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Modeling the Value of Flexible Heat Pumps

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by

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Abstract

Demand side management has been proposed as one means to improve integration of high shares of non-dispatchable renewables-based electricity generation. Among other technologies, residential heat pumps are seen as suitable application for demand side management. Thus, we present a model of an air-water heat pump combined with a thermal energy storage. This heating system supplies heat to a floor-heated building. All subsystem dynamics (thermal energy storage, heating zone, water circuit of the floor heating, heated floor) are modeled in detail. That is dynamics are described by differential equations which are based on thermodynamic first principles. In terms of heat pump operation, we derive several model-predictive control strategies following different objectives. These strategies aim at the minimization of electricity consumption, procurement prices or operational costs. The difference in operational costs of the flexible operation and of the non-optimized operation represents the value of this flexibility option. The model is designed to assess manifold technical set-ups and economic conditions and, thereby, lays the foundation for further detailed analyses.

Keywords : Heat Pump, Model-Predictive Control, Real-Time Pricing, Demand Side Management.

JEL-Classification : Q41 (Q55).

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Contents

List of Figures	III
List of Tables	III
1. Introduction	1
1.1. Demand Side Management and Heat Pumps	1
1.2. Approaches to DSM of Space Heating Appliances	2
1.3. Modeling Approach	3
1.4. Focus and Structure of the Present Paper	4
2. Methodology	4
2.1. Model Predictive Control	4
2.2. System Behavior and Characteristics	5
2.2.1. Thermodynamic Behavior	5
2.2.2. Heat Pump Characteristics and Operation	8
2.3. Model of Future Inputs and Outputs	10
2.4. Objective Function and Control Sequence	12
2.5. Receding Strategy	14
3. Test Case	14
4. Results	16
4.1. Annual Figures	16
4.2. Thermal Behavior of the System	17
4.3. Forecasts and Operation Decisions	18
4.4. Operational Cost Savings and Investments	20
5. Conclusions	22
References	VII
A. Derivation of Stationary Temperature Profile	XII

List of Figures

1. Schematic illustration of the considered thermodynamic system.	5
2. Schematic illustration of a typical compensation curve	9
3. Sequence of forecasts and operation decision.	14
4. Illustration of varying electricity prices and the ECC for the period of January 2015.	16
5. Thermal behavior of subsystems (exemplary period from January 6th to January 25th).	17
6. (Forecast) temperatures and prices on February 16th at 0.00 am.	19
7. (Forecast) temperatures and prices on February 18th at 0.00 am.	19
8. Heat price and operation forecasts on February 16th at 0.00 am.	19
9. Heat price and operation forecasts on February 18th at 0.00 am.	19
10. Actual operation on February 16th.	19
11. Actual operation on February 18th.	19
12. Illustration of changes in electricity consumption using different strategies and different forecast qualities in comparison to the naive operation.	20
13. Illustration of changes in electricity costs using different strategies and different forecast qualities in comparison to the naive operation.	21

List of Tables

1. Summary of building characteristics.	14
2. Summary of HP characteristics. Temperatures in parentheses indicate the ambient and supply temperature at which these nominal values are achieved: $T_{amb} = 7^{\circ}C$ and $T_s = 35^{\circ}C$	15
3. Summary of technical results of the simulation of the base case.	16
4. Summary of economic results of the simulation of the base case.	17
5. Operation statistics of the simulation of the base case.	17

Abbreviations

COP	coefficient of performance
DSM	demand side management
ECC	end consumer charge
HE	(auxiliary) heat element
HP	heat pump
HVAC	heating, ventilation and air conditioning
KKT	Karush-Kuhn-Tucker
LP	linear program
MPC	model predictive control
PV	photovoltaics
RES	renewable energy sources
TES	thermal energy storage
ToU	time-of-use
VARMA	vector autoregressive moving average
VAT	value added tax

Symbols

A	coefficient matrix of state space representation
α	parameter for the price forecast
A_{f-z}	surface of heated floor subject to heat transfer to heating zone
a_j	parameters for the polynomial approximating $\dot{Q}_{hp,nom}$
A_p	cross section of piping of the floor heating
$A_{tes-sur}$	surface of TES subject to heat transfer to its surrounding
A_{w-f}	inner surface of piping of the floor heating subject to heat transfer to the heated floor
B₁	coefficient matrix for the control vector
B₂	coefficient matrix for the excitation vector
β_i	error weight for the price forecast
b_j	parameters for the polynomial approximating $P_{hp,nom}$
$\tilde{C}_t^{24}$	total expected operational costs for the next 24 hours
c_c	specific heat capacity of concrete screed
c_w	specific heat capacity of water
C_z	absolute heat capacity of heating zone
$\Delta E_{sys,t}$	term for heat demand forecast taking into account the lack of sensible heat of the system
Δt	time increment (1 hour)
$\dot{E}_z$	term for mass flow control of the circulation pump taking into account the lack of sensible heat of the building
ϵ	forecast error of the ambient temperature forecast (index T) / electricity price forecast (index p)
i	superscript indicating the look-ahead hour
I	set of hours of look-ahead horizon (including $i = 0$)

I'	set of hours of look-ahead horizon reduced by $i = 24$
η_{he}	efficiency of the heating element
f_{num}	numerical constant (= 1.2)
$\mathcal{L}$	Lagrange function
$\tilde{\lambda}_{d,t}^{i,*}$	Lagrange multiplier for the demand constraint
$\tilde{\lambda}_{he,t}^{i,*}$	Lagrange multiplier for the capacity constraints of the HE
$\tilde{\lambda}_{hp,t}^{i,*}$	Lagrange multiplier for the capacity constraints of the HP
l_p	length of piping of the floor heating
$\dot{m}_w$	mass flow of water through the piping of the floor heating
p_{el}	electricity price
P_{he}	electricity consumption of the heating element
$P_{he,nom}$	nominal electricity consumption of the heating element
P_{hp}	electricity consumption of the heating element
$P_{hp,nom}$	nominal electricity consumption of the heating element
$\dot{Q}_{z,dem}$	heat demand of the building
$\dot{Q}_{he}$	usable heat transfer from the HE to the TES
$\dot{Q}_{hp}$	usable heat transfer from the HP to the TES
$\dot{Q}_{hp,nom}$	nominal usable heat transfer from the HP to the TES
$\tilde{Q}_{sys,dem,t}^{24}$	forecast heat demand of the system for the next 24 hours
$\tilde{Q}_{tes,loss,t}^{24}$	forecast heat losses of the TES during the next 24 hours
$\tilde{Q}_{z,dem,t}^{24}$	forecast building heat demand for the next 24 hours
$\dot{Q}_{int}$	internal heat returns to the building
$\dot{Q}_{sol}$	solar heat returns to the building
ρ_c	density of concrete screed
ρ_w	density of water
t	time step of simulation
$\mathbf{T}$	vector of state variables
T_{amb}	ambient temperature
T_{comf}	comfort temperature (= set value of the zone temperature)
T_f	temperature of heated floor (concrete screed)
T_f^{ref}	reference temperature of heated floor (concrete screed)
τ_{ht}	characteristic time constant for heat transfer
τ_{pump}	time constant for mass flow control (of the circulation pump)
T_s	supply temperature of the HP
τ_{so}	time constant for convergence of $T_{w,out}$ to its stationary equilibrium
T_{sur}	temperature of the surrounding of the TES
T_{tes}	temperature of the water inside the TES
T_{tes}^{ref}	reference temperature of water inside the TES
$\bar{T}_w$	average temperature of the water running through the piping of the floor heating
$T_{w,in}$	water temperature at pipe inlet
$T_{w,out}$	water temperature at pipe outlet
T_z	heating zone temperature
$\mathbf{u}$	control vector in state space representation
U_{f-z}	heat transfer coefficient between heated floor and heating zone
$U_{tes-sur}$	heat transfer coefficient between TES and its surrounding

U_{w-f}	heat transfer coefficient between water inside the piping of the floor heating and the heated floor
$\mathbf{v}$	excitation vector in state space representation
V_c	volume of concrete screed
V_{tes}	filling volume of TES
$\sim$	indicator for forecast values

1. Introduction

During the last decades, an increased use of renewable energy sources (RES) in numerous regions of the world could be observed. The continuation of this trend is foreseen (cf. IEA 2016; BP 2017) also due to ongoing political efforts both on national and multi-national level (e.g. EEG 2017; EC 2017; UN 2015). To a high extent decentralized and non-dispatchable generation from technologies like photovoltaics (PV) and wind power has been and will have to be integrated into large-scale power systems. In some regions, this integration already imposes challenges to system operators as the location of variable supply frequently does not coincide with the place of consumption and as grid capacities are scarce. Symptoms of these challenges are preventive and curative congestion management measures. Examples of the latter ones are curtailment of RES or redispatch of conventional power plants which have increased significantly during the last years (cf. Lew et al. 2013; BNetzA and BKartA 2016; Fink et al. 2009). Further (envisaged) effects are the increase in short-term reserve requirements, the increase in balancing costs and network extension costs (cf. Holttinen 2012; dena 2012). There are several ways to face this challenge. In addition to the classical approach of reinforcing the transmission and distribution networks, one measure could be the use of demand side management (DSM) which is discussed subsequently.

