

Identification of SVAR Models by Combining Sign Restrictions With External Instruments^{*}

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Abstract

We identify structural vector autoregressive (SVAR) models by combining sign restrictions with information in proxy variables designed to instrument structural shocks. Our modeling framework allows to simultaneously identify different shocks using either sign restrictions or an external instrument approach, always ensuring that all shocks are orthogonal. We also discuss the combination of restrictions for identifying a single shock. In this scenario, exogeneity of the proxies is not required and the set of models consistent with the data is further reduced by discarding all those that imply structural shocks with no close relation to the external proxy variables. We discuss full Bayesian inference, which accounts for both, model and estimation uncertainty. We illustrate the usefulness of our method in SVARs analyzing the effects of oil market and monetary policy shocks. Our results suggest that combining sign restrictions with proxy variable information is a promising way to sharpen results from SVAR models.

Keywords: Structural vector autoregressive model, sign restrictions, external instruments

JEL classification: C32, C11, E32, E52

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1 Introduction

Ever since Sims (1980), structural vector autoregressive (SVAR) models have become a popular tool in applied macroeconomics.¹ Starting from a reduced form vector autoregressive (VAR) model, which summarizes the joint dynamics of a vector of time series variables, applied researchers impose various restrictions to identify structural shocks. Conditional on a particular identification scheme, the effects and importance of different structural shocks are summarized by impulse responses, forecast error variance decompositions or historical decompositions. Applications of this method include e.g. the analysis of monetary policy shocks, demand and supply shocks, fiscal shocks, oil price shocks, and news shocks.

In this paper, we suggest to identify structural shocks in SVAR models by combining both, sign restrictions and the information in narrative series constructed to instrument a certain shock in the model (proxy variables). As we argue below, combining both approaches is useful because it mitigates some drawbacks occurring when using either sign restrictions or external instruments only.

To achieve identification different types of restrictions have been suggested in the literature. Popular traditional restrictions include short- and long-run restrictions on the effects of structural shocks, which have the disadvantage that they are often difficult to justify and not testable if they are just-identifying. Therefore, alternative methods for identification have been developed including sign restrictions and the use of external instruments.²

Sign restrictions have been introduced into the literature by Faust (1998), Canova & De Nicoló (2002) and Uhlig (2005) as an alternative to existing methods involving short and long-run restrictions. The obvious advantage is that the researcher may directly restrict the signs of responses to the structural shock of interest. Sign restrictions may be imposed on the contemporaneous response or on the responses at later response horizons. In the context of monetary policy shocks, for instance, sign restrictions have been used to avoid the so-called ‘prize-puzzle’ by restricting the response of the price level to be non-positive for a certain period after a contractionary monetary policy shock (see e.g. Uhlig (2005) and Arias, Caldara & Rubio-Ramírez (2016)) but leaving the response of interest (e.g. the response of output) unrestricted. Employing sign restrictions does not yield a unique model but leads to set identification only. As pointed out by Fry & Pagan (2011), special care has to be taken when summarizing results across multiple models. Another practical problem of sign restrictions is that they are often rather weak, resulting in a wide range of admissible models with impulse responses that are not very informative. Consequently, additional restrictions are often needed to narrow down the set of models (see e.g. Kilian & Murphy (2012)).

Another method for identifying structural shocks without imposing short- or long-run restrictions is based on external instrument variables. While the underlying economic shock of interest is unobservable to the researcher, there may be related time series available that act as instrumental variables (IV) for the unobserved structural shock. To achieve identification the researcher needs to find an instrument variable that is highly correlated

¹For a general overview of different structural VAR models see e.g. Kilian & Lütkepohl (2017).

²Another strand of the literature uses statistical identification and identifies structural shocks by exploiting changes in the error term variance (see e.g. Lütkepohl & Netsunajev (2017) and Bertsche & Braun (2018) for recent contributions).

with the structural shock of interest and uncorrelated with all other structural shocks in the system. Studies that have used external instruments include e.g. Hamilton (2003), Romer & Romer (1989, 2004), Kilian (2008b, 2008a), Mertens & Ravn (2012, 2013, 2014), Stock & Watson (2012), and Gertler & Karadi (2015). While conceptually appealing, the external IV approach has also potential drawbacks. First, the exogeneity of instruments is questionable in some applications (see e.g. the discussion in Ramey (2016) on the narrative measures of monetary policy shocks). Furthermore, the external IV approach hinges critically on the validity of instruments and is problematic when the instruments are weak. In this case, inference is non-standard, complicated and still under development (see Montiel-Olea, Stock & Watson (2015, 2016)). Finally, if the IV approach is used to identify different economic shocks at a time as e.g. in Stock & Watson (2012), the different structural shocks are not necessarily orthogonal.

Consequently, using either sign restrictions or external instruments alone may not be optimal in practical work. In this paper, we therefore suggest ways to combine these two identification strategies in order to circumvent some of the mentioned problems. Our paper makes the following contributions. First, we suggest an econometric framework to combine identification from sign restrictions with identification from using external proxy variables. This is achieved by augmenting a standard SVAR system with equations for the proxy variables to relate them with the structural shocks. Second, we discuss estimation and inference in a full Bayesian setup, which accounts for both model and estimation uncertainty.

Our framework allows to combine identifying information from sign restrictions and external proxy series in two different ways. First, it can be used to identify some shocks by sign restrictions, while other shocks are identified via instrumental variables. One set of shocks is therefore set-identified and the others are point-identified. Our modeling framework nests the pure sign restriction and the pure IV approach as special cases. Compared to the existing methodology, our framework analyzes shocks identified from sign restrictions and external instruments jointly in a system approach and leads to structural shocks that are orthogonal by construction. In addition, our Bayesian framework can handle situations in which the external proxy variables are only weak instruments. As long as one uses a proper prior, the inference requires no adjustment to be valid. Second, we discuss how our framework can be exploited to identify a single shock by simultaneously using information from sign restrictions *and* external proxy series, however, without assuming exogeneity of the latter. This case is highly relevant in empirical work where applied researchers are often in the situation that pure sign restrictions are not enough to achieve informative results and at the same time, no credibly exogenous instruments are available. For this scenario, we suggest using the external variables to further tighten the set of models obtained by sign restrictions. This combination is simple and intuitive. Essentially, we narrow down the set of admissible models by only keeping those models that imply structural shocks that show a close relation to the external proxy variable. As explained in the paper, this relation is measured by either correlations or variance contributions. The resulting identification scheme is set-identifying, and thereby could be also thought of adding additional sign restrictions based on the information contained in the external proxies. As we will show in an empirical application, these additional restrictions can help to avoid the wide and uninformative confidence intervals

around impulse responses, a typical problem observed in applied sign restriction studies.

Methodologically, our first way to combine sign restrictions with external proxy variables is related to recent formulation of Bayesian Proxy SVARs (Drautzburg; 2016; Caldara & Herbst; forthcoming; Miranda-Agrippino & Rey; 2018). Among those, we are closest to Drautzburg (2016) in that we also use a system approach to conduct inference on the reduced form parameters before proceeding with the identification of the model. In comparison to these papers, however, our methodology is more general in the sense that we allow for additional shocks to be identified by pure sign restrictions. Our second way of combining sign restrictions with external proxy variables is related to studies that exploit non-model information for identification. For example, Kilian & Murphy (2012) use microeconomic evidence to construct elasticity bounds with the goal to further tighten the credible set obtained by sign restrictions. Furthermore, Antolín-Díaz & Rubio-Ramírez (forthcoming) suggest to narrow down the set of models by discarding all models that lead to structural shocks with signs and historical decompositions that are at odds with narrative evidence. Our approach is also close to Ludvigson, Ma & Ng (2017) who, similar to Antolín-Díaz & Rubio-Ramírez (forthcoming), suggest to include information on unusual historical events to restrict the sign and size of structural shocks. In addition, they discard models whose shocks are not strongly correlated with the ‘synthetic proxy’ variables and therefore, is similar in spirit to what we suggest.³ In comparison, our paper is more general with respect to important modeling aspects: Our setup does not require to choose a threshold for the correlation with the proxy and can also be used to identify some shocks by instrumental variables only. Moreover, we properly take into account all sources of model and estimation uncertainty within our full Bayesian framework. Therefore, inference is straightforward and error bands for impulse responses are readily available, which makes our setup particularly useful for applied work. Finally, since we use information of external variables without assuming strict exogeneity, our paper is also related to a branch of the microeconomic literature that discusses (set) identification using weakly endogenous instruments (Nevo & Rosen; 2012; Conley, Hansen & Rossi; 2012).

We illustrate the usefulness of our method in two empirical applications. We identify structural shocks on the oil market and monetary policy shocks. In both applications, we find that combining both, sign restrictions and external proxy series, leads to more informative and economically more sensible impulse response patterns.

The remainder of the paper is structured as follows. Section 2 introduces the econometric modeling framework and discusses identification and inference. Section 3 illustrates the suggested methods in applications to oil market shocks and to US monetary policy shocks. Section 4 summarizes and concludes.

³See also Uhrin & Herwartz (2016) for a similar idea to narrow down set identified SVAR models.

2 Combining Sign Restrictions with External Instruments

In the following, we develop a unified SVAR framework which enables us to combine information from sign restrictions and proxy variables to identify the effect of structural shocks. In Section 2.1 we describe a general econometric framework for this purpose, which is based on a SVAR model augmented by equations for proxy variables. Section 2.2 discusses identifying restrictions for the case where multiple shocks are identified by a combination of sign restrictions and instrumental variables, a scenario which nests both approaches. Section 2.3 discusses how information on sign restrictions and that of proxy variables can be exploited jointly to identify a single shock, making weaker assumptions about the relation between the SVAR shocks and the proxy variables. This idea is similar in spirit to Ludvigson et al. (2017) and Antolín-Díaz & Rubio-Ramírez (forthcoming). Finally, we discuss how to coherently conduct inference in SVAR models subject to these restrictions in Section 2.4. Note that, whenever we use the term ‘proxy variable’ we mean any external series designed to be similar to a structural shock, while we use the term ‘instrumental variable’ only if the proxy is used for identification in the sense of classical IV moment conditions.

