F132.

- Corsetti, G., Kuester, K., and Müller, G. J. (2013b). Floats, pegs and the transmission of fiscal policy. In Céspedes, L. F. and Galí, J., editors, *Fiscal Policy and Macroeconomic Performance*, volume 17 of *Central Banking, Analysis, and Economic Policies*, chapter 7, pages 235–281. Central Bank of Chile.
- Davig, T. and Leeper, E. (2007a). Generalizing the Taylor principle. *American Economic Review*, 97(3):607–635.
- Davig, T. and Leeper, E. (2007b). Fluctuating macro policies and the fiscal theory of the price level. *NBER Macroeconomics Annual 2006*, 21:247–298.
- Davig, T. and Leeper, E. M. (2011). Monetary-fiscal policy interactions and fiscal stimulus. *European Economic Review*, 55(2):211–227.
- Domeij, D. and Flodén, M. (2006). The labor-supply elasticity and borrowing constraints: Why estimates are biased. *Review of Economic Dynamics*, 9(2):242–262.
- ECB (2013). Annual report 2012.
- Eichenbaum, M. and Fisher, J. D. (2007). Estimating the frequency of price re-optimization in calvo-style models. *Journal of Monetary Economics*, 54(7):2032–2047.
- Farmer, R. E., Waggoner, D. F., and Zha, T. (2009). Understanding markov-switching rational expectations models. *Journal of Economic Theory*, 144(5):1849–1867.
- Farmer, R. E., Waggoner, D. F., and Zha, T. (2011). Minimal state variable solutions to markov-switching rational expectations models. *Journal of Economic Dynamics and Control*, 35(12):2150–2166.
- Flood, R. P. and Garber, P. M. (1984). Collapsing exchange-rate regimes : Some linear examples. *Journal of International Economics*, 17(1-2):1–13.
- Galí, J. and Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, 72:707–734.
- Gibson, H. D., Hall, S. G., and Tavlas, G. S. (2012). The greek financial crisis: Growing imbalances and sovereign spreads. *Journal of International Money and Finance*, 31:498–516.
- IMF (2012). World Economic Outlook Database, October 2012.
- IMF (2013). World Economic Outlook, April 2013.

- Krugman, P. (1979). A Model of Balance-of-Payments Crises. *Journal of Money, Credit and Banking*, 11(3):311–325.
- Lahiri, A. and Végh, C. A. (2003). Delaying the inevitable: Interest rate defense and balance of payments crises. *Journal of Political Economy*, 111(2):404–424.
- Lane, P. R. (2012). The European Sovereign Debt Crisis. *Journal of Economic Perspectives*, 26(3):49–68.
- Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*, 27(1):129–147.
- Nakamura, E. and Steinsson, J. (2008). Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics*, 123:1415–1464.
- Obstfeld, M. (1996). Models of currency crises with self-fulfilling features. *European Economic Review*, 40(3-5):1037–1047.
- Reinhart, C. M. and Rogoff, K. S. (2011). The forgotten history of domestic debt. *Economic Journal*, 121(552):319–350.
- Shambaugh, J. C. (2012). The euro's three crises. *Brookings Papers on Economic Activity*, (Spring):157 – 231.
- Sims, C. A. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic Theory*, 4(3):381–99.
- Sims, C. A. (1997). Fiscal foundations of price stability in open economies. mimeo, Yale University.
- Sims, C. A. (1999). The precarious fiscal foundations of emu. DNB Staff Reports (discontinued) 34, Netherlands Central Bank.
- Uribe, M. (2006). A fiscal theory of sovereign risk. *Journal of Monetary Economics*, (53):1857–1875.
- Woodford, M. (1995). Price-level determinacy without control of a monetary aggregate. *Carnegie-Rochester Conference Series on Public Policy*, 43(1):1–46.
- Woodford, M. (1996). Control of the public debt: A requirement for price stability? NBER Working Papers 5684, National Bureau of Economic Research, Inc.
- Zettelmeyer, J., Trebesch, C., and Gulati, M. (2012). The greek debt exchange: An autopsy. Technical report.

A Non-linear model

In what follows, we present the non-linear model, along with first order and market clearing conditions, as well as details on the log-linearization. Our exposition draws on Corsetti et al. (2013b), focusing on the domestic economy and its interaction with the rest of the world, ROW, for short.

A.1 Final Good Firms

The final consumption good, C_t , is a composite of intermediate goods produced by a continuum of monopolistically competitive firms both at home and abroad. We use $j \in [0, 1]$ to index intermediate good firms as well as their products and prices. Final good firms operate under perfect competition and purchase domestically produced intermediate goods, $Y_{H,t}(j)$, as well as imported intermediate goods, $Y_{F,t}(j)$. Final good firms minimize expenditures subject to the following aggregation technology

$$C_t = \left[(1 - \omega)^{\frac{1}{\sigma}} \left(\left[\int_0^1 Y_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \right)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} \left(\left[\int_0^1 Y_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (\text{A.1})$$

where σ measures the trade price elasticity. The parameter $\epsilon > 1$ measures the price elasticity across intermediate goods produced within the same country, while ω measures the weight of imports in the production of final consumption goods—a value lower than 1/2 corresponds to home bias in consumption.