1.1. Demand Side Management and Heat Pumps

Strbac 2008 describes the benefits of DSM as reducing the generation capacity margin, improving transmission grid investment and operation efficiency and improving distribution network investment. In simple words, these effects can be achieved by peak load shaving (i.e. avoiding expensive peak load production and not having to reserve peak capacity) or valley filling (i.e. making use of low-cost, possibly surplus energy) or combinations of both¹. The potentials of different DSM applications are assessed in e.g. Gils 2014 for Europe, dena 2010, Nabe et al. 2011, UBA 2015 and P. D. Lund et al. 2015 for Germany and Petrović and Karlsson 2016 for Denmark. Recurrently heat pumps (HPs) are named therein. For several reasons HPs are of interest: (i) They are relatively resource-efficient. Complemented by the future electricity mix tending to be more renewables-based, HPs represent a promising option for contributing to climate change mitigation (cf. Carvalho et al. 2015). (ii) The share of HPs in the heating market is foreseen to increase significantly. Industry sales numbers and prospects (cf. Nowak and Westring 2015; BWP 2011) are equally positive as the long-term forecasts of governmental institutions and researchers (cf. Prognos et al. 2014; Bauermann et al. 2014; Fehrenbach et al. 2014; Petrović and Karlsson 2016). Market share forecasts for HPs in the German heating market range from 10 % to slightly above 20 % in 2030 and from around 30 % to 50 % in 2050. In absolute terms, a market share of 50 % would mean an installed aggregate peak capacity of 67 GW_{el} in Germany (cf. Fehrenbach et al. 2014). Likewise HPs are expected to become the predominant heating technology among the individual heating sources in Denmark (cf. Petrović and Karlsson 2016). (iii) By installing a TES, the electricity consumption of an HP can be decoupled from space heating demand. In the case of well-insulated houses, the building may even offer enough buffer capacity so that the HP has a certain degree of flexibility even without TES. Some of the benefits of the inherent or added storage capacity are demonstrated in e.g. ETG 2015, Schubert and Sensfuß 2014, Papadaskalopoulos and Strbac 2013, Papadaskalopoulos, Strbac, et al. 2013 and Miara et al. 2014.

¹See e.g. Eid et al. 2016 for a more comprehensive depiction.

1.2. Approaches to DSM of Space Heating Appliances

Researchers have looked at DSM by space heating appliances from different perspectives. These perspectives can be clustered roughly into three categories of which the first two can be subdivided into two subcategories:

Technical system analyses: The first subgroup of technical system analyses considers DSM in distribution networks and usually models network operation (e.g. including load flows, voltage profiles, system losses) and its constraints with a high level of detail. As means of coordination usually technical signals are chosen. These can either be exchanged collaboratively (e.g. Zhang et al. 2016) or provided by a central controller (e.g. Brandstetter et al. 2017). Other methods such as modeling adjusted residual load profiles and assessing the resulting changes in load flows can also be found (cf. Veldman et al. 2013). The second subgroup of analyses considers a broader system. Eventually they incorporate the electricity, heating and transport sector of one or more countries (e.g. B. V. Mathiesen and H. Lund 2009, B. Mathiesen et al. 2015). In these analyses, the technical system cannot be modeled in such detail as in the first subgroup of analyses, but their strength is to grasp the interdependencies of sectors. The results of the first subgroup of these analyses tend to be very positive on the use of DSM by residential HPs and other appliances (peak demand reductions of up to 40 % and reductions of distribution grid investment costs of up to 72 % (cf. Veldman et al. 2013). The second subgroup sees individual HPs having a role in the RES integration, but, at least in the cited Danish case studies, assesses it to be smaller than the role of large-scale HPs connected to district heating grids.

Market-based analyses: The first subgroup of market-based analyses consists of studies solving short-term scheduling problems (commonly assessing a period of one year). A usual approach is to model a market clearing process which, under certain assumptions, can be formulated as a system cost minimization. DSM devices can either be integrated in the optimization problem (cf. Papadaskalopoulos and Strbac 2013, Papadaskalopoulos, Strbac, et al. 2013, Schubert and Sensfuß 2014 and Papaefthymiou et al. 2012) or scheduled sequentially (cf. Patteeuw et al. 2016). Variations of these analyses are presented in Steen et al. 2016 and Felten et al. 2018. Latter variant studies include a similarly-detailed representation of the distribution network as the first-mentioned technical type of analyses and solve scheduling problems of agents in a distribution network under certain pricing schemes². The second subgroup contains investment models which minimize total discounted system costs (cf. Bauermann et al. 2014, Fehrenbach et al. 2014, Petrović and Karlsson 2016, Hedegaard and Münster 2013). In order to make welfare-optimal investment decisions, these models use a reduced representation of a scheduling model³. The highly positive prospects of HP installations according to this method are described in sec. 1.1. The model types of both subgroups can be used to forecast the benefits of certain technology types. E.g. *ibid.* conclude that approximately 12 % of the Danish heat and power cost could be saved in 2030 by having deployed individual HPs. The contribution of their flexible operation is stated to be 0.9 %. Papaefthymiou et al. 2012 obtain similar results for Germany in 2030. Depending on the expected penetration levels of RES and other constraints, they calculate system cost reductions ranging from 0.4 to 1.5 %. To put these numbers into context, 0.5 % of German system costs are 50 M€/yr (cf. *ibid.*). In turn, Patteeuw et al. 2016's analysis yields higher

²The difference between both studies is that Steen et al. 2016 use exogenous prices (including energy-based and power-based tariffs) and Felten et al. 2018 solve a local market clearing problem which results in locational marginal prices.

³A common way of reducing complexity is e.g. the simulation of a set of representative weeks instead of a full year.

relative figures for a Belgian case with potential system cost reductions of up to 5.5 % and CO₂ emission reductions of up to 6.7 %.

Single-agent analyses: This kind of analyses considers a single space heating system (possibly complemented by a local RES generator) and uses certain control algorithms or heuristics based on different rationales. These rationales can be predominantly technical: E.g. Miara et al. 2014 use daily normalized residual load signals effecting set value of the temperature of the TES. Dar et al. 2014 propose a control algorithm aiming at maximizing self-consumption of PV generation. The exploitation of more economically-based signals, e.g. time-of-use (ToU) tariffs, is assessed by e.g. Arteconi et al. 2013 and Verhelst, Logist, et al. 2012. Under the employed static ToU, it is well-feasible to obtain substantial savings by applying an adequate control strategy. If the control algorithm includes objectives different from cost minimization (e.g. aiming at load shifting), this may come at the expense of higher electricity bills and investment costs (cf. Miara et al. 2014). Few authors have assessed flexible operation against (constructed) real-time prices (e.g. Oldewurtel et al. 2010; Georges et al. 2014; Dar et al. 2014).

In the context of real-time pricing, it is uncertain how time-variant tariffs will look like in the future. One uncertainty is imposed by the wholesale price of electricity, another by regulatory components (grid charges, levies and cost allocations). Several proposals have been made to replace time-invariant taxes and levies by so-called dynamic charges (e.g. Frontier and BET 2016 and Eid et al. 2016). In addition, DSM appliances may participate in various markets – at least if capacities are gathered by an aggregator: Day-ahead, intraday and reserve markets. For all of the afore-mentioned options, it is relevant how well a market participant can anticipate prices and weather. I.e. if either one of the HP operator’s forecasts is weak, his strategy is likely not to yield best outputs (and the beneficial DSM effects may be diminished).

All in all, there are manifold aspects which influence the business case of flexible HPs. Typically large-scale system models as summarized under the first two categories need to use relatively strong assumptions. On the technical side, appliances’ features, constraints and stochastic effects may be neglected or stylized. On the economic side, an efficient, non-discriminatory market is commonly presumed. This is partly due to computational efforts which also make it hardly manageable to calculate a larger set of scenarios. Hence in this paper, we develop a coherent single-agent approach how a smart HP owner (or aggregator⁴) is likely to use the HP’s flexibility. We derive this approach under first principles (explained in sec. 2). In the subsequent part, we give insights into the general idea and summarize precedent work in this field.