2.1 Proxy Augmented SVAR Model

Let $y_t = (y_{1t}, \dots, y_{nt})'$ be a $n \times 1$ vector of endogenous time series generated by the SVAR model:

$$y_t = c + \sum_{i=1}^p A_i y_{t-i} + u_t, \quad u_t \sim (0, \Sigma_u), \quad (2.1)$$

$$u_t = B\varepsilon_t, \quad \varepsilon_t \sim (0, I_n), \quad (2.2)$$

where (2.1) corresponds to the reduced form VAR(p) model with c being an $n \times 1$ vector of intercepts and the $n \times n$ matrices A_i for $i = 1, \dots, p$ capturing the impact of lagged vectors of time series up to horizon p . Equation (2.2) contains the structural relations of the model, linking the reduced form errors u_t to structural shocks ε_t linearly by the $n \times n$ structural impact matrix B , which implies a reduced form error covariance matrix $\Sigma_u = BB'$. It is well known that the structural model (2.2) is not identified from the data alone.⁴ Therefore, a set of restrictions must be imposed on B in order to uniquely pin down the coefficients of the structural model. We propose a simple method to combine identification from sign restrictions and proxy variables. For this purpose, let $m_t = (m_{1t}, \dots, m_{kt})'$ be a $k \times 1$ vector of proxy series designed to provide identifying information about a subset of $k < n$ structural shocks of the SVAR model. Our approach is based on augmenting equation (2.2) by equations for the proxies m_t :

$$\underbrace{\begin{pmatrix} u_t \\ m_t \end{pmatrix}}_{\tilde{u}_t} = \underbrace{\begin{pmatrix} B & 0_{n \times k} \\ \Phi & \Sigma_\eta^{1/2} \end{pmatrix}}_{\tilde{B}} \underbrace{\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix}}_{\tilde{\varepsilon}_t}, \quad \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim (0, I_{n+k}). \quad (2.3)$$

⁴This can be seen that by simply multiplying matrix B by any $n \times n$ orthogonal matrix Q with property $QQ' = I_n$. We then end up with the same reduced form covariance matrix $\Sigma_u = (BQ)(BQ)' = BQQ'B' = BB'$.

The augmented model has a measurement error interpretation similar to Mertens & Ravn (2013). The k proxy variables m_t are modeled as a linear function of the structural errors ε_t with $k \times n$ regression coefficients Φ , plus a zero mean measurement error η_t , which is assumed to be orthogonal to the structural shocks ε_t , i.e. $\eta_t \perp \varepsilon_t$. A $n \times k$ block of zeros ensures that the measurement error η_t is also orthogonal to the reduced form errors u_t and avoids any impact on the dynamics of y_t . The augmented system has the reduced form covariance matrix

$$\Sigma = \text{Cov} \begin{pmatrix} u_t \\ m_t \end{pmatrix} = \begin{pmatrix} \Sigma_u & \Sigma_{mu} \\ \Sigma_{mu'} & \Sigma_m \end{pmatrix},$$

and through restrictions on Φ , identifying information can be imposed to pin down values of B in the spirit of an IV regression. In the following we discuss possible restrictions in detail and provide Bayesian algorithms for inference.

2.2 Identifying Multiple Shocks with a Combination of Sign Restrictions and Instrumental Variables

We first describe a scenario where either instrumental variables or sign restrictions are used to identify distinct structural shocks. Without loss of generality, assume that out of n structural shocks, the researcher wants to identify the first $k \leq n$ via an IV and the last $q \leq (n-k)$ shocks via sign restrictions. Given the following partition of the structural shocks $\varepsilon_t = [\varepsilon'_{1t} : \varepsilon'_{2t} : \varepsilon'_{3t}]'$ this corresponds to identifying ε_{1t} and ε_{3t} via the IV and sign restrictions, respectively.

To state the sign restrictions explicitly, we follow the notation of Arias, Rubio-Ramírez & Waggoner (2018). Assume that the researcher has prior information on the contemporaneous impact of the i th structural shock on the endogenous variables y_t and therefore on elements of the i th column of the impact matrix B , denoted as $B_{\bullet i}$ in the following.⁵ Let S_j be a $s_j \times n$ selection matrix with rank $\text{rk}(S_j) = s_j$ and let e_j be the j th column of the identity matrix I_n . Then, we gather all sign restrictions in the augmented SVAR system (2.3) as

$$S_j B e_j > 0, \quad j = n - q + 1, \dots, n.$$

With respect to the first k shocks (ε_{1t}), which are to be identified via instrumental variables, assume that a set of k instrumental variables are available for this purpose, implying the following restrictions:

$$\text{E}(m_{it}, \varepsilon_{i,t}) \neq 0, \quad i = 1, \dots, k, \tag{2.4}$$

$$\text{E}(m_{it}, \varepsilon_{j,t}) = 0, \quad i \neq j. \tag{2.5}$$

Equations (2.4) and (2.5) are known as relevance and exogeneity conditions, respectively. For local identification of the structural shock $\varepsilon_{i,t}$, it is thus required that the corresponding

⁵More generally, prior information can be available for any function $F(B, A_i)$ of the structural coefficients B and lag matrices $A_i, i = 1, \dots, p$, including for example restrictions on higher horizons of the impulse responses.

proxy variable m_{it} is correlated with this shock and uncorrelated with all other shocks of the SVAR system.

As discussed by Mertens & Ravn (2013) and Stock & Watson (2012), these conditions imply linear restrictions that identify the respective column of the structural impact matrix $B_{\bullet i}$ up to sign and scale. To be more specific about the exact form of the restriction, we follow Mertens & Ravn (2013) by partitioning $B = [\beta_1 : \beta_2 : \beta_3]$ as well as $\Phi = [\phi_1 : \phi_2 : \phi_3]$. Equations (2.4) and (2.5) imply that

$$\Sigma_{mu'} = E(m_t u_t') = E(m_t \varepsilon_t' B') = \phi_1 \beta_1'.$$

Further partitioning of $\Sigma_{mu'}$ and β_1 yields

$$[\Sigma_{mu'_1} : \Sigma_{mu'_2}] = \phi_1 [\beta'_{11} : \beta'_{12}].$$

This, in turn, translates into the following linear restrictions for the matrix β_1 :

$$\beta_{11} = (\Sigma_{mu'_2}^{-1} \Sigma_{mu'_1})' \beta_{12}. \quad (2.6)$$

For $k = 1$, equation (2.6) identifies β_1 up to sign and scale, while the additional restriction $\beta_1' \Sigma_u^{-1} \beta_1 = 1$ normalizes the shock to one standard deviation. For $k > 1$, additional restrictions must be specified to achieve identification, see e.g. Mertens & Ravn (2013) and Angelini & Fanelli (2018).

Note that an alternative representation of these restriction can be obtained by simply setting zero restrictions in the augmented impact matrix \tilde{B} as stated in equation (2.3). In particular, equations (2.4) and (2.5) imply that in addition to the zero block already mentioned in \tilde{B} , also $\phi_2 = 0$ and $\phi_3 = 0$. See also Angelini & Fanelli (2018) for a more extensive discussion. The described combination of sign restrictions with external instruments can therefore be also thought of combining zero and sign restrictions.

Example 1 Consider a simple three variable macro model as in Fry & Pagan (2011), involving the output gap z_t , prices π_t and an interest rate i_t . Assume that the system is driven by a monetary policy shock ε_t^{mp} identified via an external instrument m_t , as well as cost push (ε_t^c) and demand shocks (ε_t^d) identified with standard sign restrictions. Specifically, the cost push shock is assumed to decrease output and increase prices and interests on impact, while the demand shock is assumed to increase all variables. These identifying assumptions can be stated as follows:

$$\begin{pmatrix} u_t^z \\ u_t^\pi \\ u_t^i \end{pmatrix} = \begin{pmatrix} b_{11} & - & + \\ b_{21} & + & + \\ b_{31} & + & + \end{pmatrix} \begin{pmatrix} \varepsilon_t^{mp} \\ \varepsilon_t^c \\ \varepsilon_t^d \end{pmatrix}, \quad \begin{pmatrix} \varepsilon_t^{mp} \\ \varepsilon_t^c \\ \varepsilon_t^d \end{pmatrix} \sim (0, I_3),$$

$$b_{11} = \frac{Cov(u_{1t}, m_t)}{Cov(u_{3t}, m_t)} b_{31}, \quad b_{21} = \frac{Cov(u_{2t}, m_t)}{Cov(u_{3t}, m_t)} b_{31}, \quad 1 = \beta_1 \Sigma_u^{-1} \beta_1'.$$

2.3 Identifying a Single Shock Using Information from Sign Restrictions and an External Proxy

In our second scenario, we describe how sign restrictions and identifying information from external proxy variables are combined to trace down the effects of a single shock. This

idea is also proposed by Ludvigson et al. (2017) who start their SVAR analysis with a set identified model based on sign restrictions. To further narrow down the set of admissible models and thereby reduce the model uncertainty, they discard all those models where the corresponding structural shock exhibits a correlation with the external proxy less than a certain threshold value \bar{c} . Only relevance of the external proxy (equation (2.4)) is assumed, but not its exogeneity (equation (2.5)). The assumptions needed are therefore weaker than in a pure instrumental variables approach.