Expenditure minimization implies the following price indices for domestically produced intermediate goods and imported intermediate goods, respectively,

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}, \quad P_{F,t} = \left(\int_0^1 P_{F,t}(j)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \quad (\text{A.2})$$

By the same token, the consumption price index is

$$P_t = \left((1 - \omega) P_{H,t}^{1-\sigma} + \omega P_{F,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (\text{A.3})$$

Regarding the ROW, we assume an isomorphic aggregation technology. Further, the law of one price is assumed to hold at the level of intermediate goods such that

$$P_{F,t} \mathcal{E}_t = P_t^*, \quad (\text{A.4})$$

where \mathcal{E}_t is the nominal exchange rate (the price of domestic currency in terms of foreign currency). P_t^* denotes the price index of imports measured in foreign currency. It corresponds

to the foreign price level, as imports account for a negligible fraction of ROW consumption. We also define the terms of trade and the real exchange rate as

$$S_t = \frac{P_{H,t}}{P_{F,t}}, \quad Q_t = \frac{P_t \mathcal{E}_t}{P_t^*} \quad (\text{A.5})$$

respectively. Note that while the law of one price holds throughout, deviations from purchasing power parity (PPP) are possible in the short run, due to home bias in consumption.

A.2 Intermediate Good Firms

Intermediate goods are produced on the basis of the following production function: $Y_t(j) = H_t(j)$, where $H_t(j)$ measures the amount of labor employed by firm j . Intermediate good firms operate under imperfect competition. We assume that price setting is constrained exogenously à la Calvo. Each firm has the opportunity to change its price with a given probability $1 - \xi$. Given this possibility, a generic firm j will set $P_{H,t}(j)$ in order to solve

$$\max E_t \sum_{k=0}^{\infty} \xi^k \rho_{t,t+k} [Y_{t,t+k}(j) P_{H,t}(j) - W_{t+k} H_{t+k}(j)], \quad (\text{A.6})$$

where $\rho_{t,t+k}$ denotes the stochastic discount factor and $Y_{t,t+k}(j)$ denotes demand in period $t+k$, given that prices have been set optimally in period t .

A.3 Households

The domestic economy is inhabited by a representative household that ranks sequences of consumption and labour effort, $H_t = \int_0^1 H_t(j) dj$, according to the following criterion

$$E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\gamma}}{1-\gamma} - \frac{H_{t+k}^{1+\varphi}}{1+\varphi} \right). \quad (\text{A.7})$$

The household trades a complete set of state-contingent securities with the rest of the world. Letting Ξ_{t+1} denote the payoff in units of domestic currency in period $t+1$ of the portfolio held at the end of period t , the budget constraint of the household is given by

$$W_t H_t + \Upsilon_t - T_t - P_t C_t = E_t \{ \rho_{t,t+1} \Xi_{t+1} \} - \Xi_t, \quad (\text{A.8})$$

where T_t and Υ_t denotes lump-sum taxes and profits of intermediate good firms, respectively.

A.4 Monetary and Fiscal Policy

In case the economy is not part of a currency union, domestic monetary policy is specified by an interest rate feedback rule. Defining the riskless one period interest rate as $R_t \equiv 1/E_t(\rho_{t,t+1})$, we assume

$$\log(R_t) = \log(R) + \phi_\pi (\Pi_{H,t} - \Pi_H), \quad (\text{A.9})$$

where $\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ measures domestic inflation and (here as well as in the following) variables without a time subscript refer to the steady-state value of a variable. Conversely, if the country is part of a currency union the exchange rate is exogenously fixed at unity, $\mathcal{E}_t = 1$.

As regards fiscal and budget policy, we posit that the government levies lump sum taxes, T_t , and issues one-period risky debt, D_t . Debt becomes risky as in any period, the government may default on a fraction $\delta_t \in [0, 1]$ of its outstanding liabilities. The period budget constraint of the government then reads as follows:

$$I_t^{-1} D_t = (1 - \delta_t) D_{t-1} - T_t, \quad (\text{A.10})$$

where I_t denotes the gross interest rate which the government pays on newly issued debt. The following no-arbitrage condition must hold in equilibrium:

$$I_t^{-1} = E_t(\rho_{t,t+1}(1 - \delta_{t+1})). \quad (\text{A.11})$$

It links the interest rate to the expected loss due to default. Next, defining $D_t^r := D_t/P_{H,t}$ and $T_t^r := T_t/P_{H,t}$ as a measure of real debt and tax revenues, we posit that

$$T_t^r - T^r = \psi(D_{t-1}^r - D^r) - \varepsilon_t^d. \quad (\text{A.12})$$

ε_t^d measures an exogenous iid shock to taxes, or, equivalently a “deficit shock”.