1.3. Modeling Approach

For assessing the behavior of a smart HP owner applying a cost-minimizing strategy, model predictive control (MPC) has proven to be an appropriate technique. Being part of control theory and having been developed for technical applications (cf. Camacho 2004) MPC is able to capture the underlying technical process (here: building dynamics, HP characteristics, etc.). However it can incorporate cost-minimizing objectives and thus be used to model smart agents. MPC has been applied successfully to heating, ventilation and air conditioning (HVAC) applications. Early works include Zaheer-Uddin 1992 and Zaheer-Uddin et al. 1997 who propose MPC algorithms for the optimal control of an HP with TES and for a boiler supplying an embedded-piping floor heating system respectively. Chen 2002 uses a predictive control method

⁴For brevity, we will only refer to the HP owner henceforth.

for a floor radiant heating system in a full-scale test room and compares different control methods by means of simulation models. He concludes that the predictive control is superior to two alternative models. More recent works are presented by Širok et al. 2011, Freire et al. 2008, Verhelst, Logist, et al. 2012, Verhelst, Degrauwe, et al. 2012, Georges et al. 2014 and Oldewurtel et al. 2010. Širok et al. 2011 present an MPC strategy and test it during a two-month experiment of buildings equipped with ceiling radiant heating systems. The energy savings potential is assessed to be significant (15 % to 28 %). Freire et al. 2008 propose a predictive control method for an HVAC system. They show that the system is capable of promoting thermal comfort and reducing energy consumption while respecting comfort constraints at all times. Verhelst, Logist, et al. 2012 describe four types of MPC formulations for an air-water HP (without TES) and compare these to simulations based on actual characteristic maps of the manufacturer. They use a combined objective function which minimizes heating costs and thermal discomfort. For calculating the heating costs, they use a day-night electricity tariff. In a similar paper, Verhelst, Degrauwe, et al. 2012 rather focus on the advantages of nonlinear optimization of the HP operation. Oldewurtel et al. 2010 use MPC to optimize the operation of standard cooling and heating appliances of several building types in Switzerland. They propose the usage of a time-varying, hourly electricity tariff which is constructed from the spot price of electricity, the load level and the city concession. They conclude that the use of this constructed tariff slightly affects costs of Swiss standard houses and significantly reduces the costs of passive houses (28.4 % to 31.2 %).

1.4. Focus and Structure of the Present Paper

As can be seen from the summary above, different authors have addressed ‘smart’ operation of HPs in different ways. In this paper, we take the single-agent perspective. Thereby we can describe the HP operation and the building dynamics by a model based on first principles and with an appropriate level of detail. In a nutshell, the model serves for assessing the operational cost savings of flexible HP operation. The modeling framework is designed to be capable of assessing the influence of manifold set-ups and conditions. This shall serve for extensive sensitivity analyses of operational cost savings of flexible HP operation.

In the subsequent section, we derive the used MPC algorithm. Thereby the control behavior and specific trade-offs are identified. The test case description is given in sec. 3. The validation of the test case and the results are illustrated in sec. 4. Finally in sec. 5, the results are discussed and the relevant conclusions are drawn.

2. Methodology

2.1. Model Predictive Control

According to Camacho 2004, MPC refers to no specific control strategy but a rather ample range of control methods which have several common characteristics. These are (i) the explicit use of a model to predict the process output at future time steps, (ii) the calculation of a control sequence minimizing an objective function, and (iii) a receding horizon strategy. Thus we structure the following sections along these general characteristics. Where we depart from traditional MPC designs or use certain assumptions we indicate it in the respective sections. Prior to the MPC, we describe the system dynamics and characteristics (building,

TES and HP) in sec. 2.2. Subsequently sec. 2.3 describes how the future outputs are modeled and future inputs are forecast. Sec. 2.4 describes the derivation of the control sequence.

2.2. System Behavior and Characteristics

2.2.1. Thermodynamic Behavior

We consider the system as shown in fig. 1. The system can be divided into four subsystems: Heating zone (z), the heated floor (f), water running through the pipes of the floor heating system (w) and the TES. The modeling of the system dynamics is explained subsequently.

Figure 1: Schematic illustration of the considered thermodynamic system.

Heating zone: Most importantly, this system comprises the air inside the building, but also all other items (furniture, electronic equipment, etc.) inside the house. It also comprises the walls (interior and exterior) and the roof. It does not contain the floors which are heated by the floor heating system. Thus the subsystem boundaries are the interfaces between the outer walls / roof to the ambient air (with temperature T_{amb}) and any connection between the inside of the building and the heated floor. We use the simplification that the temperature is homogeneous throughout the heating zone (lumped-capacity approximation). Using the first law of thermodynamics, the differential equation for the zone temperature T_z can be derived.

$$\frac{dT_z}{dt} = \frac{U_{f-z}A_{f-z}}{C_z} (T_f - T_z) - \frac{U_{z-amb}A_{z-amb}}{C_z} (T_z - T_{amb}) + \frac{1}{C_z} (\dot{Q}_{solar} + \dot{Q}_{int}) \quad (1)$$

Here U_{f-z} symbolizes the heat transfer coefficients (between floor and heating zone) and A_{f-z} the respective surfaces. C_z is the absolute heat capacity of the heating zone. $\dot{Q}_{sol}$ and $\dot{Q}_{int}$ are the solar respectively internal heat gains of the building.

Heated floor: This system mainly comprises the cement screed surrounding the pipes of the floor heating system. It also includes the floor cover and the floor heating pipes themselves, but without the water running inside these pipes. We assume that the thermal capacities of the floor cover and the piping are small compared to the thermal capacity of the cement screed. We use the lumped capacity approximation for the floor (T_f being homogeneous). Applying the first law of thermodynamics results in the following differential equation for the floor temperature T_f .

$$\frac{dT_f}{dt} = \frac{U_{w-f}A_{w-f}}{\rho_c V_c c_c} (\bar{T}_w - T_f) - \frac{U_{z-f}A_{z-f}}{\rho_c V_c c_c} (T_f - T_z) \quad (2)$$

U_{w-f} and A_{w-f} are used in analogy to equ. 1 with the difference that one heat transfer is from the water inside the pipes to the floor. ρ_c , V_c , and c_c are the density, volume and specific heat capacity of the concrete screed. $\bar{T}_w$ is the average water temperature in the pipes of the floor heating for which some more explanations are given in the subsequent paragraph.

Water running through the piping of the floor heating system: At the pipe inlet, the water has the temperature $T_{w,in}$. At the outlet, it has the temperature $T_{w,out}$. Its volumetric average temperature is symbolized by $\bar{T}_w$. We assume that pipes have a constant diameter and that the water temperature at any given piping cross section is uniform. Hence $\bar{T}_w$ is also the average temperature along the piping length. These assumptions (together with the ones further above) allow expressing the heat transfer using a simple temperature difference. Fig. 1 also illustrates the piping between floor heating and TES. These pipes are assumed to be perfectly insulated. We obtain the following equation.

$$\frac{d\bar{T}_w}{dt} = -\frac{U_{w-f}A_{w-f}}{l_p A_p \rho_w c_w} (\bar{T}_w - T_f) + \frac{\dot{m}_w}{l_p A_p \rho_w} (T_{w,in} - T_{w,out}) \quad (3)$$

l_p and A_p are the length and the cross section of piping respectively. $\dot{m}_w$ is the mass flow of water through the piping of the floor heating. ρ_w and c_w are the density and specific heat capacity of the water. If the floor heating is in operation ($\dot{m}_w > 0$), the inlet temperature is determined by the upper TES temperature (see next paragraph). For the same case, the outlet temperature of the piping is modeled to converge to its temperature profile in stationary operation. If the floor heating is not in operation, heat transfer from the concrete floor to the water (and vice versa) will assure that $\bar{T}_w$ converges to T_f (cf. appendix A for underlying assumptions and derivation of time constants). Both cases are implemented by using differential equations of first order.

$$\frac{dT_{w,out}}{dt} = \begin{cases} \frac{1}{\tau_{so}} \left(-T_{w,out} + T_{w,in} + \frac{U_{w-f}A_{w-f}}{c_w \dot{m}_w} (T_f - \bar{T}_w) \right) & \text{if the floor heating is in operation} \\ \frac{1}{\tau_{ht}} (T_f - T_{w,out}) & \text{otherwise} \end{cases} \quad (4)$$

τ_{ht} is the characteristic time constant of heat transmission. For τ_{so} (the characteristic time constant for the differential equation describing the convergence to the stationary output temperature), we use the time constant for heat transport multiplied by a numerical constant f_{num} . Equ. 5 and 6 illustrate the calculation.

$$\tau_{ht} = \frac{\rho_w c_w A_p l_p}{U_{w-f} A_{w-f}} \quad (5)$$

$$\tau_{so} = f_{num} \frac{\rho_w A_p l_p}{\dot{m}_w} \quad (6)$$

The water mass flow $\dot{m}_w$ is assumed to be controlled in accordance with the heat demand of the building $\dot{Q}_{z,dem}$ (e.g. by a high efficiency circulation pump).

$$\begin{aligned} \frac{d\dot{m}_w}{dt} &= \frac{1}{\tau_{pump}} \left(-\dot{m}_w + \frac{\dot{E}_z + \dot{Q}_{z,dem}}{c_w (T_{w,in} - T_{w,out})} \right) \\ &= -\frac{\dot{m}_w}{\tau_{pump}} + \frac{\max \left(\frac{C_z}{\Delta t} (T_z - T_{comf}) + U_{z-amb} A_{z-amb} (T_{comf} - T_{amb}) - \dot{Q}_{solar} - \dot{Q}_{int}, 0 \right)}{\tau_{pump} c_w (T_{w,in} - T_{w,out})} \end{aligned} \quad (7)$$

T_{comf} is the comfort temperature, i.e. the set value of the heating zone temperature T_z . τ_{pump} is the characteristic time constant of the mass flow control. The term $\dot{E}_z$ assures that the mass flow tends to compensate the absolute lack in sensitive heat stored by the building within the next time step Δt (= 1 hour).