Without loss of generality, assume that the goal is to identify the first structural shock ε_{1t} by a combination of the two identifying sources, that is sign restrictions and information from a proxy variable. Let us first gather the inequality restrictions and assume that some information on the sign of elements in $B_{\bullet 1}$ is available. As in Section 2.2, we can make the restriction explicit with the help of the selection matrix S_1 defined previously:

$$S_1 B e_1 > 0. \quad (2.7)$$

Besides the sign restrictions of equation (2.7), assume that a set of k external proxy variables m_t is available containing information on the structural shock, that is:

$$E(m_{it}\varepsilon_{1t}) \neq 0, \quad i = 1, \dots, k.$$

We propose the following restrictions that exploit different degrees of identifying information from the external proxy variable without imposing its exogeneity:

1. The correlation between the i th proxy and the structural shock is positive:

$$\text{Corr}(m_{it}, \varepsilon_{1t}) = \frac{E(m_{it}\varepsilon_{1t})}{\sqrt{\text{Var}(m_{it})}} > 0.$$

From an economic point of view, this means that we are confident that the proxy variable is at least positively correlated with the structural shock it has been designed for. Note that this restriction does little harm if the proxy is only loosely associated with the structural shock and might be interesting for variables, which are assumed to be weak instruments.

2. The correlation between the i th proxy and the structural shock exceeds \bar{c}_i :

$$\text{Corr}(m_{it}, \varepsilon_{1t}) = \frac{E(m_{it}\varepsilon_{1t})}{\sqrt{\text{Var}(m_{it})}} > \bar{c}_i.$$

This restriction has been applied by Ludvigson et al. (2017). Of course, it is more restrictive than only a sign restriction on the correlation in that it rules out more models from the set of admissible SVARs. However, choosing \bar{c}_i is difficult and hard to justify in practice.

To circumvent the problem of choosing a threshold \bar{c}_i , we discuss two other possibilities:

3. First, among all structural shocks of the SVAR model ($\varepsilon_{jt}, j = 1, \dots, n$), the shock to be identified ε_{1t} has the largest correlation with the i th proxy variable.

$$\text{Corr}(m_{it}, \varepsilon_{1t}) > \text{Corr}(m_{it}, \varepsilon_{jt}), \quad j = 2, \dots, n. \quad (2.8)$$

This restriction imposes that among all shocks in the SVAR, the shock to be identified shows the highest correlation with the external variables.

4. Second, among all structural shocks of the SVAR model $(\varepsilon_{jt}, j = 1, \dots, n)$, the shock to be identified ε_{1t} explains most of the variation of the i th proxy variable m_{it} . To trace down the restriction, recall the regression equation for m_t :

$$m_t = \Phi \varepsilon_t + \Sigma_{\eta}^{\frac{1}{2}} \eta_t, \quad \eta_t \sim (0, I_k).$$

Since the regressors ε_t are orthogonal by assumption, the contribution of the j th structural shock to the variance of the i th proxy m_{it} is $\psi_{ij} = \phi_{ij}^2 / \text{Var}(m_{it})$. Therefore, the restriction is given as:

$$\psi_{i1} > \psi_{ij} \quad j = 2, \dots, n. \quad (2.9)$$

In words, imposing this restrictions rules out all SVAR models where other shocks explain more variation of the i th proxy m_{it} than the one to be identified (ε_{1t}).

Note that in order to implement the last two restriction, the other shocks $\varepsilon_{jt}, j = 2, \dots, n$ have to be identified. If they are not identified by economic reasoning, it is still possible to use a statistical identification, e.g. by orthogonal rotations of arbitrarily identified shocks. In that case, the prior implicitly used to statistically identify these shocks is of particular relevance must be chosen carefully. We further discuss this possibility in the empirical application.

Note that similar to the observation made in Section 2.2, the just described restrictions relating the external proxy and the structural model can be reinterpreted as imposing additional sign restrictions on the augmented SVAR model given in equation (2.3).

Example 2 *Reconsider Example 1 with the three variable macro model involving the output gap z_t , prices π_t and an interest rate i_t . Assume that we are particularly interested in identifying the monetary policy shock ε_t^{mp} but that no credible instrument is available in terms of exogeneity. However, information about the sign of the impact of the shock is readily available, as well as an imperfect external variable m_t designed to proxy the policy shock. To state specific restrictions, assume that a monetary policy shock has an immediate positive effect on the interest rate i_t and decreases prices π_t and output z_t on impact. Moreover, among all three structural shocks, ε_t^{mp} explains most of the variation of the proxy variable m_t . This can be stated as follows in the SVAR framework:*

$$\begin{pmatrix} u_t^z \\ u_t^\pi \\ u_t^i \end{pmatrix} = \begin{pmatrix} - & b_{11} & b_{12} \\ - & b_{12} & b_{22} \\ + & b_{13} & b_{23} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t}^{mp} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix}, \quad \begin{pmatrix} \varepsilon_{1t}^{mp} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \sim (0, I_3),$$

$$\frac{\phi_1^2}{\text{Var}(m_{it})} > \frac{\phi_j^2}{\text{Var}(m_{it})}, \quad j = 2, 3.$$

2.4 Bayesian Estimation and Inference

In the following we outline how to conduct Bayesian inference in our framework to coherently summarize both, modeling and sampling uncertainty. As in Drautzburg (2016), we specify priors on the reduced form proxy augmented VAR model which allows usage of an efficient

textbook style Gibbs sampler on these parameters. In a second step, we identify the model by generating orthogonal matrices that relate the reduced form parameters to the structural representation of the model and satisfy the parameter restrictions discussed previously.

Given our Bayesian setup, we assume a standard Gaussian likelihood function for the proxy augmented SVAR:

$$y_t = Ax_t + u_t, \quad \begin{pmatrix} u_t \\ m_t \end{pmatrix} \sim \mathcal{N} \left(0, \Sigma = \begin{pmatrix} \Sigma_u & \Sigma_{mu} \\ \Sigma_{mu'} & \Sigma_m \end{pmatrix} \right),$$

where $A = [c, A_1, \dots, A_p]$ and $x_t = [1, y'_{t-1}, \dots, y'_{t-p}]'$. We link the reduced form to the structural form through the contemporaneous impact matrix:

$$B = PQ, \tag{2.10}$$

where P is the Cholesky decomposition of the VAR block reduced form error variance such that $PP' = \Sigma_u$. The matrix Q is orthogonal, that is $Q'Q = I_n$, and links the reduced form model to the structural representation as in equation (2.10). We assume conditionally conjugate prior distributions for the reduced form coefficients Σ^{-1} and A , which take the form of Normal and Wishart distributions respectively:

$$\Sigma^{-1} \sim \mathcal{W}(d, \Psi), \tag{2.11}$$

$$\text{vec}(A) \sim \mathcal{N}(\alpha_0, V_0). \tag{2.12}$$

Before discussing the prior distribution for the orthogonal matrix Q , we first characterize the posterior of the reduced form model. Unfortunately, this density is of no known form. However, random draws of the posterior can be efficiently generated by a standard Gibbs sampler iterating between the conditional posteriors $p(\Sigma|Y, M, A)$ and $p(A|Y, M, \Sigma)$. To find these distributions, it is instructive to rewrite the model in a Seemingly Unrelated Regression (SUR) representation (Drautzburg; 2016). Conditional on presample values, the model can be written as:

$$\tilde{y} = Z\alpha + \tilde{u}, \quad \tilde{u} \sim \mathcal{N}(0, \Sigma \otimes I_T),$$

where $\alpha = \text{vec}(A)$, $X = [x'_1, \dots, x'_T]'$, $Z = [(I_n \otimes X)' : 0_{n(np+1) \times Tk}]'$, $Y = [y'_1, \dots, y'_T]'$, $M = [m'_1, \dots, m'_T]'$, $\tilde{y} = [\text{vec}(Y)', \text{vec}(M)']'$, $U = [u'_1, \dots, u'_T]'$, $\tilde{U} = [U', M']'$ and $\tilde{u} = \text{vec}(\tilde{U})$. The density function of the observables \tilde{y} is then given as:

$$p(\tilde{y}|\alpha, \Sigma) \propto |\Sigma|^{-\frac{1}{2}} \exp \left(-\frac{1}{2} \text{tr}(S\Sigma^{-1}) \right),$$

where $S = \tilde{U}'\tilde{U}$. Based on Geweke (2005, Chapter 5), the conditional posterior distributions of SUR models with priors (2.11) and (2.12) are of the following convenient form:

$$\alpha|\tilde{y}, \Sigma \sim \mathcal{N}(\bar{\alpha}, \bar{V}), \tag{2.13}$$

$$\Sigma^{-1}|\tilde{y}, \alpha \sim \mathcal{W}((\Psi + S)^{-1}, d + T), \tag{2.14}$$

where $\bar{V} = (V_0^{-1} + Z'(\Sigma^{-1} \otimes I_T)Z)^{-1}$ and $\bar{\alpha} = \bar{V} (V_0^{-1}\alpha_0 + Z'(\Sigma^{-1} \otimes I_T)\tilde{y})$. Based on these conditionals, it is straightforward to implement a Gibbs sampler to numerically characterize the posterior distribution of the reduced form parameters.

In the remainder of the section, we discuss the prior and posterior of the orthogonal matrix Q , which according to equation (2.10), will provide a mapping to the structural parameters of the model. Furthermore, we will discuss algorithms to generate draws from its posterior distribution. The details will depend on whether we are identifying the model as in Section 2.2 or 2.3. We start with the case described in Section 2.2 where a subset of shocks ($q \leq (n - k)$) are set identified via sign restrictions and that, for simplicity, just one additional shock ($k = 1$) is point identified via instrumental variables. Given a (prior or posterior) draw of the reduced form covariance matrix Σ , the linear restrictions from equation (2.6) along with the normalization $\beta_1' \Sigma_u^{-1} \beta_1 = 1$ enables us to point identify β_1 , the first column of the structural impact matrix B .⁶ Given this observation, it is convenient to decouple the orthogonal matrix Q into a fixed (\tilde{Q}) and a stochastic (orthogonal) component (\bar{Q}):

$$Q = \tilde{Q}\bar{Q}.$$

Given a value of the augmented reduced form matrix Σ , \tilde{Q} will provide the rotation of P such that the first columns of B are β_1 . To get an explicit expression, let $\tilde{q}_k = P^{-1}\beta_1$. Then, we define $\tilde{Q} = [N_\perp, \tilde{q}_k]$ where N_\perp is any orthonormal basis for the null space of \tilde{q}_k such that $N_\perp' \tilde{q}_k = 0$ and $N_\perp' N_\perp = I$. With respect to the stochastic matrix \bar{Q} , we define it as follows:

$$\bar{Q} = \begin{pmatrix} I_k & 0 \\ 0 & Q_{n-k}^* \end{pmatrix}.$$

where Q_{n-k}^* is random and given the structure of \bar{Q} , will not affect β_1 but only relate the reduced form parameters to those structural parameters that are set-identified. We specify a uniform prior distribution (Haar prior) $\pi(Q_{n-k}^* | \Sigma, A)$ over the space of $n - k$ dimensional orthogonal matrices subject to the sign restrictions.⁷ This enables us to easily draw from the (conditional) posterior distribution of Q as follows (Algorithm 1).