A.5 Market clearing

At the level of each intermediate good, supply equals demand stemming from final good firms and the ROW:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} \left((1 - \omega) \left(\frac{P_{H,t}}{P_t} \right)^{-\sigma} C_t + \omega \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \right), \quad (\text{A.13})$$

where $P_{H,t}^*$ and C_t^* denote the price index of domestic goods expressed in foreign currency and ROW consumption, respectively. It is convenient to define an index for aggregate domestic output: $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$. Substituting for $Y_t(j)$ using (A.13) gives the aggregate relationship

$$Y_t = (1 - \omega) \left(\frac{P_{H,t}}{P_t} \right)^{-\sigma} C_t + \omega \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\sigma} C_t^*. \quad (\text{A.14})$$

We also define the trade balance in terms of steady-state output as follows:

$$\frac{1}{Y} \left(Y_t - \frac{P_t}{P_{H,t}} C_t \right). \quad (\text{A.15})$$

A.6 Equilibrium Conditions and the Linearized Model

In the following, lower-case letters denote percentage deviation from steady-state values, ‘hats’ denote (percentage point) deviations from steady-state scaled by steady state output. Variables in the ROW are assumed constant, and we normalize $P_t^* = 1$.

Price indices The terms of trade, the law of one price, the CPI, CPI inflation and the real exchange rate can be written as

$$s_t = p_{H,t} - p_{F,t} \quad (\text{A.16})$$

$$p_{F,t} = -e_t \quad (\text{A.17})$$

$$p_t = (1 - \omega)p_{H,t} + \omega p_{F,t} = p_{H,t} - \omega s_t \quad (\text{A.18})$$

$$\pi_t = \pi_{H,t} - \omega \Delta s_t \quad (\text{A.19})$$

$$q_t = (1 - \omega)s_t \quad (\text{A.20})$$

Intermediate good firms The demand for a generic good (j) is given by

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t, \quad (\text{A.21})$$

so that

$$\int_0^1 Y_t(j) dj = \zeta_t Y_t, \quad (\text{A.22})$$

where $\zeta_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj$ measures price dispersion. Aggregation gives

$$\zeta_t Y_t = \int_0^1 H_t(j) dj = H_t. \quad (\text{A.23})$$

A first order approximation is given by $y_t = h_t$.

The first order condition to the price setting problem is given by

$$E_t \sum_{k=0}^{\infty} \xi^k \rho_{t,t+k} \left[Y_{t,t+k}(j) P_{H,t}(j) - \frac{\varepsilon}{\varepsilon - 1} W_{t+k} H_{t+k} \right] = 0. \quad (\text{A.24})$$

In the steady state, we have a symmetric equilibrium:

$$P_H = \frac{\varepsilon}{\varepsilon - 1} \frac{WH}{Y} = \frac{\varepsilon}{\varepsilon - 1} MC^n, \quad (\text{A.25})$$

where the second equation defines nominal marginal costs.

Linearizing (A.24) and using the definition of price indices, one obtains a variant of the New Keynesian Phillips curve (see, e.g., Galí and Monacelli, 2005):

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa m c_t^r, \quad (\text{A.26})$$

where $\kappa := (1 - \xi)(1 - \beta\xi)/\xi$ and marginal costs are defined in real terms, deflated with the domestic price index

$$m c_t^r = w_t - p_{H,t} = w_t^r - \omega s_t. \quad (\text{A.27})$$

Here $w_t^r = w_t - p_t$ is the real wage (deflated with the CPI).

Households The first order conditions in deviations from steady state are familiar

$$w_t^r = w_t - p_t = \gamma c_t + \varphi h_t, \quad (\text{A.28})$$

$$c_t = E_t c_{t+1} - \frac{1}{\gamma} (r_t - E_t \pi_{t+1}). \quad (\text{A.29})$$

Risk sharing implies that consumption is tightly linked to the real exchange rate (see, e.g., Galí and Monacelli, 2005)

$$\gamma c_t = -q_t. \quad (\text{A.30})$$

Government Rewriting the interest rate feedback rule in terms of percentage deviation from steady state gives immediately

$$r_t = \phi \pi_{H,t}, \quad (\text{A.31})$$

and similarly for the case of membership to a currency union, where $e_t = 0$.

Scale the tax rule (A.12) by steady state output and rewrite to obtain

$$\hat{t}_t^r = \psi \hat{d}_{t-1}^r - \varepsilon_t^d. \quad (\text{A.32})$$

Similarly, scale the flow budget constraint (A.10) by producer prices and steady state output, and linearize around zero default to obtain

$$\beta \hat{d}_t^r = \hat{d}_{t-1}^r + \zeta (\beta i_t - \delta_t - \pi_{H,t}) - \hat{t}_t^r, \quad (\text{A.33})$$

where $\zeta := \frac{D}{\bar{P}\bar{Y}}$ defines debt in steady state.