Thermal energy storage: This system contains the water inside the cylindrical storage container and the container itself. The container's heat capacity is considered to be small compared to the water's heat capacity. It does not include the piping of the HP which is used to transfer the heat to the water inside the TES. Thus its system boundary is the outer surface of the container to its surrounding (with temperature T_{sur}), the pipe connections for the floor heating water and the surfaces of the HP piping. Again we use the first law of thermodynamics to derive the describing differential equation for the TES temperature T_{tes} .

$$\frac{d\bar{T}_{tes}}{dt} = -\frac{U_{tes-sur} A_{tes-sur}}{\rho_w c_w V_{tes}} (\bar{T}_{tes} - T_{sur}) - \frac{\dot{m}_w}{\rho_w V_{tes}} (T_{w,in} - T_{w,out}) + \frac{\dot{Q}_{hp} + \dot{Q}_{he}}{\rho_w c_w V_{tes}} \quad (8)$$

$U_{tes-sur}$ and $A_{tes-sur}$ are used in the same way as above. V_{tes} is the filling volume of the TES. We consider a stratified storage (as illustrated by e.g. Arteconi et al. 2013). We use the simplification that the temperature averaged over the heat loss areas $A_{tes-sur}$ is equal to the volume averaged temperature. This implies certain conditions on the temperature profiles and geometries, but represents a good approximation for the given configuration. $\dot{Q}_{hp}$ and $\dot{Q}_{he}$ are the heat inputs from the HP and the heat element (HE) respectively. These are explained in sec. 2.2.2.

Equ. 1 to 8 can be written as a system of differential equations of first order⁵.

$$\dot{\mathbf{T}} = \mathbf{A}\mathbf{T} + \mathbf{B}_1\mathbf{u} + \mathbf{B}_2\mathbf{v} \quad (9)$$

The vector of state variables $\mathbf{T}$ contains the five temperatures T_z , T_f , $\bar{T}_w$, $T_{w,out}$ and T_{tes} in this order. $\mathbf{T} \in \mathbb{R}^{5 \times 5}$ is the coefficient matrix giving the state variables' influence on their speed of change. $\mathbf{u}$ is the control vector ($\mathbf{u} = (\dot{Q}_{hp}, \dot{Q}_{he})^T$). $\mathbf{v}$ is the excitation vector which contains the external influencing parameters ($\mathbf{v} = (T_{sur}, T_{amb}, T_{w,in} = f(T_s), \dot{Q}_{sol}, \dot{Q}_{int})^T$). $T_{w,in} = f(T_s)$ indicates that the inlet temperature of the floor heating depends on the supply temperature T_s of the HP. $\mathbf{B}_1 \in \mathbb{R}^{5 \times 2}$ and $\mathbf{B}_2 \in \mathbb{R}^{5 \times 5}$ are the coefficient matrices for the control and excitation vector respectively. It should be noted that some of the elements of $\mathbf{A}$ and $\mathbf{B}_2$ have a time dependency which is due to the operational status of the floor heating (e.g. $\dot{m}_w > 0$ or $\dot{m}_w = 0$). Several of the excitation vector's variables are straightforward. E.g. a well-known method for determining the solar gains $\dot{Q}_{sol}$ from irradiance data and building properties is given in Quaschnig 1996. This method is implemented and the used parameters for this calculation are given in sec. 3. For

⁵It can be interpreted as state space representation with an identity matrix as output matrix.

the internal gains $\dot{Q}_{int}$, a constant value per square meter according to DIN 4108-6: 2003 is applied. The surrounding temperature of the thermal storage tank T_{sur} is chosen to be constant in analogy to Schmidt et al. 2010. The ambient temperature T_{amb} is the prevalent temperature time series at the considered location. The elements of the control vector $\mathbf{u}$ can be manipulated (operation decision of the HP and HE). The values that they can take depend on the HP characteristics and operation conditions which are addressed in sec. 2.2.2.

2.2.2. Heat Pump Characteristics and Operation

As stated above, the HP is assumed to be an air-water HP⁶. This type of HP uses the ambient air as heat source and heats a heat transfer fluid. The heated fluid is compressed by an electrically-driven compressor. This compression lifts the temperature level of the heat transfer fluid to a range where it is utilizable for heating a conventional building heating circuit. Thus part of the sensible heat of the heat transfer fluid is usually transferred to the water circuit of the building heating. In our case, the heating circuit contains a TES which decouples the operation of the floor heating from the HP operation, but principally does not make a difference in HP modeling. After transferring the heat to the water circuit, the heat transfer fluid is expanded which shifts the temperature level below ambient temperature T_{amb} and which makes it capable of receiving heat from the environment. The technical process is well-known and documented e.g. by Bonin 2012. However it implies some characteristics which are important for the present analysis:

- The lower the ambient temperature the higher are the required temperature lift and the corresponding pressure lift. This implies that the required compressor power increases relative to the heat transfer to the building's heating circuit.
- For the same reason, it is useful to operate at low supply temperatures T_s . Floor heating systems have relatively big heating surfaces (compared to conventional radiator heating systems). Hence the required average temperature difference between the floor heating and the heating zone ($T_f - T_z$) can be chosen to be small and so can the supply temperature T_s .
- Even though floor heating systems require lower supply temperatures, the heating circuit is typically laid out for a maximum circulation rate of water. In order to meet the demand of the building at periods with low ambient temperatures, a higher temperature difference ($T_f - T_z$) is required. These circumstances are commonly taken into account by a so-called compensation curve (cf. Schmidt et al. 2010 or Arteconi et al. 2013). A typical example of such a compensation curve is shown in fig. 2.

⁶According to BWP 2011 air-water HPs are expected to become the predominantly-installed HP technology in Germany and already are in other markets (Sweden and Switzerland).

Figure 2: Schematic illustration of a typical compensation curve (cf. Schmidt et al. 2010 and Arteconi et al. 2013).

- Further non-linearities originate from other technicalities (e.g. compressor efficiencies at different loads).

The ratio of the usable heat provided to the heating circuit / TES ($\dot{Q}_{hp}$) and the electricity consumption of the HP⁷ (P_{hp}) is expressed by the coefficient of performance COP.

$$COP = \frac{\dot{Q}_{hp}}{P_{hp}} \approx f(T_s, T_{amb}) \quad (10)$$

As indicated in equ. 10, we take into account the most relevant conditions influencing the COP, i.e. the supply temperature T_s and the ambient temperature T_{amb} . Likewise the nominal inputs and outputs depend on these conditions. I.e. in general, the maximum possible inputs / outputs vary with time (with varying operation conditions). We model this dependency by polynomials of second order (cf. equ. 11 and 12).

$$\dot{Q}_{hp,nom} = a_1 + a_2 T_{amb} + a_3 T_s + a_4 T_{amb} T_s + a_5 T_{amb}^2 + a_6 T_s^2 \quad (11)$$

$$P_{hp,nom} = b_1 + b_2 T_{amb} + b_3 T_s + b_4 T_{amb} T_s + b_5 T_{amb}^2 + b_6 T_s^2 \quad (12)$$

We do not consider the effect of load changes or start-ups. In order not to overstate the flexibility of the HP, we only provide the possibility of making operation decisions on an hourly basis. Furthermore the efficiency of heat transfer to the TES is assumed to be constant. The approximation of the characteristic map is similar to those in Verhelst, Degrauwe, et al. 2012 and Gayeski 2010⁸. The parameters a_j to b_j are determined from manufacturer data which are illustrated in more detail in sec. 3. As explained above, we model a stratified storage. The upper temperature of the TES and thus the inlet water temperature of the floor heating $T_{w,in}$ are assumed to be slightly lower than the mean supply temperature T_s during the last hours of HP operation. The HP can also be operated in bivalent mode. I.e. an auxiliary HE can be used to increase the heat supply to the TES.

$$\dot{Q}_{he} = \eta_{he} P_{he} \quad (13)$$

η_{he} is the efficiency of the HE which is assumed to be constant. P_{he} is the electricity consumption of the HE.

⁷I.e. mainly of its compressor.

⁸Note that in (some of) their algorithms Verhelst, Degrauwe, et al. 2012 and Gayeski 2010 explicitly consider the compressor frequency which we do not.

2.3. Model of Future Inputs and Outputs

In order to minimize operational costs of the HP, a smart operator needs to have knowledge of the current system state, best-possible forecasts of the future external conditions (input model) and an understanding of the consequences of his operation decisions (output model) and of their constraints. The first requirement is assumed to be fulfilled (e.g. by use of sensors). The second and third one are explained in the following paragraphs. For all forecasts, we use the following convention in terms of notation: Forecast values are indicated by a $\sim$ -operator. The time index is the information time, i.e. the time up to which all operation states and external influences and parameters are known. If this index is t , we refer to a forecast made at the actual time step of the simulation. $t - 1$ indicates a forecast made one time step before (and so forth). Henceforth we always index time-dependent values with t ⁹. The superscript (e.g. i) indicates the look-ahead time, i.e. for how many hours after the information time the value is forecast. The look-ahead horizon is 24 hours.