1. Given a draw of the reduced form parameters Σ , solve for β_1 using equation (2.6) along with the normalization $\beta_1' \Sigma_u^{-1} \beta_1 = I_k$ described in Section 2.2. Furthermore, compute P such that $PP' = \Sigma_u$.
2. Compute $\tilde{q}_k = P^{-1}\beta_1$ and define the $n \times n$ matrix $\tilde{Q} = [N_\perp, \tilde{q}_k]$ where N_\perp is an orthonormal basis for the null space of \tilde{q}_k such that $N_\perp' \tilde{q}_k = 0$ and $N_\perp' N_\perp = I$.
3. Draw Q_{n-k}^* from the uniform distribution over the space of all $(n - k) \times (n - k)$ dimensional orthogonal matrices with an algorithm of your choice and define $\bar{Q} = \begin{pmatrix} I_k & 0 \\ 0 & Q_{n-k}^* \end{pmatrix}$.
4. Compute $Q = \tilde{Q}\bar{Q}$ and its associated candidate structural impact matrix $B = PQ$. Accept the draw of Q if all sign restrictions are satisfied.

⁶In case of $k > 1$, recall that additional restrictions will be necessary to identify β_1 (Mertens & Ravn; 2013).

⁷A candidate draw from this prior can be easily obtained, e.g. via a Householder based algorithm (Rubio-Ramírez, Waggoner & Zha; 2010) or a sequence of random Givens matrices (Uhlig; 2005; Fry & Pagan; 2011).

If the task is to identify a single shock combining sign restrictions with the identifying information of a set of k non-exogenous proxies as in Section 2.3, all elements in B are set-identified. In this case, we can directly specify the uniform prior distribution $\pi(Q|\Sigma, A)$ on the n dimensional orthogonal matrix Q as usually done in the literature. Then, given a posterior draw of the reduced form parameters, a draw of the (conditional) posterior of Q can be obtained in a standard fashion (Algorithm 2):

1. Given a draw of the reduced form parameters Σ , draw a candidate matrix Q from the uniform distribution over the space of $n \times n$ dimensional orthogonal matrices with an algorithm of your choice. Furthermore, compute P such that $PP' = \Sigma_u$.
2. Compute the following quantities: $E(m_t \varepsilon_t') = Q' P^{-1} \Sigma_{mu'}$ as well as
 - $\text{Corr}(m_{it}, \varepsilon_{1t}) = E(m_{it}, \varepsilon_{1t}) / \sigma_i$ for $i = 1, \dots, k$ where $\sigma_i^2 = \Sigma_{m,ii}$.
 - $\phi_{ij} = E(m_{it}, \varepsilon_{1t})$ for $i = 1, \dots, k$ as well as the associated contributions to the variance of m_{it} , $\psi_{ij} = \phi_{ij}^2 / \sigma_i^2$.
3. Compute the candidate structural impact matrix $B = PQ$ and accept the draw if all sign and proxy induced moment restrictions (computed in step 2) are satisfied.

To sum up, inference can be decoupled into drawing reduced form parameters and generating draws of rotation matrix that satisfy the proposed restrictions and therefore, we take proper account of both sampling and model uncertainty. On a high level, the Gibbs sampler algorithm can be summarized as follows:

1. (Sampling uncertainty) Iteratively draw a set of reduced form parameters from their conditional posterior distribution as in equations (2.13) and (2.14).
2. (Model uncertainty) Draw Q subject to the sign restrictions and the proxy induced moment restrictions by either Algorithm 1 or 2.⁸

Note that both approaches can also be combined to identify a subset of shocks with IV and sign restrictions, and further shocks with a combination of both approaches.

Given the recent critique by Baumeister & Hamilton (2015) on the implications of using the Haar prior in SVAR models, we highlight that one might also use different priors for $\pi(Q_{n-k}^*|\Sigma, A)$ and $\pi(Q|\Sigma, A)$. However, this complicates efficient drawing of Q as discussed in Arias et al. (2018). Alternatively, one might also apply the Robust Bayes approach by Giacomini & Kitagawa (2015) to conduct inference on the identified set that is robust to the choice of prior for Q_{n-k}^* and Q .

Another highly relevant point for the practitioner is that many instruments are available only for a rather short sample period. For example, the shock series used in Gertler & Karadi (2015) are only available starting in the late 80s and early 90s. If the sample of all macroeconomic data in the VAR is not adjusted to the same length, a missing data problem arises. More generally, it might occur that the external variable is not observed in every period of time. Missing data can be handled in a straightforward way within a Bayesian

⁸Note that the conditional posteriors of the reduced form parameters do not depend on Q . Therefore, if desired, step one can be also run in isolation before draws of Q are generated in step two.

setting. It involves a simple imputation of the missing observations through an additional step in the Gibbs sampler. We refer to Appendix A for the details.

3 Empirical Applications

We demonstrate the usefulness of our methodological framework in two empirical applications. In Section 3.1, we use instrumental variables and sign restrictions separately to identify multiple shocks in an SVAR model for the international oil market. This corresponds to the type of restrictions described in Section 2.2. In our second application in Section 3.2, we identify a single shock by combining information from sign restrictions and an external proxy variable. More specifically, we use the restrictions introduced in Section 2.3 to identify a monetary policy shock and investigate its effects on key macroeconomic variables, in particular on the US credit market. Note that for both applications, we specify an uninformative prior with little influence on the posterior of the reduced form parameters.⁹

3.1 The Effects of Oil Price Shocks

In this section, we illustrate how the external instrument approach and sign restrictions can be combined to simultaneously identify a set of different oil market specific shocks. In particular, we illustrate how an oil supply shock may be identified from using a proxy variable related to exogenous disruptive oil supply shortages, while at the same time the demand shocks may be identified from a set of sign restrictions.

In recent years, structural VAR models have been popular tools in investigating and understanding the dynamics of oil price shocks and their effect on macroeconomic variables. Different identifying assumptions have been used to disentangle oil supply demand shocks. While some studies rely only on short-run exclusion restrictions (e.g. Kilian (2009), Stock & Watson (2016)), there are also a number of studies using sign restrictions on either contemporaneous impacts (see Kilian & Murphy (2012) and Baumeister & Hamilton (2015)) or on multiple horizons (see e.g. Peersman & Van Robays (2012)). As pointed out by Kilian & Murphy (2012), the use of sign restrictions alone may not be enough to properly identify the oil market shocks.

An alternative way to identify an oil supply shock has been suggested by Hamilton (2003) and Kilian (2008b). Both papers construct an ‘exogenous’ time series based on disruptive events, which are typically related to wars in oil producing countries. Kilian (2008b), for instance, computes a measure of an oil supply shock based on a counter-factual evolution of oil production. If this measure is truly exogenous, the series from Kilian (2008b) may also serve as an external instrument for an oil supply shock. The external instrument approach in the context of oil prices shocks approach has been used in e.g. Stock & Watson (2012) using quarterly US data. Their results suggest that both measures, Hamilton’s and Kilian’s series, may only be weak instruments for the underlying structural oil price shock and consequently using standard IV inference is potentially problematic.

⁹We use $\alpha_0 = 0$, $V_0 = 10 \cdot I$, $\Psi = I$ and $d = n + k + 1$.

Instead of relying on either sign restrictions or external instruments alone, we combine both approaches within our modeling framework. We identify an oil supply shock using a Kilian (2008b) type proxy variable as an instrument. At the same time, we identify the aggregate demand and the oil-specific demand shock using the sign restrictions suggested by Kilian & Murphy (2012). Handling all three shocks simultaneously has the advantage that all shocks are mutually orthogonal by construction. Moreover, the information in the proxy variable allows us to be agnostic on the signs of the responses to an oil supply shock and may also help to narrow down the set of admissible impulse responses.

We use the 3-variable oil-market VAR of Kilian & Murphy (2012) and start with a reduced form VAR for $y_t = (\Delta\text{prod}_t, \text{rae}_t, \text{rpo}_t)'$, where Δprod_t is the change in the log of oil production), rae_t is a measure of (world) real economic activity, and rpo_t is the real price of oil. The data for these three series have been taken from Kilian & Murphy (2012). We fit a reduced form VAR with $p = 12$ lags over the sample period 1973M02 to 2008M09 as in Kilian & Murphy (2012).¹⁰

We identify three structural shocks: An adverse oil supply shock ε_t^s , an aggregate demand shock ε_t^d and an oil-specific demand shock ε_t^{od} . The oil supply shock ε_t^s will be identified by an external instrument m_t . As an instrument we use a monthly series of exogenous oil price shocks constructed as in Kilian (2008b). Since the monthly series from Kilian (2008b) was not available to us, we have constructed the proxy series for the oil supply shock using our own calculations based on the US Energy Information Administration (EIA) production data and following the description in Kilian (2008b). We show the corresponding time series plot of the instrument in the lower right hand panel of Figure B.1.¹¹ Following e.g. Stock & Watson (2012), we measure the strength of this instrument by regressing the instrument m_t on the reduced form VAR(12) residuals \hat{u}_t and by computing the corresponding F -statistic. Using the monthly series, we find an F -statistic of 8.13. This value is much larger than what Stock & Watson (2012) have found for the corresponding quarterly series indicating that the monthly instrument series may be not a weak instrument.¹² Note that we only use observations on the proxy for a sample between 1973M02 to 2004M09, which corresponds to the sample used in Kilian (2008b). Consequently, the instrument and the VAR time series are observed over different sample periods. As explained in Section 2.4, our method can easily handle ‘missing data’ on the proxy variable.