Next, using that $\rho_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}}$, linearize (A.11) to obtain

$$c_t = E_t c_{t+1} - \frac{1}{\gamma} (i_t - E_t \delta_{t+1} - E_t \pi_{t+1}), \quad (\text{A.34})$$

which (compare it to (A.29)) establishes that $i_t = r_t + E_t(\delta_{t+1})$.

Equilibrium Linearizing the good market clearing condition (A.14) yields

$$y_t = -(2 - \omega)\sigma\omega s_t + (1 - \omega)c_t. \quad (\text{A.35})$$

The trade balance becomes

$$\hat{t}b_t = y_t - c_t + \omega s_t. \quad (\text{A.36})$$

Some key equations We finally show how to obtain equations (2.1)-(2.3) from the main text (which are the dynamic IS curve, the New Keynesian Phillips curve and a risk sharing condition).

Combine good market clearing (A.35), risk sharing (A.30) and the definition of the real exchange rate (A.20) to obtain

$$y_t = -\frac{1}{\gamma} \underbrace{(1 + \omega(2 - \omega)(\sigma\gamma - 1))}_{:=\varpi} s_t. \quad (\text{A.37})$$

Rearrange to obtain

$$s_t = -\frac{\gamma}{\varpi} y_t, \quad (\text{A.38})$$

which is (2.3) in the main text.

Rewrite the Euler equation (A.29)

$$c_t = E_t c_{t+1} - \frac{1}{\gamma} (r_t - E_t(\pi_{H,t+1} - \omega\Delta s_{t+1})) \quad (\text{A.39})$$

$$= E_t c_{t+1} - \frac{1}{\gamma} (r_t - E_t \pi_{H,t+1} - \frac{\omega\gamma}{\varpi} E_t \Delta y_{t+1}), \quad (\text{A.40})$$

where we use $\pi_t = \pi_{H,t} - \omega\Delta s_t$ in the first line and (A.38) in the second.

Combine (A.38) with (A.30) and (A.20) to obtain

$$c_t = \frac{1 - \omega}{\varpi} y_t. \quad (\text{A.41})$$

Use this expression to substitute for consumption in (A.40)

$$y_t = E_t y_{t+1} - \frac{\varpi}{\gamma} (r_t - E_t \pi_{H,t+1}), \quad (\text{A.42})$$

which is (2.2) in the main text.

Finally, use (A.28), (A.38), (A.41) and production technology $y_t = h_t$ to rewrite marginal cost

$$mc_t^r = w_t^r - \omega s_t = \gamma c_t + \varphi h_t - \omega s_t = \left(\frac{\gamma}{\varpi} + \varphi\right) y_t. \quad (\text{A.43})$$

Insert into the Phillips curve (A.26) to obtain (2.2) in the main text.

B Model solution

In what follows, we present details regarding the model solution. Markov-Switching Linear Rational Expectations models (MS-LRE) in general are discussed in Farmer et al. (2009) and Farmer et al. (2011). We consider mean square stable solutions which we obtain by applying the method of undetermined coefficients.

An MS-LRE in general has the following structure:

$$\Gamma_{\varsigma_t} x_t = E_t x_{t+1} + \Psi_{\varsigma_t} \varepsilon_t \quad \forall \varsigma_t, \quad (\text{B.1})$$

with x_t being a vector of endogenous random variables, ε_t being a vector of white noise structural errors, and where Γ_{ς_t} and Ψ_{ς_t} are matrices containing the model's deep parameters. They evolve over time, following a discrete time Markov Chain $\{\varsigma_t\}$, with transition matrix $P = [p_{ij}] = [Prob(\varsigma_t = j; \varsigma_{t-1} = i)]$.

A candidate solution looks as follows:

$$x_t = F_{\varsigma_t} x_{t-1} + G_{\varsigma_t} \varepsilon_t \quad \forall \varsigma_t, \quad (\text{B.2})$$

and it is mean square stable (thus constitutes a rational expectations equilibrium to (B.1)) if and only if all eigenvalues of

$$(P' \otimes I_{n^2}) \text{diag}(F_{\varsigma_1} \otimes F_{\varsigma_1}, \dots, F_{\varsigma_h} \otimes F_{\varsigma_h}) \quad (\text{B.3})$$

lie within the unit circle. Here n is the number of variables considered, h denotes the number of regimes, \otimes is the Kronecker-product and ‘‘diag’’ stacks matrices in a bigger diagonal matrix.

Specifically, in the full model with default there are four distinct regimes, with transitions governed by

$$P = \begin{pmatrix} \mu & (1-\mu)\lambda & 0 & (1-\mu)(1-\lambda) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (\text{B.4})$$

Regime 2 from section 2 is split up into two regimes (call them 2' and 2), the former being purely transitory to be left for the latter immediately. Recall that the model features two endogenous state variables (\hat{d}_t^r and $p_{H,t}$) and one shock (ε_t^d). In what follows we outline the derivation of the solution (B.2) for the state variables only, so that $n = 2$.