Forecasting inputs: In terms of input modeling, the elements of the excitation vector need to be taken into account ($T_{sur}, T_{amb}, T_{w,in} = f(T_s, \dot{Q}_{sol}, \dot{Q}_{int})$). The forecast of T_{sur} is trivial as it is constant. The forecast of T_{amb} is described in the subsequent paragraph. $T_{w,in}$ depends on T_s which can be derived from the forecast of T_{amb} (cf. compensation curve in sec. 2.2.2). For $\dot{Q}_{sol}$ and $\dot{Q}_{int}$, we use a perfect foresight assumption, but in principle these values could be forecast in a similar manner as T_{amb} . For the ambient temperature, a vector autoregressive moving average (VARMA) process of order (1,1) is proposed:

$$\tilde{T}_{amb,t}^i = \begin{cases} T_{amb,t} & \text{for } i = \{0, 24\} \\ \tilde{T}_{amb,t-1}^{i+1} + \epsilon_{T,t-1}^1 & \text{for } i \in [1, 23] \end{cases} \quad (14)$$

$\epsilon_{T,t-1}^1$ is the error of the temperature forecast made at $t - 1$ for the next hour ($i = 1$). At information time t this error is known ($\epsilon_{T,t-1}^1 = T_{amb,t} - \tilde{T}_{amb,t-1}^1$) and is used for forecast adjustment. In a VARMA process these errors are assumed to be normally distributed ($\sim \mathcal{N}(0, \sigma^2)$). In matrix notation, the dimensions of the process become apparent.

$$\begin{pmatrix} \tilde{T}_{amb,t}^{24} \\ \tilde{T}_{amb,t}^{23} \\ \vdots \\ \tilde{T}_{amb,t}^1 \\ \tilde{T}_{amb,t}^0 \end{pmatrix} = \begin{pmatrix} 0 & \cdots & 0 & 0 & 0 \\ 1 & \ddots & \ddots & \vdots & \vdots \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & 1 & 0 & \vdots \\ 0 & \cdots & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{T}_{amb,t-1}^{24} \\ \tilde{T}_{amb,t-1}^{23} \\ \vdots \\ \tilde{T}_{amb,t-1}^1 \\ \tilde{T}_{amb,t-1}^0 \end{pmatrix} + \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \\ 0 \end{pmatrix} \epsilon_{T,t-1}^1 + \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} T_{amb,t} \quad (15)$$

In addition to the entries of the excitation vector (which describe the external influences on system dynamics), there are more inputs to the optimization strategy. I.e. the efficiencies and maximum requestable power of HP and HE must be anticipated. The COP and the nominal electricity consumptions $P_{hp,nom}$ are forecast under use of the T_{amb} forecast, the compensation curve (cf. 2.2.2) and equ. 10 to 12. The HE's efficiency η_{he} and its nominal electricity consumption $P_{he,nom}$ are constant and do not require elaborate

⁹For readability, t has been omitted in the equations of previous sections.

forecasting ($\tilde{P}_{he,nom,t}^i = P_{he,nom}$). This is different to the hourly electricity prices p_{el} which are forecast in a manner similar to the T_{amb} forecast.

$$\tilde{p}_{el,t}^i = \begin{cases} p_{el,t} & \text{for } i = \{0, 24\} \\ \tilde{p}_{el,t-1}^{i+1} + \underbrace{\epsilon_{p,t-1}^1 \frac{(1-\alpha)\alpha^i}{(1-\alpha^{i+1})}}_{\beta_i} & \text{for } i \in [1, 23] \end{cases} \quad (16)$$

Analogously $\epsilon_{p,t-1}^1$ is the price forecast error for the next hour ($= p_{el,t} - \tilde{p}_{el,t-1}^1$). α is a constant parameter which is used to calculate the error weights β_i for the price forecasts. α is chosen to be 0.9. Again we provide the process in matrix notation.

$$\begin{pmatrix} \tilde{p}_{el,t}^{24} \\ \tilde{p}_{el,t}^{23} \\ \vdots \\ \tilde{p}_{el,t}^1 \\ \tilde{p}_{el,t}^0 \end{pmatrix} = \begin{pmatrix} 0 & \cdots & 0 & 0 & 0 \\ 1 & \ddots & \ddots & \vdots & \vdots \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & 1 & 0 & \vdots \\ 0 & \cdots & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{p}_{el,t-1}^{24} \\ \tilde{p}_{el,t-1}^{23} \\ \vdots \\ \tilde{p}_{el,t-1}^1 \\ \tilde{p}_{el,t-1}^0 \end{pmatrix} + \begin{pmatrix} \beta_{24} \\ \beta_{23} \\ \vdots \\ \beta_1 \\ 0 \end{pmatrix} \epsilon_{t-1}^1 + \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} p_{el,t} \quad (17)$$

For both forecasts ($\tilde{T}_{amb,t}^i$ and $\tilde{p}_{el,t}^i$), perfect foresight solutions are also implemented and tested. These sensitivities are discussed and assessed in sec. 4.3 and 4 respectively. Within the literature presented in sec. 1.3, there are few MPC algorithms which use (imperfect) predictions. E.g. from the literature presented in sec. 1, Širok et al. 2011 is the only simulation-based test case which uses weather predictions. Usually price predictions are not necessary because static ToU tariffs are considered. Only Oldewurtel et al. 2010 use a least-square support vector machine in order to forecast the constructed hourly tariff.

Output model: Probably the most relevant output assumption is that the operation decisions of the HP have little effect on the control of the operation of the floor heating system (cf. equ. 7). This is due to the fact that the control of the floor heating system is decoupled from the operation of the HP through the TES. Thus a good approximation of the zone temperature T_z is the comfort temperature $T_{com,f}$ (independent of the HP operation). With this output assumption, the future heat demand of the system during the next 24 hours $\tilde{Q}_{sys,dem,t}^{24}$ can be forecast as follows:

$$\begin{aligned} \tilde{Q}_{sys,dem,t}^{24} &= \tilde{Q}_{z,dem,t}^{24} + \tilde{Q}_{tes,loss,t}^{24} + \Delta E_{sys,t} \\ &= \sum_{i=0}^{23} \left(U_{z-amb} A_{z-amb} \left(T_{com,f} - \tilde{T}_{amb,t}^i \right) - \tilde{Q}_{solar,t}^i - \tilde{Q}_{int,t}^i \right) \Delta t \\ &\quad + \sum_{i=0}^{23} U_{tes-sur} A_{tes-sur} \left(\tilde{T}_{tes,t}^i - T_{sur} \right) \Delta t + \Delta E_{sys,t} \end{aligned} \quad (18)$$

$\tilde{Q}_{z,dem,t}^{24}$ is the forecast heat demand of the building. Assuming that the set value of the zone temperature can be achieved to a reasonable extent, it is sufficient to forecast the ambient temperature and the solar and internal gains (see above explanation). The time increment Δt is 1 hour. $\tilde{Q}_{tes,loss,t}^{24}$ are the forecast heat losses of the TES. They depend on the expected storage temperature. As $\tilde{Q}_{z,dem,t}^{24} \gg \tilde{Q}_{tes,loss,t}^{24}$ for the purpose of forecasting $\tilde{Q}_{sys,dem,t}^{24}$, T_{tes} can be assumed to stay at a medium level. $\Delta E_{sys,t}$ is a system energy deficit which describes the difference to the set value or a reference value (T_{tes}^{ref} and T_f^{ref}) respectively. This

term renders periodic boundary conditions (as e.g. used by Verhelst, Logist, et al. 2012) unnecessary. It can be expressed as follows:

$$\begin{aligned} \Delta E_{sys,t} = & \rho_w c_w V_{tes} (T_{tes}^{ref} - T_{tes,t}) \\ & + \rho_c c_c V_c (T_f^{ref} - T_{f,t}) + C_z (T_{comf} - T_{z,t}) + c_w \rho_w A_{plp} (\bar{T}_w^{ref} - \bar{T}_{w,t}) \end{aligned} \quad (19)$$

2.4. Objective Function and Control Sequence

We simulate the price-responsive operation of an HP. That is the operation decisions are taken with the aim of variable cost minimization given the available information at time t . Variable costs mainly arise (and we assume only arise) from electricity costs¹⁰. The total operational costs during the actual hour t and the 23 following hours of the forecast horizon expected at time t are symbolized by $\tilde{C}_t^{24}$.

$$\tilde{C}_t^{24} = \sum_{i=0}^{23} \tilde{p}_{el,t}^i (\tilde{P}_{hp,t}^i + \tilde{P}_{he,t}^i) \Delta t \quad \text{with } \tilde{P}_{hp,t}^i \in [0, \tilde{P}_{hp,nom,t}^i] \text{ and } \tilde{P}_{he,t}^i \in [0, P_{he,nom}] \quad (20)$$

$\tilde{P}_{hp,nom,t}^i$ is the nominal electricity consumption expected at time t for i hours ahead. $P_{he,nom}$ is the nominal electricity consumption of the heating element (which is constant over time). By allowing the HP and HE to operate close to 0, no explicit technical minimum load constraint is implemented. However the upcoming derivation and the results in sec. 4.1 show that the optimization only yields very few occasions where such minimum load constraint would be relevant. In addition, part load operation can also be seen as operation at nominal load for a duration shorter than 1 hour (which is the time step for the operation decision, cf. sec. 2.2.2).