The aggregate demand ε_t^d and oil-specific demand shock ε_t^{od} are identified by a set of sign restrictions on their contemporaneous impact. Here we simply use the sign restrictions suggested in Table 1 of Kilian & Murphy (2012). Consequently, in the 3-variable oil market

¹⁰We have also tried a lag length of $p = 24$ as in Kilian & Murphy (2012) and $p = 3$ as suggested by the Akaike information criterion. The results are very similar to those reported for $p = 12$.

¹¹Visual inspection of the series shows a close resemblance to Figure 7 of Kilian (2008b).

¹²We have rerun this regression using quarterly oil market data and quarterly versions of our exogenous shock series as well as Kilian’s quarterly series. The corresponding F -statistics are 1.1 and 0.05. Thus it seems that the weak instrument problem is a consequence of losing information by converting the external instrument series to quarterly frequency.

system our identification scheme can be represented by

$$\begin{pmatrix} u_t^{\Delta\text{prod}} \\ u_t^{\text{rea}} \\ u_t^{\text{rpo}} \end{pmatrix} = B \begin{pmatrix} \varepsilon_t^s \\ \varepsilon_t^d \\ \varepsilon_t^{od} \end{pmatrix},$$

where the impact matrix is given as

$$B = \begin{pmatrix} \text{IV} & + & + \\ \text{IV} & + & - \\ \text{IV} & + & + \end{pmatrix}. \quad (3.1)$$

The ‘IV’ in the first column of (3.1) indicates that the supply shock is identified from using m_t as an instrumental variable only, while the signs in column 2 and 3 show the impact sign restrictions for identifying the two demand shocks. No further restrictions have been imposed on the model.

[– insert Figure 1 about here –]

Figure 1 compares the impulse responses up to horizon $h = 36$ obtained from combining IV and sign restriction identification to results obtained from using sign restrictions alone.¹³ The solid line is the posterior median and the shaded area correspond to 68% posterior credibility sets of impulse responses obtained from the combined IV and sign restriction identification. The dotted lines are obtained from using sign identification only and correspond to the model of Kilian & Murphy (2012).¹⁴

A number of interesting results emerge. First, we observe that identifying the oil supply shock by the instrumental variable approach changes the magnitude of oil price responses to oil supply and oil-specific demand shocks. Compared to the sign restriction only model, we find a smaller increase of the oil price after an adverse oil supply shock ε_t^s and a more pronounced increase following an oil-specific demand shock ε_t^{od} . This change of impulse response pattern is reasonable according to Kilian & Murphy (2012). In fact, they find similar changes in responses once they impose their elasticity bounds. Second, it is interesting to see that identifying the oil supply shock by the proxy variable has also substantial impact on the oil-specific demand shock. In particular, compared to the sign restriction only model, the demand shock ε_t^{od} also leads to a much less pronounced increase in oil production. Taken together with the larger price response, this implies a much smaller impact elasticity of oil supply with respect to the real price of oil (b_{13}/b_{33}).¹⁵ Given the consensus of a small short-run oil supply elasticity, we find that the IV identification for the oil supply shock shifts the SVAR results to a much more reasonable region even without imposing elasticity bounds.

We also find that replacing the sign restrictions for the supply shock with IV identification leads to a response in economic activity which is no longer significantly different from zero.

¹³The panels in the first row show accumulated responses on Δprod_t and consequently show the effects on the log-level of production.

¹⁴In the sign restriction only identification, we follow Kilian & Murphy (2012) and use $(-, -, +)'$ for the first column of B in equation (3.1). Also note that we only use sign restrictions, i.e. no additional restrictions in the form elasticity bounds are imposed. The results shown in Figure 1 are based on a VAR with 12 lags.

¹⁵As summary statistics of the posterior distribution of this elasticity we report the 16th and 84th quantile. For the sign only model we find [0.22; 3.14], while for the IV/sign restriction model we find [0.028; 0.42].

In other words, using the IV method we do not find support for the sign restriction on real economic activity imposed by Kilian & Murphy (2012). Compared to the sign restriction only model, the adverse supply shock identified by the IV approach also leads to a somewhat sharper drop in oil production. Together with the less pronounced increase in the real price of oil, this implies a somewhat larger demand elasticity (b_{11}/b_{31}).¹⁶ We point out, however, that the implied values are still around the lower bound of -0.8 discussed for quarterly data in Baumeister & Peersman (2013). Finally, we note that the aggregate demand shock ε_t^d is virtually unaffected by using an IV approach for the supply shock.

The oil market example illustrates the usefulness of combining the external instrumental approach for identification with sign restrictions in SVAR analysis. Exploiting the information from an instrument allows to relax some of the sign restrictions while at the same time leading to response patterns that are more in line with evidence on oil price elasticities.

3.2 The Effects of Monetary Policy Shocks

Since the seminal paper of Sims (1980), the effects of monetary policy shocks on economic activity have been extensively studied using SVAR models (see Ramey (2016) for a recent review of the literature). In the early literature, surprises to monetary policy have been identified by using a Cholesky decomposition of the reduced form VAR covariance matrix, with the policy instrument ordered below the real variables, see e.g. Christiano, Eichenbaum & Evans (1999) or Bernanke, Boivin & Eliasch (2005). This identifying assumption implies that the central bank can respond instantaneously to movements in the real sector of the economy, while the real variables may only respond with one lag to the policy shock. Such an identification is in line with macroeconomic models subject to nominal rigidities (Christiano, Eichenbaum & Evans; 2005). However, it is not yet clear how the effects of monetary policy shocks can be identified under the presence of fast moving financial variables such as credit costs or equity prices. A simple recursiveness assumption is unrealistic no matter of the ordering, since it can be assumed that both, monetary policy and financial markets respond immediately to any innovation in the system. Therefore, alternative identification schemes have emerged in recent years that avoid the recursiveness assumption. One strand of the literature draws on sign restrictions on the impulse responses with respect to the policy shock (Uhlig; 2005; Faust; 1998). These restrictions are derived from conventional wisdom, such that a monetary policy tightening should be associated with an increase in the interest rates but not in consumer prices nor liquidity. Unfortunately, because of the implied set identification, this identification procedure often leads to wide confidence intervals around impulse responses such that results are often not informative enough to allow for policy conclusions. An alternative branch of the literature uses narrative measures of monetary policy shocks for identification. Among the most prominent measures are shock series based on readings of Federal Open Market Committee (FOMC) minutes (Romer & Romer; 2004; Coibion; 2012; Miranda-Agrippino & Rey; 2018) and factors based on changes in high frequency future prices around FOMC meetings (Faust, Swanson & Wright; 2004; Gertler & Karadi; 2015;

¹⁶The 16th and 84th quantile of the posterior distribution of this elasticity is -1.34 and -0.41 in our IV/sign restriction model.

Barakchian & Crowe; 2013; Nakamura & Steinsson; 2013). However, it is a very difficult task to construct convincing exogenous instruments for monetary policy shocks. With respect to Romer & Romer (2004), the authors themselves claim that their series is only ‘relatively free of endogenous and anticipatory movement’. To ensure against remaining endogeneity they exclude the possibility of a contemporaneous response of the macroeconomic variables to the narrative series. The exogeneity of instruments based on high frequency future data is also questionable. Ramey (2016), for example, finds that the main instrument of Gertler & Karadi (2015) suffers from a nonzero mean, significant autocorrelation and predictability by Greenbook forecasts.¹⁷

Our methodology provides a simple framework to combine identification from sign restrictions and such proxy variables. We illustrate that problems arising if either of the methods is used individually are mitigated to some extent. We start with an identification scheme based on sign restrictions similar in spirit to Uhlig (2005). To further narrow down the set of models we restrict the covariance of the implied structural shock and a narrative proxy series, thereby discarding all models that imply only a loose relation between the SVAR shock and the narrative series. This sharpens inference of the set identified model, while at the same time avoiding the potentially wrong assumption of exogeneity of the proxy series.

For our explorations, we use a monthly VAR for key US macroeconomic variables, i.e. we use a VAR for $y_t = (\text{ip}_t, \text{cpi}_t, \text{nbr}_t, \text{EBP}_t, R_t^s)'$, where ip_t is the log of industrial production, cpi_t the log of the consumer price index, nbr_t the log of non-borrowed reserves, EBP_t the ‘Excess Bond Premium’, a measure of credit market tightness developed by Gilchrist & Zakrajek (2012), and R_t^s the federal funds rate. Given the availability of the Excess Bond Premium series and the recent period at the zero lower bound of interest rates, we use a sample period from 1973M07 until 2007M12.¹⁸ We include $p = 6$ lags to account for sufficient dynamics of the time series vector. With respect to the narrative series, we use $m_t = \text{rr}_t$, the Romer and Romer (R&R) narrative shock series updated by Wieland & Yang (2016). We focus specifically on the effects of monetary policy shocks on credit markets, which is an aspect difficult to analyze within a recursive identification scheme. Consider the following set of identifying restrictions in our structural analysis, where ε_t^{mp} is the monetary policy shock:

- R1: Sign restrictions on the contemporaneous impulse responses.

$$\frac{\partial \text{E}(\text{ip}_t | \Omega_t)}{\partial \varepsilon_t^{mp}} \leq 0, \quad \frac{\partial \text{E}(\text{cpi}_t | \Omega_t)}{\partial \varepsilon_t^{mp}} \leq 0, \quad \frac{\partial \text{E}(\text{nbr}_t | \Omega_t)}{\partial \varepsilon_t^{mp}} \leq 0, \quad \frac{\partial \text{E}(R_t^s | \Omega_t)}{\partial \varepsilon_t^{mp}} \geq 0.$$

R1 imposes that a contractionary monetary policy shock does not have (contemporaneous) positive effects on output, prices and non-borrowed reserves, and no negative effect on the federal funds rate.