We repeat the model equilibrium conditions for convenience:

$$y_t = E_t y_{t+1} - \frac{\varpi}{\gamma} (r_t - E_t \pi_{H,t+1}) \quad (\text{B.5})$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa \left(\varphi + \frac{\gamma}{\varpi} \right) y_t \quad (\text{B.6})$$

$$y_t = -\frac{\varpi}{\gamma} s_t \quad (\text{B.7})$$

$$s_t = p_{H,t} + e_t \quad (\text{B.8})$$

$$\beta \hat{d}_t^r = \hat{d}_{t-1}^r + \zeta (\beta i_t - \pi_{H,t} - \delta_t) - \hat{t}_t^r \quad (\text{B.9})$$

$$i_t = r_t + E_t (\delta_{t+1}) \quad (\text{B.10})$$

$$\hat{t}_t^r = \psi_{\varsigma_t} \hat{d}_{t-1}^r - \varepsilon_t^d \quad (\text{B.11})$$

$$\delta_t = \zeta^{-1} \delta_{\varsigma_t} \hat{d}_{t-1}^r \quad (\text{B.12})$$

$$r_t = \phi_\pi \pi_{H,t} \text{ or } e_t = 0, \quad (\text{B.13})$$

with inflation being defined by $\pi_{H,t} = p_{H,t} - p_{H,t-1}$.

Union PF–Regime 2

We start by obtaining F_{ς_2} and G_{ς_2} . Combine equations (B.5),(B.7),(B.8) to obtain the UIP-condition, combine equations (B.6),(B.7),(B.8) to obtain a second order difference equation in the producer price:

$$r_t = -E_t (\Delta e_{t+1}) \quad (\text{B.14})$$

$$\beta E_t (p_{H,t+1}) = \underbrace{\left(1 + \beta + \frac{\kappa \varphi \varpi}{\gamma} + \kappa \right)}_{\phi_{aux}} p_{H,t} - p_{H,t-1}. \quad (\text{B.15})$$

Union PF is absorbing, thus $E_t (\Delta e_{t+1}) = 0$ and so $r_t = 0$. Prices are solved by $p_{H,t} = \phi^{PF} p_{H,t-1}$, with $\phi^{PF} = \phi_{aux}/2\beta - \sqrt{\phi_{aux}^2/4\beta^2 - 1/\beta} \in (0, 1)$, where ϕ_{aux} is specified in (B.15). As there is no default in Union PF, $i_t = r_t = 0$ (B.10), and so

$$\beta \hat{d}_t^r = (1 - \psi) \hat{d}_{t-1}^r - \zeta \pi_{H,t} + \varepsilon_t^d,$$

where we suppress the regime-dependence of ψ for expositional clarity (thus $\psi_{\varsigma_2} = \psi$, and accordingly for the other regimes below). More compactly:

$$\underbrace{\begin{bmatrix} p_{H,t} \\ \hat{d}_t^r \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} \phi^{PF} & 0 \\ \frac{\zeta(1-\phi^{PF})}{\beta} & \frac{1-\psi}{\beta} \end{bmatrix}}_{F_{\varsigma_2}} \underbrace{\begin{bmatrix} p_{H,t-1} \\ \hat{d}_{t-1}^r \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{\beta} \end{bmatrix}}_{G_{\varsigma_2}} \varepsilon_t^d.$$

Default–Regime 2'

Given that regime Default is purely transitory, and hence $E_t(\delta_{t+1}) = 0$ also here (yielding again $i_t = 0$), regimes 2' and 2 differ only in how debt evolves:

$$\underbrace{\begin{bmatrix} p_{H,t} \\ \hat{d}_t^r \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} \phi^{PF} & 0 \\ \frac{\zeta(1-\phi^{PF})}{\beta} & \frac{1-\psi-\delta}{\beta} \end{bmatrix}}_{F_{\zeta_2'}} \underbrace{\begin{bmatrix} p_{H,t-1} \\ \hat{d}_{t-1}^r \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{\beta} \end{bmatrix}}_{G_{\zeta_2'}} \varepsilon_t^d.$$

Float AF–Regime 3

In regime Float AF, there is an independent central bank and no outright default ($i_t = r_t$). Insert the Taylor-rule into (B.5) and (B.9) to obtain a three-by-three system in $(y_t, \pi_{H,t}, \hat{d}_t^r)$:

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{\varpi}{\gamma} (\phi_\pi \pi_{H,t} - E_t \pi_{H,t+1}) \\ \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa \left(\varphi + \frac{\gamma}{\varpi} \right) y_t \\ \beta \hat{d}_t^r &= (1 - \psi) \hat{d}_{t-1}^r + \zeta (\beta \phi_\pi - 1) \pi_{H,t} + \varepsilon_t^d. \end{aligned}$$