A cost-efficient operation strategy aims at minimizing operation costs given the information at time t while fulfilling the restriction of providing sufficient heat to the building during the next 24 hours. This can be written as linear program (LP).

$$\begin{aligned} & \min_{\tilde{P}_{hp,t}^i, \tilde{P}_{he,t}^i} \tilde{C}_t^{24} \\ s.t. & \sum_{i=0}^{23} (\tilde{Q}_{hp,t}^i + \tilde{Q}_{he,t}^i) \Delta t = \tilde{Q}_{sys,dem,t}^{24} \\ & \tilde{P}_{hp,t}^i, \tilde{P}_{he,t}^i \geq 0 \quad \forall i \in I' = \{0, \dots, 23\} \\ & \tilde{P}_{hp,t}^i \leq \tilde{P}_{hp,nom,t}^i \quad \forall i \in I' \\ & \tilde{P}_{he,t}^i \leq P_{he,nom} \quad \forall i \in I' \end{aligned} \quad (21)$$

I indicates the set of hours of look-ahead horizon (including $i = 0$). I' is the same set reduced by $i = 24$. Hence we have 48 decision variables. Equ. 21 represents a special case of classical MPC objective functions. Usually these contain one summand which increases with the deviation of the output vector from its set values. This summand is weighted for each time step of the look-ahead horizon as is a second summand that increases with the control action (cf. Camacho 2004). In our case, we set the weights of the first summand to 0 as we have a fast-reacting control of the floor heating system which is decoupled from the HP control. The weights of the second summand are the forecast electricity prices. Such formulation

¹⁰E.g. we do not consider variable operation and maintenance costs, start-up costs, etc.

is quite similar to common economic optimization problems. We use equ. 21, formulate the Lagrangian function $\mathcal{L}$ and derive the Karush-Kuhn-Tucker (KKT) conditions.

$$\mathcal{L} = \sum_{i=0}^{23} \tilde{p}_{el,t}^i \left(\tilde{P}_{hp,t}^{i,*} + \tilde{P}_{he,t}^{i,*} \right) \Delta t + \tilde{\lambda}_{d,t}^* \left(\tilde{Q}_{sys,dem,t}^{24} - \sum_{i=0}^{23} \left(\tilde{P}_{hp,t}^{i,*} C\tilde{O}P_t^i + \tilde{P}_{he,t}^{i,*} \eta_{he} \right) \Delta t \right) + \sum_{i=0}^{23} \tilde{\lambda}_{hp,t}^{i,*} \left(\tilde{P}_{hp,t}^{i,*} - \tilde{P}_{hp,nom,t}^i \right) + \sum_{i=0}^{23} \tilde{\lambda}_{he,t}^{i,*} \left(\tilde{P}_{he,t}^{i,*} - \tilde{P}_{he,nom} \right) \quad (22)$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{P}_{hp,t}^{i,*}} = \tilde{p}_{el,t}^i \Delta t - \tilde{\lambda}_{d,t}^* C\tilde{O}P_t^i \Delta t + \tilde{\lambda}_{hp,t}^{i,*} \geq 0 \quad \perp \quad \tilde{P}_{hp,t}^{i,*} \geq 0 \quad \forall i \in I' \quad (23)$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{P}_{he,t}^{i,*}} = \tilde{p}_{el,t}^i \Delta t - \tilde{\lambda}_{d,t}^* \eta_{he} \Delta t + \tilde{\lambda}_{he,t}^{i,*} \geq 0 \quad \perp \quad \tilde{P}_{he,t}^{i,*} \geq 0 \quad \forall i \in I' \quad (24)$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{\lambda}_{d,t}^*} = \tilde{Q}_{sys,dem,t}^{24} - \sum_{i=0}^{23} \left(\tilde{P}_{hp,t}^{i,*} C\tilde{O}P_t^i + \tilde{P}_{he,t}^{i,*} \eta_{he} \right) \Delta t = 0 \quad \perp \quad \tilde{\lambda}_{d,t}^* \quad (25)$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{\lambda}_{hp,t}^{i,*}} = \tilde{P}_{hp,nom,t}^i - \tilde{P}_{hp,t}^{i,*} \geq 0 \quad \perp \quad \tilde{\lambda}_{hp,t}^{i,*} \geq 0 \quad \forall i \in I' \quad (26)$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{\lambda}_{he,t}^{i,*}} = \tilde{P}_{he,nom} - \tilde{P}_{he,t}^{i,*} \geq 0 \quad \perp \quad \tilde{\lambda}_{he,t}^{i,*} \geq 0 \quad \forall i \in I' \quad (27)$$

In equ. 22 to 27, $\tilde{\lambda}_{hp,t}^{i,*}$, $\tilde{\lambda}_{he,t}^{i,*}$ and $\tilde{\lambda}_{d,t}^*$ are the Lagrange multipliers (shadow prices) of the capacity constraints of the HP, the capacity constraints of HE and of heat demand constraint respectively. Following case distinction can be made for the HP operation during hour i :

Case 1:	$\tilde{\lambda}_{hp,t}^{i,*} = 0$	
Case 1.1:	with equ. 23 $\wedge \tilde{P}_{hp,t}^{i,*} = 0$	(HP not in operation) $\Rightarrow \tilde{\lambda}_{d,t}^* \leq \frac{\tilde{p}_{el,t}^i}{C\tilde{O}P_t^i}$
Case 1.2:	with equ. 23 $\wedge \tilde{P}_{hp,t}^{i,*} > 0$	(HP in operation) $\Rightarrow \tilde{\lambda}_{d,t}^* = \frac{\tilde{p}_{el,t}^i}{C\tilde{O}P_t^i}$
Case 2:	$\tilde{\lambda}_{hp,t}^{i,*} > 0$	with equ. 26 $\Rightarrow$ HP in operation at maximum load
		with equ. 23 $\Rightarrow \frac{\tilde{p}_{el,t}^i}{C\tilde{O}P_t^i} < \tilde{\lambda}_{d,t}^* = \frac{\tilde{p}_{el,t}^i \Delta t + \tilde{\lambda}_{hp,t}^{i,*}}{C\tilde{O}P_t^i \Delta t}$

An analogous case distinction can be made for the HE. Thus we derive the expected shadow price of heat demand $\tilde{\lambda}_{d,t}^*$.

$$\tilde{\lambda}_{d,t}^* = \begin{cases} \frac{\tilde{p}_{el,t}^i}{C\tilde{O}P_t^i} & \text{if } i \text{ is the hour of marginal heat supply by the HP} \\ \frac{\tilde{p}_{el,t}^i}{\eta_{he}} & \text{if } i \text{ is the hour of marginal heat supply by the HE} \\ 0 & \text{otherwise} \end{cases} \quad (28)$$

Consequently the shadow price defines the threshold for the operation decision of the HP and HE at time t which can be formulated as follows:

If $\frac{\tilde{p}_{el,t}}{C\tilde{O}P_t} < \tilde{\lambda}_{d,t}^*$ (or analogously $\frac{\tilde{p}_{el,t}}{\eta_{he}} < \tilde{\lambda}_{d,t}^*$):	HP (HE) in operation at maximum load
If $\frac{\tilde{p}_{el,t}}{C\tilde{O}P_t} = \tilde{\lambda}_{d,t}^*$ (or analogously $\frac{\tilde{p}_{el,t}}{\eta_{he}} = \tilde{\lambda}_{d,t}^*$):	HP (HE) in operation meeting the remaining demand
If $\frac{\tilde{p}_{el,t}}{C\tilde{O}P_t} > \tilde{\lambda}_{d,t}^*$ (or analogously $\frac{\tilde{p}_{el,t}}{\eta_{he}} > \tilde{\lambda}_{d,t}^*$):	HP (HE) not in operation

In above distinction, equality usually implies part load operation.

2.5. Receding Strategy

Fig. 3 shows the interplay of the forecast algorithms explained in sec. 2.3 and the cost-minimizing operation strategy described in sec. 2.4. The only operation decision that is realized based on the forecasts is the one at information time t . For the following hours (e.g. $t + 1$), the forecasts are updated and the operation decision may be different to the anticipated one at time t (following the same example, for look-ahead hour $i = 1$). More details are provided in sec. 4.1.

Figure 3: Sequence of forecasts and operation decision.

3. Test Case

As test case, a building with characteristics according to the sample building of DIN 12831: 2003 is chosen. It is a detached house with a habitable area of 110.5 m². The main building characteristics are summarized in table 1.

Table 1: Summary of building characteristics.

habitable area	design transmission losses	design ventilation losses	window areas
110.5 m ²	6.650 kW	1.817 kW	14.2 m ²

Ventilation losses are modeled in the same way as transmission losses (i.e. no detailed ventilation behavior is modeled¹¹). Window orientations are also chosen according to DIN 12831: 2003. These are used as input parameters for the solar heat gain simulation according to Quaschnig 1996. Dimensions of the floor heating, physical properties of building materials and further building characteristics are taken from Schild and Willems 2011, DIN 13790: 2008, DIN 1264-2: 2013 and Schmidt et al. 2010.