- R2: $\text{Corr}(\varepsilon_t^{mp}, \text{rr}_t) > 0$, imposing that ε_t^{mp} is positively correlated with rr_t .
- R3: $\text{Corr}(\varepsilon_t^{mp}, \text{rr}_t) > \text{Corr}(\varepsilon_{jt}, \text{rr}_t)$ for all $j \neq mp$, imposing that the correlation between ε_t^{mp} and rr_t is the largest among all shocks in the SVAR.

¹⁷Greenbook forecasts are those published by the central bank in their FOMC minutes and therefore assumed to be in the information set of the central bank.

¹⁸An exact description of data sources and corresponding time series plots are provided in Appendix B.2

- R4: $\text{Corr}(\varepsilon_t^{mp}, \text{rr}_t)^2 > \text{Corr}(\varepsilon_{jt}, \text{rr}_t)^2$ for all $j \neq mp$, imposing that ε_t^{mp} contributes most to the variance of rr_t among all shocks in the SVAR. Note that if $\varepsilon_t^{mp} = \varepsilon_{1t}$, this is equivalent to imposing that $\psi_{i1} > \psi_{ij}, j = 2, \dots, n$ as denoted in Section 2.3.
- R5: $E(\varepsilon_{jt}, \text{rr}_t) = 0$ for all $j \neq mp$, which corresponds to identification via instrumental variables.

[– insert Figure 2 about here –]

Restrictions R1 and R5 correspond to identification via sign restrictions and instrumental variables, respectively. Restrictions R2 to R4 will be used in combination with the sign restrictions in order to narrow down the set of admissible models without imposing exogeneity. Note that R4 is typically more restrictive than R3, while R3 is more restrictive than R2. Note that in order to implement R3 and R4, ε_{jt} for $j \neq mp$ need to be identified. Since we do not attempt to identify these shocks economically, we do this in a statistical sense and let them be randomly identified through the rotations generated under the uniform prior on Q .

Figure 2 provides the impulse responses obtained from using five different identification schemes for a horizon up to four years. The first row corresponds to scheme R1, that is identifying the policy shock solely with sign restrictions. In line with the prior about the impulse response function, unexpected tightening is associated with a decrease in industrial production, prices, liquidity and an increase in the federal funds rate. However, with exception of output, these effects are almost never significant. With respect to the effects of ε_t^{mp} on the credit market (EBP_t), nothing can be said since the zero line is contained in the 68% posterior credibility set at any point of time. The last row of Figure 2 shows the responses from identification scheme R5, which corresponds to a Bayesian proxy SVAR based on pure instrumental variable identification. Some puzzling results emerge in the impulse response functions. First, output increases significantly for about 6 months in response to a policy tightening, which is certainly at odds with macroeconomic theory. Furthermore, consumer prices rise sharply and remain significant for up to 3 years. Such a ‘price puzzle’ is frequently found in recursively identified VARs and often attributed to an omitted variable bias (Christiano et al.; 1999). Sometimes, including commodity prices in the VAR can mitigate the puzzle but unfortunately this is not the case for our specification. Ramey (2016) finds similar responses based on a proxy SVAR with the R&R shocks as instruments. She finds that including more information in the VAR does not solve the puzzles and argues that they are likely to be caused by endogeneity of the instrument. This would certainly invalidate the use of the R&R shock as an instrumental variable, however, does not pose a problem in our framework.

Rows two, three and four correspond to combining sign restrictions with information of the R&R series, however, without the need of its exogeneity. Each model adds an additional restriction and therefore is based on tighter constraints by construction, always narrowing down the set of admissible models to a somewhat larger extent. We find that adding a simple sign restriction on the correlation (R2) does not change the impulse response functions substantially. Minor effects are found in the persistence of the interest rates and the significance of the response of industrial production. Unfortunately, still nothing can be

said on the effects on credit market tightness, which remains insignificant. This implies that the set of monetary policy shocks of the sign restriction only model are already positively correlated with the R&R shock which is why adding R2 does not sharpen inference. However, additionally including restriction R3 makes a considerable difference (row 3 of Figure 2). The effects on industrial production and the interest rate are more pronounced, while for prices and liquidity no significant effects are found. With respect to credit markets, the response is positive and a significant tightening is found after six months. As expected, adding restrictions R4 (in addition to R2 and R3) further discards some models, retaining only those where the monetary policy shock additionally contributes most to the variance of the R&R series. The impulse responses in row 4 of Figure 2 indicate that the effects on industrial production and interest rates have similar magnitudes as in the point identified proxy SVAR (R5), with the main difference that the former variable is not subject to the increase in the first months. Also the prize puzzle is found to be less pronounced and the increase is insignificant in most of the periods. Interestingly, the response of the excess bond premium shows a significant tightening of the credit market similar in magnitude to the proxy SVAR.

In Figure C.3 of Appendix C, we provide additional plots of impulse response functions that arise if only R2, R2-R3 and R2-R4 are used to identify the model. We find that R2 and R2-R3 in isolation yield too little information to draw meaningful economic conclusions. Further including R4 tightens the identified set by a large extent, however, the same puzzling positive short run response on output as well as a pronounced price puzzle arise as in point-identification by IV (R5). Therefore, we believe that the combination with sign restrictions (R1) might be more sensible in this scenario since economic theory allows to exclude this patterns a priori.

Summarizing our results, we illustrate that combining sign restrictions with restrictions based on external narrative series can sharpen inference substantially and lead to economically meaningful results. In this context, we also propose a set of restrictions which differ in their strength but are all automatic in a sense that no threshold value (on e.g. correlations) has to be chosen by the researcher. From an empirical point of view, we find that based on our identification scheme, there is evidence for a significant tightening of credit markets in response to a monetary policy shock. This highlights the importance of a credit channel in the transmission mechanism. Such evidence supports the finding of Gertler & Karadi (2015) and implies that theoretical models should pay special attention to this feature.

4 Conclusion

For the identification of structural shocks within SVAR models, we suggest to combine sign restrictions with the information in time series that act as proxy variables for the underlying structural shocks. We propose an econometric framework that incorporates the information on the proxy variables by augmenting the SVAR with equations that relate the proxy variables to the structural shocks.

Our econometric framework is fairly general in the following sense: First, the framework allows to simultaneously identify different shocks using either sign restrictions (set-

identification) or an external instrument approach (point-identification), always ensuring that all structural shocks are orthogonal. Second, the setup also allows to (set) identify a single shock by combining sign restrictions and the information of external proxy variables, without assuming their exogeneity. Compared to a pure sign restriction approach, the additional information from the external proxy series may help to narrow down the set of admissible models and leads to sharper results, in e.g. impulse response analysis. We essentially discard models that imply structural shocks that have no close relation to the external proxy time series. We measure this relation e.g. by the variance contribution of the shock. Third, the setup also nests the pure sign restriction approach and the pure external instrument variable case. Estimation and inference is done in a full Bayesian setup, which accounts for both, model and estimation uncertainty. An additional advantage of the Bayesian setup is that the inference framework requires no modifications to handle the case of weak instruments and allows for straightforward handling of missing data.

We illustrate the usefulness of our method in two empirical applications. In the first application, we use a standard oil market model from the literature and identify simultaneously an oil supply shock using an external proxy variable as an instrument, and two demand shocks by using sign restrictions. Employing this identification leads to impulse responses that imply much more reasonable oil supply elasticities than a pure sign restriction model. In the second application, we analyze the effects of monetary policy shocks focusing on the credit market. We find that using sign restrictions alone is not informative enough with wide error bands around the response of the excess bond premium. Once we combine the sign restrictions with external information coming from a proxy variable of monetary policy shocks, we find a significant tightening of credit markets.

Overall, our paper suggests that combining sign restrictions and external proxy variables for structural shock identification is a promising way to sharpen results from SVAR models.