Now guess that $\pi_{H,t} = \phi_{\pi,d}^{AF} \hat{d}_{t-1}^r + \phi_{\pi,\varepsilon}^{AF} \varepsilon_t^d$ and $y_t = \phi_{y,d}^{AF} \hat{d}_{t-1}^r + \phi_{y,\varepsilon}^{AF} \varepsilon_t^d$.

$$\begin{aligned} \pi_{H,t} &= \underbrace{\frac{\phi_{\pi,d}^{AF} (1 - \psi) + \phi_{y,d}^{AF} \kappa \left(\varphi + \frac{\gamma}{\varpi} \right)}{1 - \phi_{\pi,d}^{AF} \zeta (\beta \phi_\pi - 1)}}_{\phi_{\pi,d}^{AF}} \hat{d}_{t-1}^r + \underbrace{\frac{\phi_{\pi,d}^{AF} + \phi_{y,\varepsilon}^{AF} \kappa \left(\varphi + \frac{\gamma}{\varpi} \right)}{1 - \phi_{\pi,d}^{AF} \zeta (\beta \phi_\pi - 1)}}_{\phi_{\pi,\varepsilon}^{AF}} \varepsilon_t^d \\ y_t &= \underbrace{\frac{\phi_{y,d}^{AF} \left(\frac{1-\psi}{\beta} + \frac{\phi_{\pi,d}^{AF} \zeta}{\beta} (\beta \phi_\pi - 1) \right) - \frac{\phi_{\pi,d}^{AF} \varpi}{\gamma \beta} (\beta \phi_\pi - 1)}{1 + \frac{\varpi \kappa}{\gamma \beta} \left(\varphi + \frac{\gamma}{\varpi} \right)}}_{\phi_{y,d}^{AF}} \hat{d}_{t-1}^r \\ &\quad + \underbrace{\frac{\phi_{y,d}^{AF} \left(\frac{1}{\beta} + \frac{\phi_{\pi,\varepsilon}^{AF} \zeta}{\beta} (\beta \phi_\pi - 1) \right) - \frac{\phi_{\pi,\varepsilon}^{AF} \varpi}{\gamma \beta} (\beta \phi_\pi - 1)}{1 + \frac{\varpi \kappa}{\gamma \beta} \left(\varphi + \frac{\gamma}{\varpi} \right)}}_{\phi_{y,\varepsilon}^{AF}} \varepsilon_t^d \end{aligned}$$

Verify the guess first for $\phi_{\pi,d}^{AF}$ and $\phi_{y,d}^{AF}$ to obtain a quadratic equation in $\phi_{\pi,d}^{AF}$. The root which implies stable dynamics is given by $\phi_{\pi,d}^{AF} = -p/2 + \sqrt{p^2/4 - q}$, where

$$\begin{aligned} p &= - \left(\frac{1}{\beta} (\beta - 1 + 2\psi) + \frac{\varpi \kappa}{\gamma \beta} \left(\varphi + \frac{\gamma}{\varpi} \right) \right) / \frac{\zeta (\beta \phi_\pi - 1)}{\beta} \\ q &= \left(\frac{\psi}{\beta} (\beta - 1 + \psi) + \frac{\varpi \kappa}{\gamma \beta} \left(\varphi + \frac{\gamma}{\varpi} \right) (\beta \phi_\pi - 1 + \psi) \right) / \frac{\zeta^2 (\beta \phi_\pi - 1)^2}{\beta}. \end{aligned}$$

Proposition 3. *Under Float AF:* $(\phi_{\pi,d}^{AF}, \phi_{\pi,\varepsilon}^{AF}, \phi_{y,d}^{AF}, \phi_{y,\varepsilon}^{AF}) > 0$.

Proof. We now prove that $(\phi_{\pi,d}^{AF}, \phi_{\pi,\varepsilon}^{AF}, \phi_{y,d}^{AF}, \phi_{y,\varepsilon}^{AF}) > 0$, as we use this result in Proposition 1 in the main text. Remember that all deep parameters in the model are positive, and that $\beta < 1$. We will start with $\phi_{\pi,d}^{AF}$. As by assumption, $1 - \psi > \beta$ and $\phi_\pi < 1$, we see that $q < 0$. By the monotonicity of the square-root function it follows immediately that $\phi_{\pi,d}^{AF} = -p/2 + \sqrt{p^2/4 - q} > 0$.

Second, verify the guess for $\phi_{y,d}^{AF}$ to arrive at

$$\phi_{y,d}^{AF} = \frac{\phi_{\pi,d}^{AF} \varpi (1 - \beta \phi_\pi)}{\varpi \kappa (\varphi + \gamma / \varpi) + \gamma (\beta - 1 + \psi) + \phi_{\pi,d}^{AF} \zeta \gamma (1 - \beta \phi_\pi)}$$

The numerator is positive because $\phi_{\pi,d}^{AF} > 0$ as shown above (remember that $\phi_\pi < 1$). However, the denominator could possibly be negative because $1 - \psi > \beta$. We thus need to show that

$$\varpi \kappa (\varphi + \gamma / \varpi) + \gamma (\beta - 1 + \psi) + \phi_{\pi,d}^{AF} \zeta \gamma (1 - \beta \phi_\pi) > 0 \quad (\text{B.16})$$