An appropriate HP is identified by using technical data and layout criteria from Panasonic 2014a and Novelan 2013. The chosen HP is of type Panasonic WH-SDC09F3E8. Its basic characteristics are provided in table 2.

Table 2: Summary of HP characteristics. Temperatures in parentheses indicate the ambient and supply temperature at which these nominal values are achieved: $T_{amb} = 7^{\circ}C$ and $T_s = 35^{\circ}C$.

name	nominal output (7°C/35°C)	nominal input (7°C/35°C)	nominal COP (7°C/35°C)
WH-SDC09F3E8	9.00 kW _{th}	1.86 kW _{el}	4.84

A linear regression from a set of manufacturer operational data (combinations of five ambient temperatures and six supply temperatures according to Panasonic 2014c) yields the coefficients a_j and b_j (cf. equ. 11 and 12). We implement a relatively large storage with a volume of 4.4 m³. This is modeled as a multiple of the storage of type Panasonic PAW-TE50E3STD whose technical data are obtained from Panasonic 2014b. The space heating is controlled by a circulation pump with a maximum flow rate of 1,200 kg/h. The heating period spans from September 1st to May 31st. Outside the heating period, the floor heating and the HP do not operate. During this time, the temperature constraints for the building and the TES are not considered. However due to relatively high ambient temperatures, the zone temperature T_z is virtually always above the allowable minimum.

For the test case, we use 2015 price data. More precisely, we suppose a varying electricity price component based on 2015 wholesale market prices plus VAT (19 % in Germany). I.e. we use 2015 day-ahead prices for the German-Austrian market zone (cf. EPEX 2016). As constant (energy-based) price component, average 2015 household end consumer price (cf. BDEW 2017) reduced by the annual mean of the varying electricity price component are applied. Hereinafter we refer to this constant price component as end consumer charge (ECC). Fig. 4 shows the supposed electricity prices and their ECC part¹².

Temperature and irradiance data are sourced from the COSMO-EU database (cf. DWD 2017) for the location of Hannover, Germany.

The simulation program is implemented in the MATLAB programming environment (cf. Mathworks 2016).

¹¹Coefficients for transmission and ventilation are added and multiplied with the corresponding temperature difference throughout all hours of the year.

¹²Notably wholesale market prices were negative during several hours of January 2015. Therefore the varying electricity price falls below the ECC several times.

Figure 4: Illustration of varying electricity prices and the ECC for the period of January 2015.

4. Results

4.1. Annual Figures

The annual results of the simulation of the base case are given in tables 3 to 5. The specific annual heat demand of the system¹³ is 186.1 kWh/(m²yr). This corresponds to an insulation level between the characteristics of the average German building stock and non-renovated detached houses (cf. EnEV 2013).

Table 3: Summary of technical results of the simulation of the base case.

heat supplied by the HP	el. consumption of the HP	yearly performance factor	reduction of el. consumption
20,569 kWh	4,982 kWh	4.13	439.4 kWh (8.1 %)

The yearly performance factor is relatively high for an air-water HP. By using the TES, the HP can operate during hours of relatively high ambient temperatures and thus at relatively high COPs. For comparison, the model yields a yearly performance factor of 3.81 being operated without any type of optimization (see also sec. 4). In this way, the smart operation also contributes to resource efficiency. Consequently the cost savings do not only result from exploiting the lowest electricity prices. The rationale described in sec. 2.4 considers the COP. By dividing the electricity price by the COP, the level of the ECC influences the weight of the COP in the optimization. Thus the optimal operating hours may change with different ECCs. The relevance of the ECCs becomes apparent by looking at the average prices realized by the HP (i.e. the consumption-weighted prices / average procurement price) in table 4. These prices are only 0.12 ct/kWh below the yearly base prices (time-averaged prices)¹⁴. That is a price decrease of less than 0.5 %. Using another metric and relating the procurement price decrease to the average variable part of the electricity price (3.76 ct/kWh), the average price decrease corresponds to 3.2 %.

¹³The term 'system' refers to building plus TES. But as the heat losses of the TES are relatively low, we use the total supplied heat for comparison.

¹⁴If operating the HP naively, the average realized prices are slightly lower than the yearly base prices (28.67 ct/kWh).

Table 4: Summary of economic results of the simulation of the base case.

operational cost	yearly base price	average procurement price	cost savings (compared to naive operation)
1,424 €	28.70 ct/kWh	28.58 ct/kWh	131 € (8.4 %)

In terms of operational statistics, during the 6,552 hours of the heating period, the HP can be operated without temperature constraints of the TES being active during 87 % of the hours. That is the operational decision can be made freely according to the rationale described in sec. 2.4. During the remaining hours, such a decision is prevented by the control program and the HP is set into must-run / must-not-run mode. The share of part load operation hours (relative to the number of operation hours) is 11.3 %. The HE is never used as the demand can always be satisfied without using the expensive HE.

Table 5: Operation statistics of the simulation of the base case.

hours at full load	hours at part load	hours with T constraint	hours of HE operation
1739 h	222 h	826 h	0 h

4.2. Thermal Behavior of the System

Fig. 5 illustrates the thermal behavior of the different building subsystems.

Figure 5: Thermal behavior of subsystems (exemplary period from January 6th to January 25th).

One can observe that the zone temperature T_z is maintained within the allowed range around the comfort temperature T_{comf} ($= 20^\circ\text{C}$). The supply temperature T_s changes according to the ambient temperature T_{amb} . This is a consequence of the compensation curve shown in fig. 2. On relatively warm days, it is possible to keep T_s at 35°C which leads to a more efficient operation of the HP. When the average TES temperature T_{tes} comes close to T_s , this implies that T_{tes} reaches its upper limit. In some of these cases, the HP is forced to switch off. Another effect that can be seen in fig. 5 on January 10th is that the floor

heating turns off as the day is quite warm and no heating is required temporarily. The HP operation is not affected directly thereby. However as the TES can only be charged, the upper temperature limit is more likely to be reached. This even becomes more likely as these high ambient temperatures coincide with a trend towards very low electricity prices. During cold days (past January 17th), the supply temperature T_s and consequently temperatures of the floor heating system (T_f , $\bar{T}_w$ and $T_{w,out}$) rise in order to provide sufficient heat to the building. Most of the times, the HP keeps its operational state for several subsequent hours as ambient conditions and prices do not change so rapidly. When it does not, this is sometimes due to the TES reaching its upper or lower temperature bound. Then must-run / must-not-run periods are kept to a minimum necessary. In fig. 5, one can also observe that the HP operation has direct impact on T_{tes} . Even though not completely comparable¹⁵, the profiles in fig. 5 are similar to the ones shown by Cho and Zaheer-Uddin 2003 and Arteconi et al. 2013 who have used a much more specialized modeling environment for thermodynamic applications (TRNSYS, cf. UoWM 2017). This supports the validity of our more straightforward model.

4.3. Forecasts and Operation Decisions

Further insight can be gained by analyzing operation decisions / forecasts at specific information times t . Fig. 6 to 9 show forecast and actual data for February 16th and February 18th. The first two figures illustrate the electricity price and temperature forecasts. In both cases, the information time is the first hour of the day (i.e. taking the decision just at the beginning of this hour at 0.00 am). We observe that the forecasts are not perfect. But it can be concluded that general patterns are reproduced. On February 18th, the temperature peak during the early afternoon was much more pronounced than the one on February 17th. Applying a forecast algorithm which supposes similar patterns, this leads to forecast errors - especially for later look-ahead hours. In contrast, prospected temperatures fit quite well to the actuals for the first hours of the look-ahead horizon of the February 18th forecast and for large part of the February 16th forecast. The decent fit during the earlier look-ahead hours can be explained by the moving average part of the forecast algorithm. The same statement can be made for the electricity price forecast. On February 18th, the price forecast is quite close to the actual prices throughout the whole day. On February 16th, the price forecast is less adequate. This can be explained by the fact that February 16th was a Monday. Using the known prices of the foregoing Sunday (under similar periodicity assumptions as for the temperature forecast) apparently leads to inadequacies. Fig. 8 and 9 show the resulting heat prices for the HP and HE. In both cases, the heat price of the HE is much higher which leads to the fact that the HE is never forecast to operate. The heat prices of the HP vary around the shadow price $\tilde{\lambda}_{d,t}^{i,*}$ illustrated by the red line. Fig. 10 and 11 illustrate the actual operation and system states (i.e. realized operation results).

¹⁵Some simulations of the cited studies do not consider a TES and thus heating system temperatures are pulsing.

Figure 6: (Forecast) temperatures and prices on February 16th at 0.00 am.

Figure 7: (Forecast) temperatures and prices on February 18th at 0.00 am.

Figure 8: Heat price and operation forecasts on February 16th at 0.00 am.

Figure 9: Heat price and operation forecasts on February 18th at 0.00 am.

Figure 10: Actual operation on February 16th.