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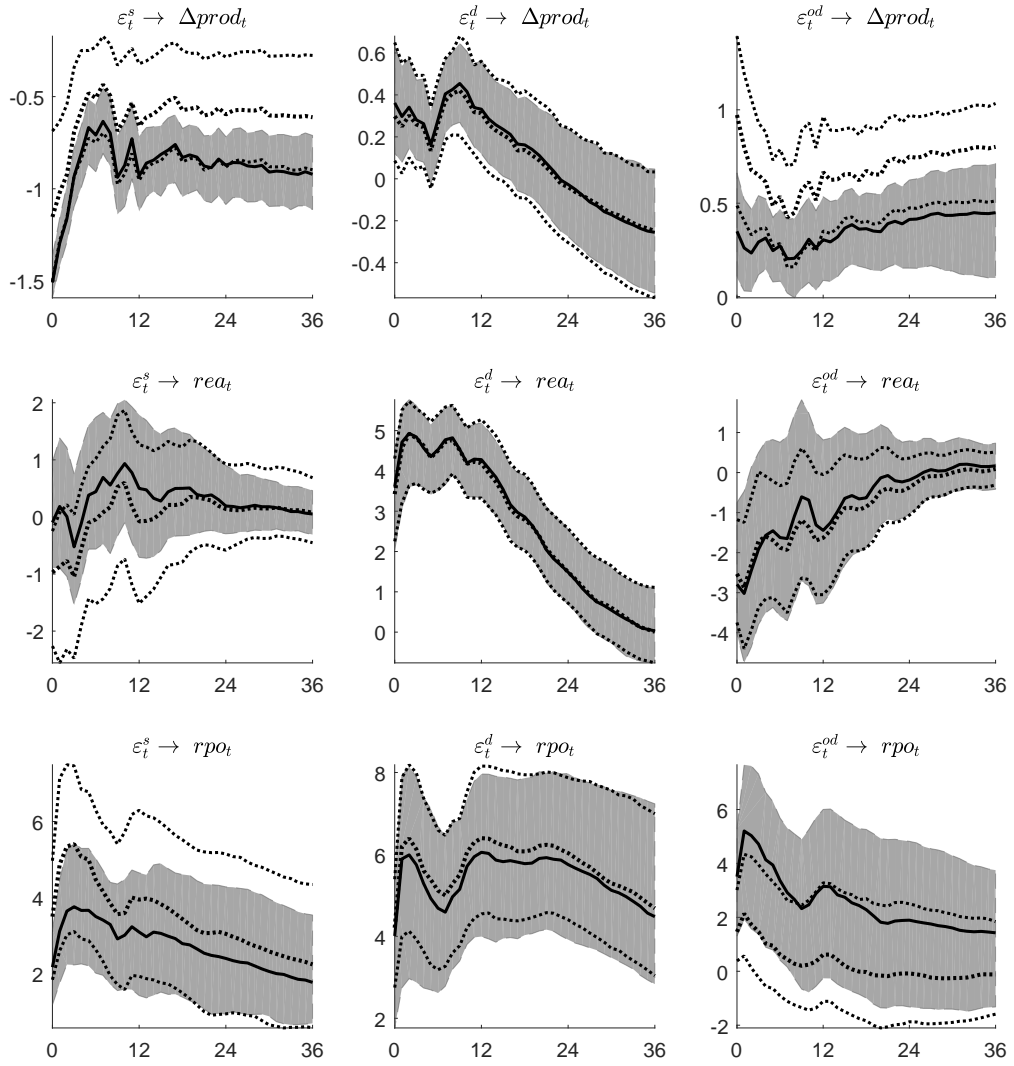


Figure 1: Impulse responses in oil market SVAR. Dotted lines: Identification with sign restrictions only. Solid lines and shaded areas: Identification by IV for oil supply shock ε_t^s and sign restrictions for shocks to aggregate demand ε_t^d and oil specific demand ε_t^{od} . Sample period: 1973M02–2008M09.

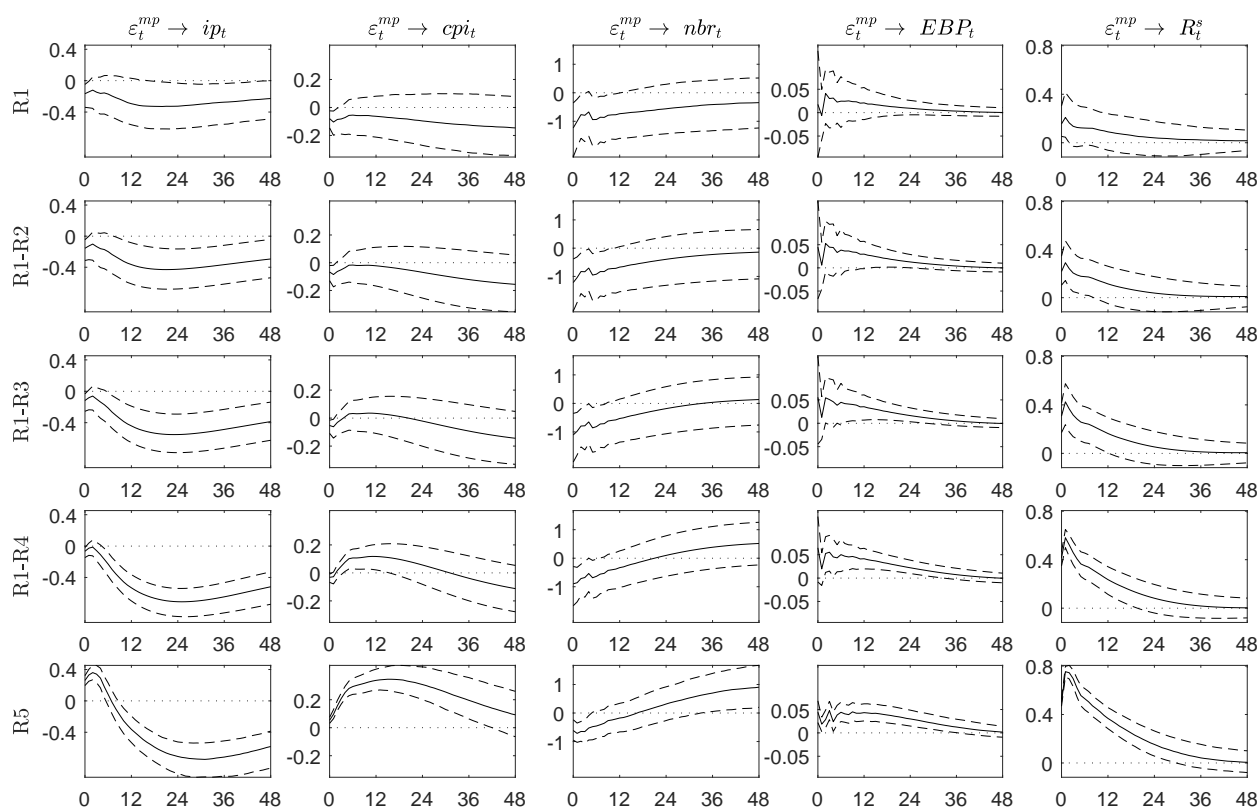


Figure 2: Impulse responses in the monetary policy SVAR obtained by using different identifying restrictions. Posterior median (solid line) and 68% posterior credibility sets (dotted lines). Sample period: 1973M07-2007M12

A Gibbs Sampler for Missing Data

Assume that some of the observations m_s are missing at random where s denotes the period of missing data. Instead of simply discarding all values of y_s , we advocate to add an additional step in the Gibbs sampler which imputes the missing values. For this purpose, we need to derive the conditional distribution $p(m_s|Y, M, \Sigma, A)$. Recall the joint likelihood of $\{y_t, m_t\}$:

$$y_t = Ax_t + u_t, \quad \begin{pmatrix} u_t \\ m_t \end{pmatrix} \sim \mathcal{N} \left(0, \begin{pmatrix} \Sigma_u & \Sigma_{mu} \\ \Sigma_{mu'} & \Sigma_m \end{pmatrix} \right).$$

Using standard results of multivariate statistics, the conditional distribution of m_t is given as $p(m_t|Y, M, \Sigma, A) \sim \mathcal{N}(\bar{m}_t, \bar{V}_t)$ where

$$\begin{aligned} \bar{m}_t &= \Sigma_{mu'} \Sigma_u^{-1} (y_t - Ax_t), \\ \bar{V}_t &= \Sigma_m - \Sigma_{mu'} \Sigma_u^{-1} \Sigma_{mu}. \end{aligned}$$

Therefore, a modified Gibbs sampler involves drawing iteratively from the following blocks to generate draws of the reduced form parameters:

1. Draw the autoregressive coefficients from $\alpha|\tilde{y}, \Sigma \sim \mathcal{N}(\bar{\alpha}, \bar{V})$ as in Section 2.4.
2. Draw the inverse of the covariance matrix $\Sigma^{-1}|\tilde{y}, \alpha \sim \mathcal{W}((\Psi + S)^{-1}, d+T)$ as in Section 2.4.
3. Impute all the missing values m_s of the proxy variables by drawing each of them from a normal distribution $m_s|\tilde{y}, \Sigma, A \sim \mathcal{N}(\bar{m}_s, \bar{V}_s)$ where

$$\begin{aligned} \bar{m}_s &= \Sigma_{mu'} \Sigma_u^{-1} (y_s - Ax_s), \\ \bar{V}_s &= \Sigma_m - \Sigma_{mu'} \Sigma_u^{-1} \Sigma_{mu}. \end{aligned}$$

B Data

B.1 Data in Oil Price Shock Example

The data for the oil market VAR have been taken from the web appendix S2 in Kilian & Murphy (2012). A quarterly time series for the ‘exogenous’ oil price shock is available on Lutz Kilian’s homepage. Since a corresponding time series on the monthly frequency is not readily available, we have constructed it from oil production data of the EIA following exactly the approach described in Kilian (2008b). A time series plot of the variables is shown in Figure B.1.