We proceed by inserting directly $\phi_{\pi,d}^{AF}$ into (B.16). Cancel terms to obtain

$$\begin{aligned} &= \frac{\gamma}{2} \left\{ \tilde{\kappa} + (\beta - 1) + \sqrt{((\beta - 1 + 2\psi) + \tilde{\kappa})^2 - 4(\psi(\beta - 1 + \psi) + \tilde{\kappa}(\beta \phi_\pi - 1 + \psi))} \right\} \\ &= \frac{\gamma}{2} \left\{ \tilde{\kappa} + (\beta - 1) + \sqrt{(\beta - 1)^2 + \tilde{\kappa}^2 + \tilde{\kappa}(4(1 - \beta \phi_\pi) - 2(1 - \beta))} \right\} \\ &> 0, \end{aligned}$$

where we abbreviate $\tilde{\kappa} := \frac{\varpi \kappa}{\gamma} (\varphi + \frac{\gamma}{\varpi})$. $\phi_\pi < 1$ guarantees that $4(1 - \beta \phi_\pi) - 2(1 - \beta) > 0$, such that, again using the monotonicity of the square-root function, the last expression is strictly positive.

It remains to show that $(\phi_{\pi,\varepsilon}^{AF}, \phi_{y,\varepsilon}^{AF})$ are positive. It turns out that $(1 - \psi) \phi_{\pi,\varepsilon}^{AF} = \phi_{\pi,d}^{AF}$, and similarly, $(1 - \psi) \phi_{y,\varepsilon}^{AF} = \phi_{y,d}^{AF}$. To see this, replace both expressions in the guess from the previous page. Thus, $(\phi_{\pi,\varepsilon}^{AF}, \phi_{y,\varepsilon}^{AF}) > 0$ also here because by assumption, $1 - \psi > \beta > 0$. \square

For the evolution of state variables while in regime Union AF, we thus obtain:

$$\underbrace{\begin{bmatrix} p_{H,t} \\ \hat{d}_t^r \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} 1 & \phi_{\pi,d}^{AF} \\ 0 & \frac{1 - \psi + \zeta(\beta \phi_\pi - 1) \phi_{\pi,d}^{AF}}{\beta} \end{bmatrix}}_{F_{\varsigma_3}} \underbrace{\begin{bmatrix} p_{H,t-1} \\ \hat{d}_{t-1}^r \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} \phi_{\pi,\varepsilon}^{AF} \\ \frac{\zeta(\beta \phi_\pi - 1) \phi_{\pi,\varepsilon}^{AF} + 1}{\beta} \end{bmatrix}}_{G_{\varsigma_3}} \varepsilon_t^d.$$

Union AF–Regime 1

Given the closed-form expressions of the solutions for all target regimes, we now solve for regime Union AF. As in Union PF above, the equilibrium is characterised by the second order difference equation in prices (B.15). Split up $E_t(p_{H,t+1})$ into conditional expectations and evaluate each of them separately:

$$E_t(p_{H,t+1}|\text{Default}) = \phi^{PF} p_{H,t} \quad (\text{B.17})$$

$$E_t(p_{H,t+1}|\text{Float AF}) = p_{H,t} + \phi_{\pi,d}^{AF} \hat{d}_t^r \quad (\text{B.18})$$

$$E_t(p_{H,t+1}|\text{Union AF}) = ?. \quad (\text{B.19})$$

The third conditional expectation depends on the solution of regime Union AF which we have not yet worked out. To obtain an expression for bond yields, use the law of iterated expectations and combine (B.10) and (B.12):

$$i_t = -(1 - \mu)(1 - \lambda)E_t(e_{t+1}|\text{Float AF}) + (1 - \mu)\lambda\zeta^{-1}\delta\hat{d}_t^r. \quad (\text{B.20})$$

Replace $E_t(e_{t+1}|\text{Float AF})$ by combining (B.7) and (B.8):

$$i_t = (1 - \mu)(1 - \lambda) \left(E_t(p_{H,t+1}|\text{Float AF}) + \frac{\gamma}{\varpi} \phi_{y,d}^{AF} \hat{d}_t^r \right) + (1 - \mu)\lambda\zeta^{-1}\delta\hat{d}_t^r. \quad (\text{B.21})$$

Now insert (B.18) into (B.21) and set $\hat{d}_t^r = \beta^{-1} \left((1 - \psi)\hat{d}_{t-1}^r + \zeta(\beta i_t - (p_{H,t} - p_{H,t-1})) + \varepsilon_t^d \right)$ to obtain an expression for the yield i_t purely as a function of today's producer price and the relevant state variables $(p_{H,t-1}, \hat{d}_{t-1}^r, \varepsilon_t^d)$:

$$i_t = \vartheta_1 p_{H,t} + \vartheta_2 p_{H,t-1} + \vartheta_3 \hat{d}_{t-1}^r + \vartheta_4 \varepsilon_t^d, \quad (\text{B.22})$$

with $\vartheta_1, \dots, \vartheta_4$ being coefficient functions of the structural parameters. Plugging back (B.22) into (B.18) yields a similar expression for $E_t(p_{H,t+1}|\text{Float AF})$:

$$E_t(p_{H,t+1}|\text{Float AF}) = \eta_1 p_{H,t} + \eta_2 p_{H,t-1} + \eta_3 \hat{d}_{t-1}^r + \eta_4 \varepsilon_t^d, \quad (\text{B.23})$$

with, again, η_1, \dots, η_4 being coefficient functions of the structural parameters.