Figure 11: Actual operation on February 18th.

At first sight, it looks like the operation forecast is fulfilled in both cases. But it can be seen that there are smaller deviations (e.g. hour $t = 1115$ which corresponds to look-ahead hour $i = 10$ at the beginning of February 16th). This can be a result of the receding horizon approach (updated information / changed forecasts) or because the TES temperature constraint affects the HP operation.

4.4. Operational Cost Savings and Investments

Fig. 12 presents the relative changes in electricity consumption using different operation strategies and forecast qualities. We distinguish between three operation strategies: (i) minimization of energy use, (ii) minimization of procurement prices and (iii) minimization of electricity costs. Strategy (iii) is the one described in sec. 2.4. The other two strategies are assessed by setting either one forecast value (forecast of T_{amb} / COP or p_{el}) to a constant value. With regard to forecast qualities, the imperfect forecast corresponds to the algorithm described in sec. 2.3. As upper limit for forecast adequacy, we consider perfect foresight. I.e. the forecasts used for MPC algorithm are identical to the exact ambient temperatures and electricity prices throughout the complete look-ahead horizon.

We always compare the values to those of a naive operation strategy, which is defined as charging and discharging cycles until the maximum / minimum temperature of the TES is reached.

Figure 12: Illustration of changes in electricity consumption using different strategies and different forecast qualities in comparison to the naive operation.

In fig. 12, it can be seen that the minimization of energy use yields the best results in terms of electricity consumption (as is to be expected). With imperfect as well as perfect foresight, the reduction is more important than for the cost minimization strategy (8.63 % and 9.46 % compared to 8.11 % and 9.07 % respectively). However the cost minimization strategy also achieves relatively good results in terms of electricity consumption. This can be attributed to the fact that the ECC makes up for a major part of the electricity bill and thus the COP has a high weight in the cost optimization. The results of the procurement price minimization (regarding electricity consumption) point at a negative correlation between p_{el} and T_{amb} , since T_{amb} is not considered for this operation strategy (and p_{el} by itself does not impact the resource efficiency). I.e. at lower electricity prices temperatures tend to be high and so do COPs. This is confirmed by a calculation of the correlation coefficient which is found to be -0.013. It is furthermore noticeable that the results of the procurement price minimization for perfect foresight are not as good as the corresponding results using imperfect foresight. However as the reduction in electricity consumption is merely triggered by correlation effects, the forecast quality impact on the results of this strategy is somewhat arbitrary. In

absolute terms, the electricity savings are 512.6 kWh (electricity consumption of 4,908.6 kWh compared to 5,421.2 kWh for the naive strategy) for the minimization of energy use under perfect foresight.

Fig. 13 provides a similar depiction of the relative changes in operation costs. In terms of cost savings, the cost minimization is obviously the best operation strategy. Under perfect foresight, it achieves cost savings of 151.4 € (9.74 %). If only minimizing the procurement prices, cost savings are significantly lower (103.5 € / 6.66 %). This underlines that sole price forecasting leads to inferior results and should be combined with temperature forecasts and the corresponding optimization strategy.

Figure 13: Illustration of changes in electricity costs using different strategies and different forecast qualities in comparison to the naive operation.

In order to place these cost savings into context, we contrast them with the annualized costs of capital which the HP owner incurs due to investments into flexibility measures. These flexibility measures are the installation of additional storage capacity (including installation and commissioning) and information, communication and control equipment for the HP. With regard to additional storage capacity, we only take into account the difference to a buffer storage capacity of 880 l. The costs for the corresponding equipment are sourced from Panasonic 2014b and Panasonic 2014c. We assume quite favorable conditions in terms of interest rates and utilization times. We use 2014 average interest rates for zero-yield AAA-rated Euro area central government bonds with maturity of 20 years (cf. ECB 2016) for our calculation (i.e. 2.23 %/yr). The payoff time is set equal to the expected utilization time of 20 years. Compared to historic averages, recent interest rates are very low. Furthermore, private investors normally would have to pay more than this risk-free central bank rate.

These inputs yield annualized capital costs of 1,307.03 €. Most of these costs correspond to the procurement and installation of additional storage capacities (1,247.32 €). A smaller part (59.71 €) is due to information, communication and control equipment for the HP. As can be seen, the cost savings (151.4 €) are - by far - not sufficient to cover additional costs.

5. Conclusions

In the present paper, we have derived a model for a floor-heated building supplied by an HP combined with a TES. The model is based on first principles. Furthermore we have designed a straightforward MPC algorithm which allows to assess different control strategies under different forecasting qualities. At the same time, the derivation of shadow prices of the optimization has illustrated the relevant trade-offs in the operation strategy.

This algorithm has been validated by a test case given in sec. 3. The validation of the thermodynamic behavior of the model has shown plausible results. The forecasting quality is moderate, but we have also assessed results under perfect foresight.

In the assessment of the results of the price-responsive HP operation for the test case, we have focused on the potential reduction of electricity consumption, the possible electricity cost savings and the economic viability. Results show that, for the implementation of an optimized HP operation, it is not only important to minimize procurement prices, but also the COP values need to be taken into account. As a matter of fact, the maximization of the COP (minimization of electricity use) has more leverage on operational cost savings than the minimization of procurement prices. Yet a combined optimization of the electricity price and the COP is to be preferred. Last but not least, for the test case (i.e. under the given technical configuration and economic conditions), the operational cost savings for the HP owner are - by far - too little for covering the annualized capital costs. With regard to this result, it shall be mentioned that there may be different technical set-ups under which an HP may be operated. This includes buildings with a higher thermal inertia in combination with control algorithms that increase / decrease temperature set points according to price signals in order to exploit the inherent thermal storage capacity of the building. This would reduce required TES capacities and lower investment costs (as especially capital cost for additional storage capacities are high). Thus the economic feasibility of further sensitivities should be investigated. Other upsides such as technical improvements or discounted grid charges are also thinkable. However the analyses of these upsides are left to future work. Nevertheless the presented results already show that long-term expectations on the penetration of distributed DSM devices such as HPs shall be regarded in detail and with some skepticism.

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A. Derivation of Stationary Temperature Profile

We consider an infinitesimally small increment of the piping of length dx at position x . The infinitesimally small increment of heat transfer from the water to the floor heating along the surface of this pipe increment is denoted $d\dot{Q}_{w-f,stat}$. The heat balance equation is provided with equ. 29.

$$0 = \dot{m}_{w,stat}c_w(T_{w,stat}(x) - T_{w,stat}(x + dx)) - d\dot{Q}_{w-f,stat} \\ = \dot{m}_{w,stat}c_w dT_{w,stat}(x) - D_p\pi dx U_{w-f}(T_{w,stat}(x) - T_{f,stat}) \quad (29)$$

$$\Leftrightarrow \frac{D_p\pi U_{w-f}}{\dot{m}_{w,stat}c_w} dx = \frac{dT_{w,stat}}{T_{w,stat}(x) - T_{f,stat}} \quad (30)$$

Equ. 29 can be solved by using the boundary conditions $T(0) = T_{w,in,stat}$. Thus we can calculate the stationary temperature profile $T_{w,stat}(x)$, the stationary average temperature $\bar{T}_{w,stat}$ and the stationary outlet temperature $T_{w,out,stat}$.

$$T_{w,stat}(x) = T_{f,stat} + (T_{w,in,stat} - T_{f,stat})e^{-\frac{U_{w-f}D_p\pi}{\dot{m}_{w,stat}c_w}x} \quad (31)$$

$$\bar{T}_{w,stat} = \frac{1}{l_p} \int_0^{l_p} T_{w,stat}(x)dx = T_{f,stat} - (T_{w,in,stat} - T_{f,stat}) \underbrace{\frac{\dot{m}_{w,stat}c_w}{U_{w-f}D_p\pi l_p}}_{r(\dot{m}_{w,stat})} \left(e^{-\frac{U_{w-f}D_p\pi l_p}{\dot{m}_{w,stat}c_w}} - 1 \right) \quad (32)$$

$$T_{w,out,stat} = T_{f,stat} + (T_{w,in,stat} - T_{f,stat})e^{-\frac{1}{r(\dot{m}_{w,stat})}} \quad (33)$$

The temperature differences in stationary equilibrium can be expressed by using the logarithmic temperature difference for reverse-flow heat exchangers.

$$\bar{T}_{w,stat} - T_{f,stat} = \frac{T_{w,in,stat} - T_{w,out,stat}}{\ln\left(\frac{T_{f,stat} - T_{w,out,stat}}{T_{f,stat} - T_{w,in,stat}}\right)} = \frac{T_{w,in,stat} - T_{w,out,stat}}{\frac{1}{r(\dot{m}_{w,stat})}} \quad (34)$$

Thus the outlet temperature $T_{w,out}$ converges towards this stationary equilibrium as long as the $\dot{m}_w$, T_f and $T_{w,in}$ are held constant.

$$T_{w,out} \rightarrow T_{w,out,stat} = -\frac{1}{r(\dot{m}_{w,stat})} (\bar{T}_{w,stat} - T_{f,stat}) + T_{w,in,stat} \quad (35)$$

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