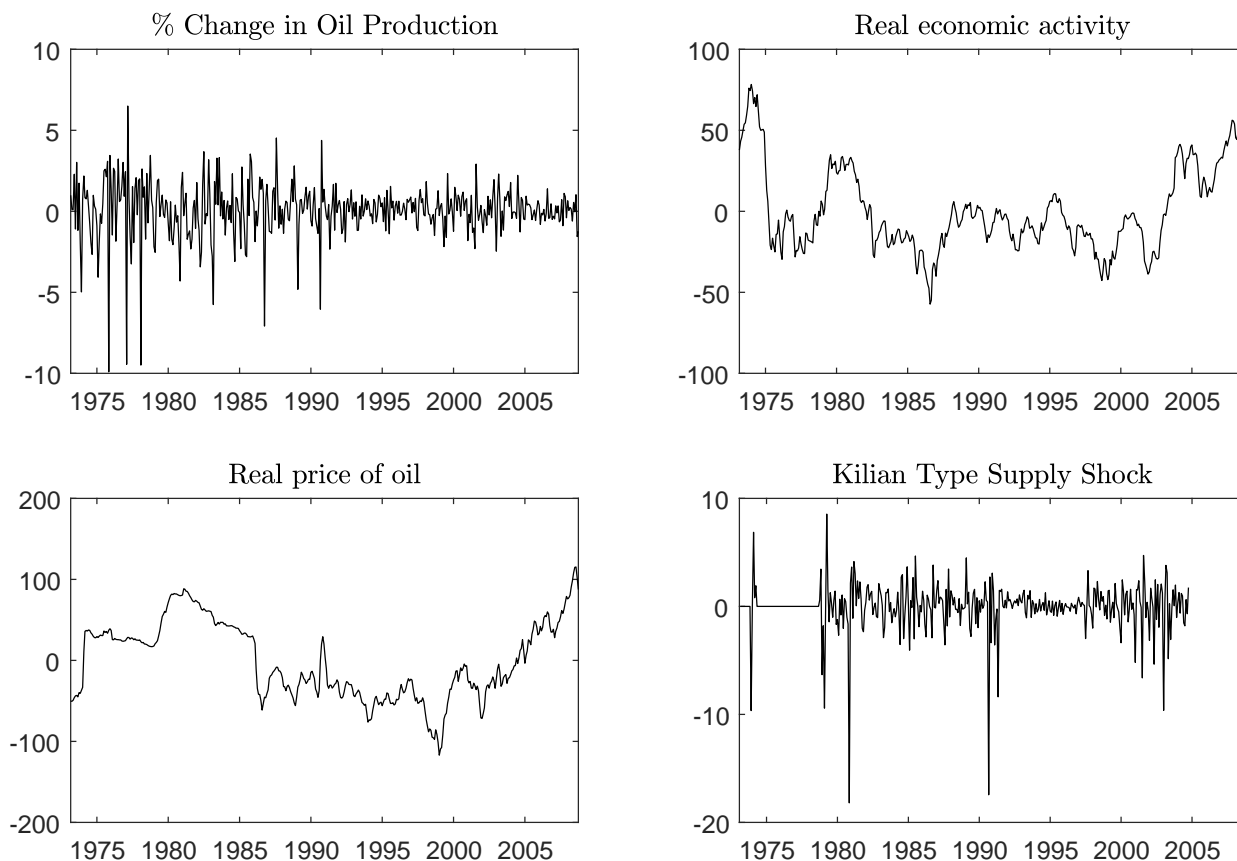


Figure B.1: Time series plots of oil market variables. Sample: 1973M01-2008M09.

B.2 Data in Monetary Policy Shock Example

The time series of the monetary policy VAR were obtained from the following sources. Industrial production, the consumer price index and the federal funds rate were obtained from FRED with series id `INDPRO`, `CPIAUCSL` and `FEDFUNDS`, respectively. The data of the Excess Bond Premium and the Romer and Romer shock were obtained from Valerie Ramey’s homepage and are part of the replication files of her recent chapter in the Handbook of Macroeconomics (Ramey; 2016). A time series plot for each of the series is given in Figure B.2.

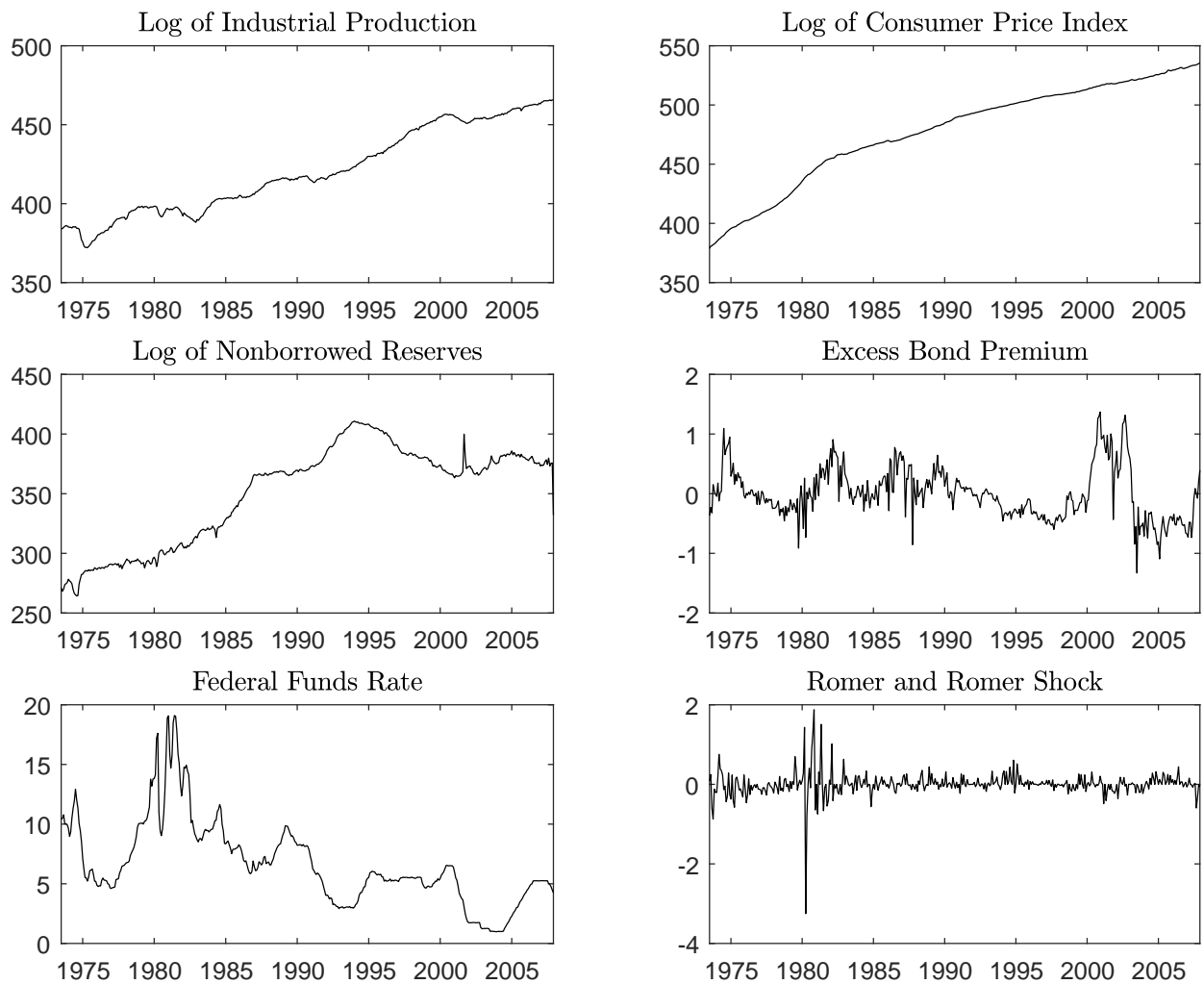


Figure B.2: Time series plots of monetary policy VAR variables. Sample: 1973M07-2007M12.

C Additional Figures

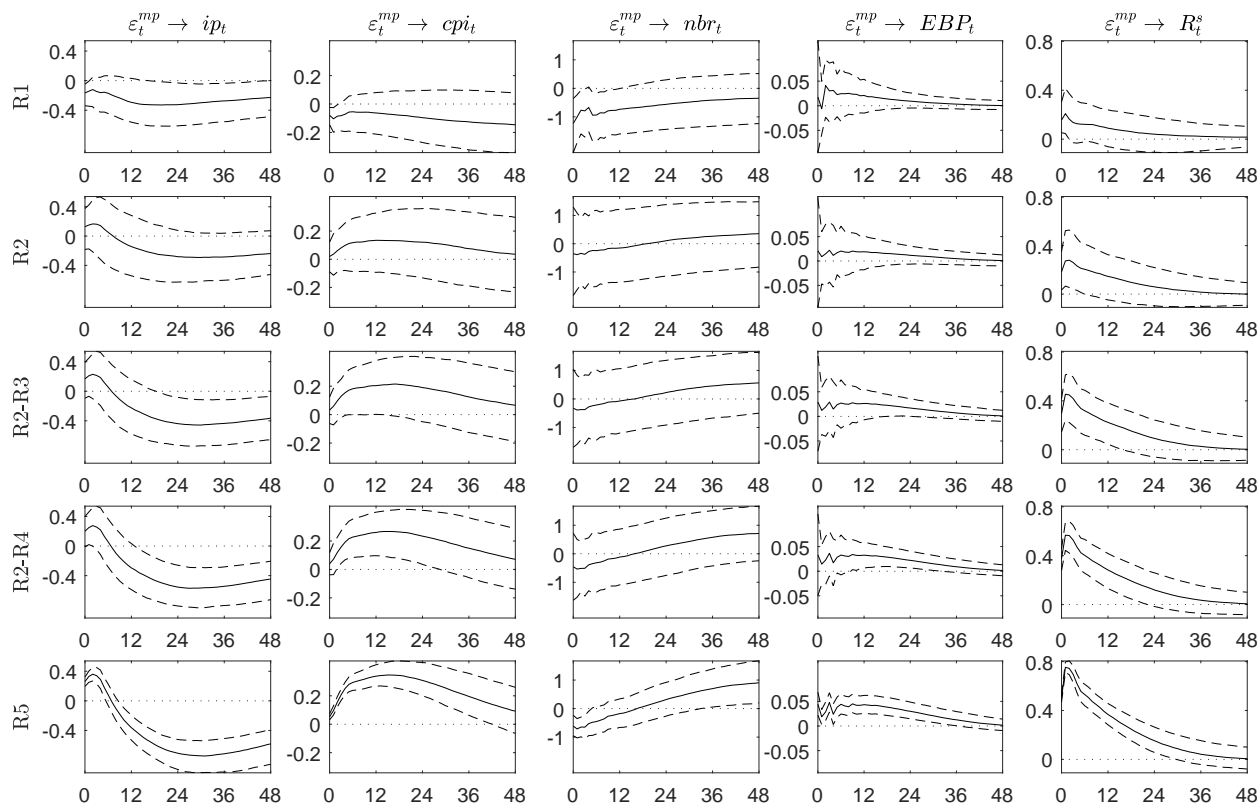


Figure C.3: Impulse responses in the monetary policy SVAR obtained by using different identifying restrictions. The rows (top to bottom) of the figure show results from using the restriction sets R1, R2, R2+R3, R2+R3+R4, and R5, respectively (see Section 3.2 for details). Posterior median (solid line) and 68% posterior credibility sets (dotted lines). Sample period: 1973M07-2007M12.