We are now in the position to apply the guess-and-verify method. Guess that, while in regime Union AF, producer prices evolve as $p_{H,t} = \phi_p^{UAF} p_{H,t-1} + \phi_d^{UAF} \hat{d}_{t-1}^r + \phi_\varepsilon^{UAF} \varepsilon_t^d$ and solve (B.19):

$$E_t(p_{H,t+1}|\text{Union AF}) = \phi_p^{UAF} p_{H,t} + \frac{\phi_d^{UAF}}{\beta} \left((1 - \psi)\hat{d}_{t-1}^r + \zeta(\beta i_t - (p_{H,t} - p_{H,t-1})) + \varepsilon_t^d \right),$$

the third conditional expectation needed to evaluate the full of $E_t(p_{H,t+1})$. Finally, replace i_t by (B.22) and rearrange (B.15) to verify the guess:

$$\begin{aligned}
p_{H,t} = & \frac{-(\mu\phi_d^{UAF}\zeta(\beta\vartheta_2 + 1) + (1 - \mu)(1 - \lambda)\beta\eta_2 + 1)}{\underbrace{\mu(\beta\phi_p^{UAF} + \phi_d^{UAF}\zeta(\beta\vartheta_1 - 1)) + (1 - \mu)(\beta\lambda\phi^{PF} + \beta(1 - \lambda)\eta_1) - \phi_{aux}}_{\phi_p^{UAF}}} p_{H,t-1} \\
& + \frac{-(\mu\phi_d^{UAF}(1 - \psi + \zeta\beta\vartheta_3) + \beta(1 - \mu)(1 - \lambda)\eta_3)}{\underbrace{\mu(\beta\phi_p^{UAF} + \phi_d^{UAF}\zeta(\beta\vartheta_1 - 1)) + (1 - \mu)(\beta\lambda\phi^{PF} + \beta(1 - \lambda)\eta_1) - \phi_{aux}}_{\phi_d^{UAF}}} \hat{d}_{t-1}^r \\
& + \frac{-(\mu\phi_d^{UAF}(\beta\zeta\vartheta_4 + 1) + (1 - \mu)(1 - \lambda)\beta\eta_4)}{\underbrace{\mu(\beta\phi_p^{UAF} + \phi_d^{UAF}\zeta(\beta\vartheta_1 - 1)) + (1 - \mu)(\beta\lambda\phi^{PF} + \beta(1 - \lambda)\eta_1) - \phi_{aux}}_{\phi_\varepsilon^{UAF}}} \varepsilon_t^d
\end{aligned}$$

Verify the guess first for ϕ_p^{UAF} and ϕ_d^{UAF} to obtain a cubic polynomial in ϕ_d^{UAF} . The polynomial has three real roots, all of which imply explosive paths for the state variables while in Union AF. However, for the calibrated model we verify that at most one of these solution candidates satisfies mean square stability (the root lying in $[0,0.5]$). The coefficients ϕ_p^{UAF} and ϕ_ε^{UAF} follow unambiguously from ϕ_d^{UAF} .

We thus obtain:

$$\begin{aligned}
\underbrace{\begin{bmatrix} p_{H,t} \\ \hat{d}_t^r \end{bmatrix}}_{x_t} = & \underbrace{\begin{bmatrix} \phi_p^{UAF} & \phi_d^{UAF} \\ \frac{\zeta(\beta(\vartheta_1\phi_p^{UAF} + \vartheta_2) - (\phi_p^{UAF} - 1))}{\beta} & \frac{1 - \psi + \zeta(\beta(\vartheta_1\phi_d^{UAF} + \vartheta_3) - \phi_d^{UAF})}{\beta} \end{bmatrix}}_{F_{\zeta_1}} \underbrace{\begin{bmatrix} p_{H,t-1} \\ \hat{d}_{t-1}^r \end{bmatrix}}_{x_{t-1}} \\
& + \underbrace{\begin{bmatrix} \phi_\varepsilon^{UAF} \\ \frac{\zeta(\beta(\vartheta_1\phi_\varepsilon^{UAF} + \vartheta_4) - \phi_\varepsilon^{UAF}) + 1}{\beta} \end{bmatrix}}_{G_{\zeta_1}} \varepsilon_t^d.
\end{aligned}$$